Statistical Analysis of Local Volume Change, with an Application to Brain Growth

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Introduction

The Hotelling's T^2 random field based on the displacement vector field has been used as a measure of structural shape differences between two groups of MR scans [1,4]. By using the displacement velocity field rather than the displacement itself, regions of brain growth over time can be detected. Based on the displacement velocity field, we present and compare three statistics to detect regions of brain growth. Furthermore, it can be shown that these three statistics are sufficient enough to characterize the brain growth process when the displacement is relatively small. Then the methods were applied to the detection of regions of brain growth or atrophy in MRI scans of 29 subjects (age =13.5 ± 4.1) over 4.5 ± 1.0 year time span.

Detecting Local Translation

Let $U(x,t)=(U_1,U_2,U_3)$ be the 3D displacement vector field required to move the structure at position $x=(x_1,x_2,x_3)$ at reference time 0 of a subject's brain to the corresponding position after time t. The displacement field $U(x,t)$ at a fixed time t is usually computed via volume-based non-linear registration [2] on the two images taken at time 0 and at time t. Then we propose the following **Linear Model**:

$$\frac{\partial U}{\partial t}(x,t) = \mu(x) + e(x),$$

where $\mu(x)$ is the mean displacement velocity of n subjects and $e(x)$ is a mean zero Gaussian random vector field.
Null Hypothesis $H_0: \mu(x) = 0$, no displacement.
Test Statistic: $T_T^2(x) = n\, M_T{}^t(x) V_T{}^{-1}(x) M_T(x)$, Hotelling's T^2 random field.
$M_T(x)$: the sample mean of the displacement velocity,
$V_T(x)$: the sample covariance matrix of the displacement velocity.

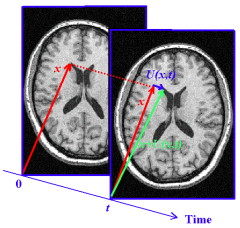

Figure 1. The structure at position x deforms to $x+U(x,t)$ after time t.

Detecting Local Volume Change

The Jacobian of the 2D deformation field has been used to measure gender-specific shape differences of the cross-sectional area of the corpus callosum [3] and the local volume gain or loss of a single subject [5]. We present a statistical analysis based on the rate of the Jacobian over time. The deformation in the Lagrangian coordinate system at time t

$$x \rightarrow x + U(x,t).$$

The local volume change is the Jacobian of the deformation

$$J(x,t) = det(I + \nabla U(x,t)),$$

where $\nabla U(x,t)$ is the displacement gradient matrix of $U(x,t)$ and I is an identity matrix. Then neglecting the higher order terms, the rate of local volume change is

$$\frac{\partial J}{\partial t}(x,t) \approx \frac{\partial^2 U_1}{\partial t \partial x_1} + \frac{\partial^2 U_2}{\partial t \partial x_2} + \frac{\partial^2 U_3}{\partial t \partial x_3} = \frac{\partial K}{\partial t}(x,t)$$

where $K(x,t) = tr(\nabla U(x,t))$ is called the *dilatation* .
Linear Model:

$$\frac{\partial K}{\partial t}(x,t) = \kappa(x) + e_K(x),$$

where $\kappa(x)$ is the mean dilatation rate field and $e_K(x)$ is a mean zero Gaussian random field whose components are independent.
Null Hypothesis $H_0: \kappa(x) = 0$, i.e. no growth or atrophy.
Test Statistic: $T(x) = n^{1/2} M(x)/S(x)$, t random field
$M(x)$: sample mean of the dilatation rate.
$S(x)$: sample standard deviation of the dilatation rate.

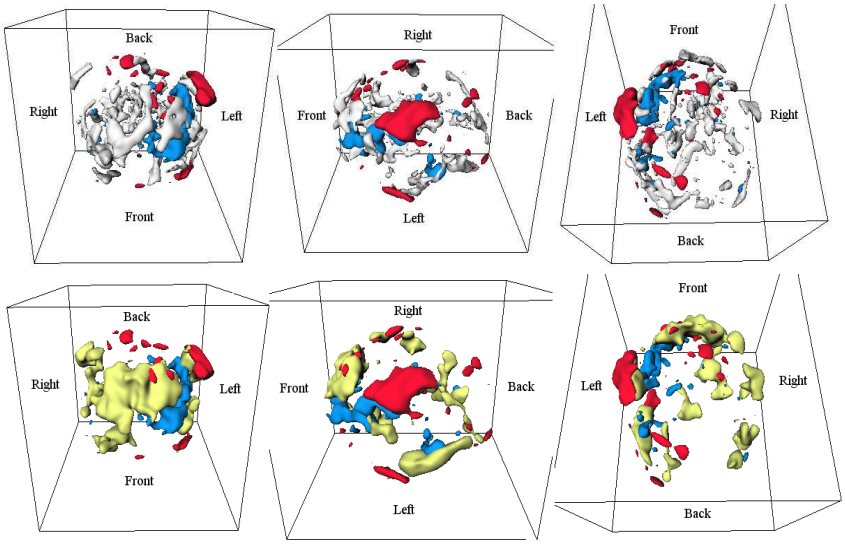

Figure 2. SPMs showing dominant local volume increase in certain regions of the left hemisphere.
Red: Volume increase $T(x) > 6.4$ ($p < 0.025$, corrected [6])
Blue: Volume decrease $T(x) < -6.4$ ($p < 0.025$, corrected)
Grey: Vorticity $T_R^2(x) > 58.9$ ($p < 0.05$, corrected [1])
Yellow: Translation $T_T^2(x) > 58.9$ ($p < 0.05$, corrected)

Detecting Local Rotation Change

The *vorticity tensor field* Ω_{ij} is given by

$$\Omega_{ij}(x,t) = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right).$$

In 3D, the vorticity vector $w = (\Omega_{23}, \Omega_{31}, \Omega_{12})$ is the angular velocity. Assuming that the displacement velocity components are i.i.d. mean zero isotropic Gaussian fields with smooth isotropic covariance function, we have
Linear Model:

$$\frac{\partial w}{\partial t}(x,t) \approx \omega(x) + e_w(x),$$

where $\omega(x)$ is the mean vorticity rate vector and $e_w(x)$ is i.i.d. mean zero isotropic Gaussian field.
Null Hypothesis $H_0: \omega(x) = 0$, i.e. no local rotation.
Test Statistic: $T_R^2(x) = n\, M_R{}^t(x) V_R{}^{-1}(x)\, M_R(x)$, Hotelling's T^2 random field
$M_R(x)$: sample mean of the vorticity rate,
$V_R(x)$: sample covariance matrix of the vorticity rate.

Decomposition of the Displacement Velocity

For relatively small displacement, the displacement velocity at $x+dx$ can be decomposed into three parts: local translation , local rotation and *strain* ε.

$$\frac{\partial U}{\partial t}(x+dx,t) \approx \frac{\partial U}{\partial t}(x,t) - \frac{\partial w}{\partial t}(x,t) \times dx + \frac{\partial \varepsilon}{\partial t}(x,t)dx.$$

The principal strains are the three eigenvalues of the strain matrix ε and nonvanishing principal strain for small displacement can be shown to be the dilatation. Hence, for small displacement, three statistics completely characterize the brain growth or atrophy over time.

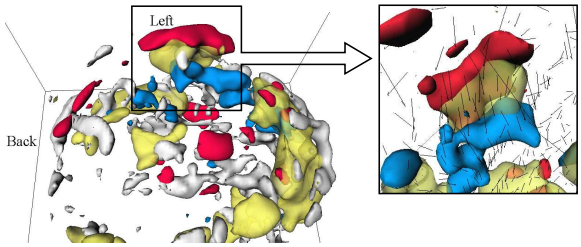

Figure 3. SPM combining all three statistics. The smaller figure is a close up of part of the outer left hemisphere where black arrows represent mean displacement velocity subsampled every 10mm and scaled by 50mm/year showing large displacement (yellow) from a region of growth (red) to a region of atrophy (blue). Note that there are not many overlapping regions indicating that the statistics measure different properties of anatomical variations.

Conclusions

1) By using the displacement velocity field instead of the displacement itself in detecting the anatomical changes, temporal variabilities in MR images for different age groups and different time intervals can be accounted for.

2) The methods presented here can localize the regions where brain volume growth or atrophy occurs over time by measuring the rate of the local translation, the local rotation and local volume changes which are sufficient to capture spatio-temporal variabilities when the deformation is relatively small.

3) The method can be applied to a general linear model, such as testing for structural shape changes between two different groups of subjects.

References

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