CPMA 521, Fall 2001
Professor Larget
Assignment #1: due September 10, 2001

0. Memorize the definitions of and understand the concepts of the following terms: *outcome*, *experiment*, *sample space*, *event*, *set union*, *set intersection*, *set union*, *set complement*, *mutaully exclusive events*, *probability*, *axioms of probability*, *combined events*, *conditional probability*, *law of total probability*, *weighted average*, *independence of events*, *mutual independence of events*, *random variables*, *discrete random variables*, *probability mass function*, *(cumulative) distribution function*, *expected value*, *moments*, *variance*, *standard deviation*, *Bernoulli trial*, *Bernoulli distributions*, *geometric distribution*, *binomial distribution*, *negative binomial distribution*. You do not need to turn anything for this problem.

1. In the card game Sheepshead, there is a deck of 32 cards. There are fourteen trumps, six hearts, six spades, and six clubs. The 32 cards are distinct — the trumps can be distinguished, etcetera. A hand of six cards is dealt. The order of these five cards does not matter.

   (a) Give an example of a single outcome.
   (b) How many different outcomes are there?
   (c) What is the probability of having exactly three trumps?
   (d) What is the probability of having exactly three clubs?
   (e) What is the probability of having exactly three trumps and exactly three clubs?
   (f) Find the probability mass function for X, the number of trumps in the hand.
   (g) Find the expected value of X
   (h) Find the standard deviation and variance of X.

2. Problem 1.6. *Recall the definition of independence on page 9.*

3. Problem 1.7. *Recall the definition of independence on page 9.*

4. Problem 1.16. *What is $P(Y > n) - P(Y > n + 1)$?*

5. Problem 1.17. *How are the events $\{\min(X, Y) > n\}$, $\{X > n\}$, and $\{Y > n\}$ related?*

6. Problem 1.34. *Let $X_1, X_2, \ldots$ be a sequence of Bernoulli trials with success probability p. Let $X = \{\min i | X_1 = 1, i = 1, 2, 3, \ldots\}$. What is X when $X_1 = 1$? What conditional distribution does $X - 1$ have when $X_1 = 0$?*

7. Problem 1.48. *Notice the following.*

$$\begin{array}{lllllllll} \mathrm{P}(X > 0) & = & \mathrm{P}(X = 1) & + & \mathrm{P}(X = 2) & + & \mathrm{P}(X = 3) & + & \cdots \\ \mathrm{P}(X > 1) & = & & & \mathrm{P}(X = 2) & + & \mathrm{P}(X = 3) & + & \cdots \\ \mathrm{P}(X > 2) & = & & & & & \mathrm{P}(X = 3) & + & \cdots \end{array}$$

   What does each column sum to?

8. Below is code of an R function that graphs the binomial distribution. The Web page includes a link to the same code with many annotations. Modify it to write a function `gnbinom` that graphs the negative binomial distribution. *Warning: S-PLUS defines the geometric and negative distributions with possible values beginning at 0. This is the random variable G described on page 14 and the random variable M_n described on page 16 of Minh.)* Your default value for high should be something other than infinity....

```
gbinom <- function(n, p, low=0, high=n,scale = F)
{
  sd <- sqrt(n * p * (1 - p))
  if(scale && (n > 10)) {
    low <- max(0, round(n * p - 4 * sd))
    high <- min(n, round(n * p + 4 * sd))
  }
  values <- low:high
  probs <- dbinom(values, n, p)
  plot(c(low,high), c(0,max(probs)), type = "n", xlab = "Possible Values",
    ylab = "Probability", main = paste("Binomial Distribution \n", "n =",
    n, ", p =", p))
  lines(values, probs, type = "h", col = 2)
  abline(h=0,col=3)
  return(invisible())
}
```