Prof. Kathy Taylor and Prof. Bret Larget Name: ______________________________
Math 315

Take-home Final Examination

The final examination is worth 200 points with 100 points for a take-home part and 100 points for an in-class part. The take-home part of the final examination is due **Monday, May 1 at 8:45 A.M.** when the in-class part begins. You are permitted to consult any textbooks, your own course notes, and any course handouts. You may use MATLAB as part of a solution to any take-home problem. The only people with whom you may discuss the examination are Professors Taylor and Larget.

Problem 1: (20 points)
Show that the set of solutions to the system of linear equations below is a vector subspace of $\mathbf{R}^4$. Find a basis for the solution space.

$$\begin{aligned} x_1 - 2x_3 - 3x_4 &= 0 \\ x_2 - 3x_3 + x_4 &= 0 \\ 2x_1 - x_2 - x_3 - 7x_4 &= 0 \end{aligned}$$

Problem 2: (20 points)
A linear mapping $L : \mathbf{R}^3 \to \mathbf{R}^3$ satisfies $L((1,2,3)^t) = (1,1,2)^t$, $L((2,3,1)^t) = (0,1,2)^t$, and $L((3,1,2)^t) = (2,1,-1)^t$.

(a) Is L uniquely determined by the information above? Justify your response.

(b) Find a 3×3 matrix A so that $L(\boldsymbol{x}) = A\boldsymbol{x}$ for all $\boldsymbol{x} \in \mathbf{R}^3$.

(c) Is L an invertible mapping? Explain fully.

(d) Is A a change of basis matrix? Explain fully.

Problem 3: (20 points)
Solve the initial value problem $\frac{d^4x}{dt^4} - 3\frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} - 12\frac{dx}{dt} = 4t + 3e^t$ where $x(0) = 1$, $x'(0) = 0$, $x''(0) = -1$, and $x'''(0) = 0$.

Problem 4: (20 points)
Find the general solution to the system of linear equations

$$\begin{aligned} \frac{dx_1}{dt} &= -4x_1 - x_2 \\ \frac{dx_2}{dt} &= 3x_2 + x_3 \\ \frac{dx_3}{dt} &= 18x_1 + 6x_3 \end{aligned}$$

Problem 5: (20 points)
Consider the matrix $A = \begin{pmatrix} -12 & -3 & -14 \\ -5 & 2 & -8 \\ 12 & 3 & 14 \end{pmatrix}$.

(a) Find matrices B and S so that $B = S^{-1}AS$ is in real Jordan normal form.

(b) Find matrices D and T so that $D = T^{-1}AT$ is in Jordan normal form.