

A sample of 100 people were surveyed to determine they were heavy smokers and if they frequently coughed in the morning. The results are tabulated here.

	Smoking	
	Heavy	Nonsmoker
Cough frequently	63	12
Do not cough frequently	7	18

- (a) What proportion of heavy smokers in the sample cough frequently in the morning? What proportion of nonsmokers in the sample cough frequently in the morning?

Solution: The proportion of heavy smokers that cough frequently in the morning is $63/70 = 0.9$.
The proportion of nonsmokers that cough frequently in the morning is $12/30 = 0.4$.

- (b) Consider a directional test of association between smoking and morning coughing with the alternative hypothesis that smokers are more prone to coughing in the morning. State hypotheses, compute a χ^2 test statistic, find a range for the p -value from a χ^2 table, and interpret the result in the context of the problem. (Recall that the p -value from the χ^2 table is nondirectional, as it accounts for deviation from the null hypothesis in either direction.)

Solution:

Let p_1 be the population proportion of heavy smokers that cough and p_2 be the population proportion of nonsmokers that cough.

$$H_0: p_1 = p_2$$

$$H_A: p_1 > p_2$$

Expected counts are:

	[,1]	[,2]
[1,]	52.5	22.5
[2,]	17.5	7.5

The test statistic is 28.

The p -value is less than 0.00005 (1.2e-07).

There is very strong evidence that the proportion of frequent morning coughers among heavy smokers is higher than among nonsmokers ($p < 0.00005$, directional χ^2 test of independence). However, in this small observational study we would not be justified in concluding that heavy smoking is the primary causal factor.

- (c) Fisher's exact test for this hypothesis test gives a p -value of 5×10^{-7} . Fill in the blanks.

Solution:

In a random sample of 70 balls from a bucket of 100 total balls of which 25 are black and 75 are white, the probability that 7 or fewer are black is 5×10^{-7} .