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Connection Probability in Diffusion Tensor Imaging via Anisotropic Gaussian Kernel Smoothing

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1. Motivation

We present a novel approach of obtaining white fiber anatomical connection probability in diffusion tensor imaging (DTI) via anisotropic Gaussian kernel smoothing. Our approach is compatible to other probabilistic approach such as solving an anisotropic diffusion equation (Batchelor *et al.,* 2001) or Monte-Carlo random walk simulation (Kosh *et al.,* 2002). Our formulation is simpler than solving the diffusion equation and deterministic in a sense that it avoids using Monte-Carlo random walk simulation in constructing the connection probability so the resulting connectivity maps do not change from one computational run to another. As a further usefulness of this new method, the same computational framework can also be used in smoothing any type of data along the white fiber tracks. This poster is based on a technical report Chung *et al.* (2003).

2. Anisotropic Gaussian kernel smoothing

Anisotropic Gaussian kernel is a multivariate probability density function whose covariance matrix is not an identity. Anisotropic Gaussian kernel can provide a powerful directional smoothing technique if the covariance matrix is spatially adaptive.

$\mathbf{x} = (x_1, \cdots, x_n)' \in \mathbb{R}^n$

n-dimensional isotropic Gaussian kernel

$$K(\mathbf{x}) = \exp(-\mathbf{x}'\mathbf{x}/2)/(2\pi)^{n/2}$$

anisotropic Gaussian kernel

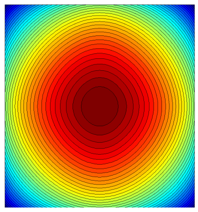
$$K_H(x) = K(H^{-1}\mathbf{x})/\det(H)$$

H bandwidth matrix

HH' covariance matrix

Anisotropic Gaussian kernel smoothing

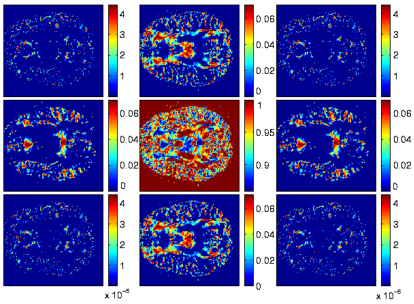
$$F(\mathbf{x}) = K_H * f(\mathbf{x})$$



This is what would happen if isotropic kernel smoothing is applied

3. Riemannian metric tensors

If the tangent vectors of the stream lines are given by the principal eigenvectors of the diffusion coefficients of DTI, the Riemannian metric tensors can be computed in terms of the components of the principal eigenvectors. By matching the covariance matrix to the Riemannian metric tensors proportionally, we have a spatially adaptive kernel in DTI. A more general approach would be to match the covariance matrix to the diffusion coefficient matrix proportionally.



Anisotropic kernel weights: weights are symmetric and the most weights are concentrated in the middle. Due to image noise, kernel weights may not be continuous. In such a case, isotropic smoothing with very small filter size on the Riemannian metric tensor can improve the performance. The above weights are unsmoothed version.

$\psi = \psi(u) = (\psi_1(u), \cdots, \psi_n(u))$

$d\psi/du = \mathbf{V}$ principal eigenvector

first fundamental form

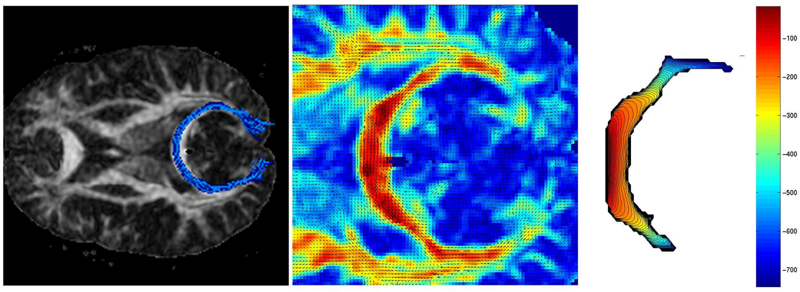
$$d\psi^2 = \sum_{j=1}^n V_j^2 dx_j^2$$

anisotropic kernel

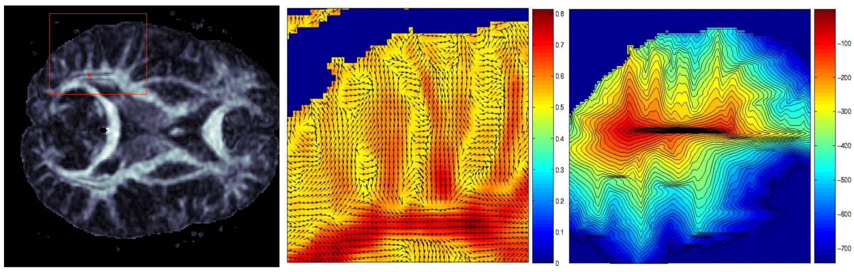
$$K_t(\mathbf{x}) = \prod_{j=1}^{n} \frac{1}{\sqrt{4\pi t}|V_j|} \exp\Big(-\frac{x_j^2}{4tV_j^2}\Big)$$

Generalization

$$K_t(\mathbf{x}) = \frac{\exp(-\mathbf{x}'D^{-1}\mathbf{x}/4t)}{(4\pi t)^{n/2}(\det D)^{1/2}}$$



Left: White fiber tracks based on the tensor deflection algorithm (Lazar *et al*., 2003) **Middle:** Arrows represent the principal eigenvectors. Color represents the corresponding eigenvalues. **Right:** the log-transitional probability of connectivity from the seed point The seed point is taken in the splenium of corpus callosum.



Left: Fractional Anisotropy (FA) map showing the seed point. Red box indicates the region of interest. **Middle:** Arrows represent the principal eigenvectors. Color represents the corresponding eigenvalues. **Right:** log-transitional probability of connectivity from the seed point. After 200 iterations, there is no visible change of the connectivity map.

4. Log-transition Probability

Our white fiber connectivity measure is based on the transition probability, which is the most natural probabilistic measure associated with diffusion process. The transition probability from point **p** to **q** is the conditional probability of going to **q** when a particle is at **p** under the diffusion. It can be shown that the transition probability can be estimated using the repeated applications of anisotropic Gaussian kernel smoothing with the bandwidth matrix determined adaptively (Chung, *et al.,* 2003). If there are one million voxels within the brain, in average, each voxel will have the connection probability of one over a million, which is extremely small. Even though the connectivity measure based on the transition probability is a mathematically sound one, it may not be a good one for visualization so we take the log-scale of the transition probability and use it as a metric for measuring the strength of the anatomical connectivity. We will term this metric as the *log-transition probability*.

$P_t(\mathbf{p},\mathbf{q})$ *transition probability*

Chapman-Kolmogorov equation

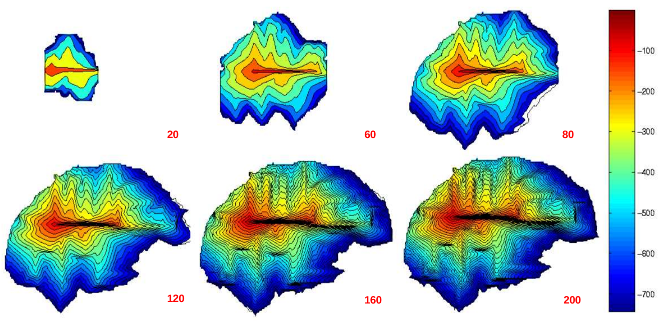
$$P_t(\mathbf{p},\mathbf{q}) = \int_{\mathbb{R}^n} P_s(\mathbf{p},\mathbf{x}) P_{t-s}(\mathbf{x},\mathbf{q})\, d\mathbf{x}$$

$$F_j(\mathbf{q}) = P_{j\Delta t}(\mathbf{p},\mathbf{q})$$

$$F_j(\mathbf{q}) = \tilde{K}_{\Delta t} * F_{j-1}(\mathbf{q}) \quad F_0(\mathbf{q}) = \delta(\mathbf{p}-\mathbf{q})$$

log-transition probility

$$\rho(\mathbf{Q}) = \log \int_{\mathbf{Q}} P_t(\mathbf{0},\mathbf{x})\, d\mathbf{x}$$



Log-transition probability: it is computed by repeatedly applying spatially adaptive anisotropic Gaussian kernel smoothing. Red numbers indicates the number of iterations.

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