

1. The chi-square test for the $r \times k$ contingency table

- Null hypothesis:

H_0 : Row variable and column variable are independent

- Calculation of expected frequencies:

$$E = \frac{(\text{Row total}) \times (\text{Column total})}{\text{Grand total}}$$

- Test statistics

$$\chi_s^2 = \sum \frac{(O - E)^2}{E}$$

- Null distribution(approximate): χ^2 distribution with

$$df = (r - 1)(k - 1)$$

where r is the number of rows and k is the number of columns in the contingency table. This approximation is adequate if $E \geq 5$ for every cell. If r and k are large, the condition that $E \geq 5$ is less critical and the χ^2 approximation is adequate if the average expected frequency is at least 5, even if some of the cell counts are smaller.

2. The ANOVA Table

| Source: | df | SS(Sum of Squares) | MS(Mean Square) |
|-----------------|-----------|--|-----------------|
| Between groups: | $k - 1$ | $\sum_{i=1}^k n_i (\bar{y}_i - \bar{\bar{y}})^2$ | SS/df |
| Within groups: | $n^* - k$ | $\sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2$ | SS/df |
| Total: | $n^* - 1$ | $\sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$ | |

where

$$\bar{y}_i = \frac{y_{i1} + y_{i2} + \dots + y_{in_i}}{n_i} = \frac{\sum_{j=1}^{n_i} y_{ij}}{n_i}$$

$$n^* = n_1 + n_2 + \dots + n_k = \sum_{i=1}^k n_i$$

and

$$\bar{\bar{y}} = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} y_{ij}}{n^*}$$