Prof. Bret Larget
Math 125
Practice Final Exam

Name: _____

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Circle **True** or **False** for Problems 1 through 10.

**Problem 1: True** or **False**:

If every individual has the same chance of being selected, a sample must be a simple random sample.

SOLUTION: False. There are random samples that are not simple random samples but everyone has the same chance of being selected.

**Problem 2: True** or **False**:

A residual plot from a regression of $y$ on $x$ shows a pattern where the residuals are positive for small $x$, negative for middle $x$ and positive again for large $x$. This indicates that $x$ and $y$ have a nonlinear relationship.

SOLUTION: True.

**Problem 3: True** or **False**:

If a correlation coefficient is exactly 1, this means the data lie exactly on a line with slope one.

SOLUTION: False. The data lie on a line with positive slope, not necessarily one.

**Problem 4: True** or **False**:

If two variables have a correlation coefficient of $-2.12$, then they are negatively associated.

SOLUTION: False. Correlation coefficients below $-1$ are impossible. This indicates a calculation error.

**Problem 5: True** or **False**:

Even if a population is strongly skewed, the sampling distribution of the sample mean of random samples drawn from the population will be fairly symmetric and approximately normal when the sample size is sufficiently large.

SOLUTION: True (central limit theorem).

**Problem 6: True** or **False**:

For any population, the sampling distribution of the sample mean will be approximately normal if the sample size is at least 30.

SOLUTION: False. For highly skewed populations, sample sizes may need to be substantially larger than 30 before the distribution is approximately normal.

**Problem 7: True** or **False**:

Among Duquesne University freshmen, 50% of males prefer pizza for dinner while 15% of females prefer pizza for dinner. In a stratified random sample of five male and five female Duquesne freshmen, the number who prefer pizza is a binomial random variable.

SOLUTION: False. The probability of selecting a person who prefers pizza for dinner is not the same for each sampled person.

**Problem 8: True** or **False**:

A 95% confidence interval for the mean weight of artificial saphires made by a new manufacturing process is from 6.54 to 6.96 carats, based on a random sample of 12 gems. This means that we can expect that 95% of all artificial saphires manufactured by the new process will weigh between 6.54 and 6.96 carats.

SOLUTION: False. A confidence interval is a probability statement about the location of the mean, not the location of the middle 95% of the population.

**Problem 9: True** or **False**:

If a variable is strongly skewed to the right, the mean will be larger than the median.

SOLUTION: True.

**Problem 10: True** or **False**:

If a p-value is 0.01, there is only a 1% chance that the null hypothesis is true.

SOLUTION: False. A p-value is not the probability that the null hypothesis is true.

**Problem 11:**

The prices of a sample of 25 brands of walking shoes in dollars are tabulated below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 59 | 109 | 70 | 76 | 55 | 50 | 55 | 69 | 58 | 59 | 40 | 46 | 62 |
| 52 | 55 | 65 | 70 | 60 | 110 | 78 | 60 | 65 | 69 | 58 | 60 | |

(a) Display the data in a stem-and-leaf display.

(b) Find the median, lower quartile, and upper quartile.

(c) Display the data in a boxplot.

(d) Is the data strongly skewed or fairly symmetric?

(e) Are there any outliers?

SOLUTION:

```
 4|06
 5|025558899
 6|00025599
 7|0068
 8|
 9|
10|9
11|0
```

(b) Median = 60, first quartile = 55, third quartile = 69.5; (d) Skewed to the right because of outliers; (e) Two outliers, 109 and 110.

**Problem 12:**

Weight (in grams) and heart rate (in beats per minute) for six species of tarantula spiders are tabulated to the right. The correlation coefficient is $r = -0.872$.

| Weight (g) | Heart Rate (beats/min) |
|---|---|
| 10.75 | 11 |
| 11.10 | 13 |
| 8.01 | 14 |
| 13.80 | 10 |
| 12.60 | 11 |
| 11.40 | 12 |
| $\bar{x} = 11.277$ | $\bar{y} = 11.83$ |
| $s_x = 1.955$ | $s_y = 1.47$ |

(a) Find the regression equation to predict heart rate from weight.

(b) Fill in the blank and include units in your answer. For every gram increase in weight for a species of tarantula, the heart rate changes by _____.

(c) Predict the heart rate of a 20 gram tarantula using the regression equation.

(d) Comment on the validity of the prediction.

SOLUTION: (a) (heart rate) = 19.2 - 0.657(weight); (b) decreases by about 0.66 beats per minute; (c) 6.1 beats per minute; (d) 20 grams is well outside the range of the data. Inference based on extrapolation is unreliable.

**Problem 13:**

A newly developed test has a 95% chance of correctly predicting Alzheimer's disease in people between the ages of 65 to 85 years of age who will eventually develop the disease but whom do not yet show symptoms.

(a) If forty such people are given the test, what is the probability that exactly 39 are predicted to eventually develop the disease?

(b) If forty such people are given the test, about how many would you expect would be predicited to eventually get the disease?

SOLUTION: (a) 0.2706; (b) 38

**Problem 14:**

Glucose levels in blood and diabetes are related. For a forty year old adult, the glucose level after a 12 hour fast is approximately normally distributed with a mean on 85 and a standard deviation of 19 milligrams of glucose per deciliter. Test results less than 40 indicate a severe excess of insulin.

(a) What is the probability that a single measurement is less than 40?

(b) What is the 5th percentile of the distribution?

SOLUTION: (a) 0.0089; (b) 53.7

**Problem 15:**

Heights of 18 year old men are normally distributed with a mean of 68 inches and a standard dviation of 3 inches. What is the probability that the average height of random sample of five men exceeds 72 inches (6 feet)?

SOLUTION: 0.0014.

**Problem 16:**

The caloric content of french fries depends on how they are prepared. A random sample of eight different 3 ounce servings of french fries from different fast-food restaurants had the calories shown below.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 222 | 255 | 254 | 230 | 249 | 222 | 237 | 287 |

(a) Find the mean and standard deviation of the sample data.

(b) Find a 99% confidence interval for the mean number of calories in a 3 ounce serving of french fries at a fast-food restaurant.

SOLUTION: (a) mean = 244.5, sd = 21.7; (b) 99% confident that the mean number of calories is in the interval $244.5 \pm 26.8$.

**Problem 17:**

A machine to make twist-off caps for bottles produces caps with a mean diameter of 1.85 cm when it is working properly. If the mean diameter changes, the machine needs to be adjusted. A sample of 20 bottle caps has a mean of 1.87 cm and a standard deviation of 0.05 cm.

(a) State null and alternative hypotheses.

(b) Find the test statistic.

(c) Draw a picture of the sampling distribution of the test statistic and shade in the area that represents the p-value.

(d) Use a table to find a numerical value or a range of numerical values for the p-value.

(e) Based on your analysis, is there strong evidence that the machine needs to be adjusted?

SOLUTION: (a) $H_0 : \mu = 1.85$, $H_a : \mu \neq 1.85$. (b) $t = 1.79$. (d) 0.05 ¡ p-value ¡ 0.10. (e) There is at most mild evidence that the machine requires adjustment.