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Introducing Heat and Geodesic Kernel Smoothing on Cortical Manifolds

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Introduction

When data lie on the 2D convoluted brain cortex, data smoothing must be weighted according to geodesic distance along the surface. On the curved surface, a straight line between two points is not the shortest distance so one may incorrectly assign less weights to closer observations.
The previously developed *diffusion smoothing* formulates smoothing as the process of heat diffusion by explicitly solving an isotropic diffusion equation with the given data as an initial condition [1]. The drawback of the diffusion smoothing approach is the complexity of setting up a finite element method and making the numerical scheme stable. To address these shortcomings, we propose a simpler and more efficient method called *heat kernel smoothing* and its extension called *geodesic kernel smoothing* [2].

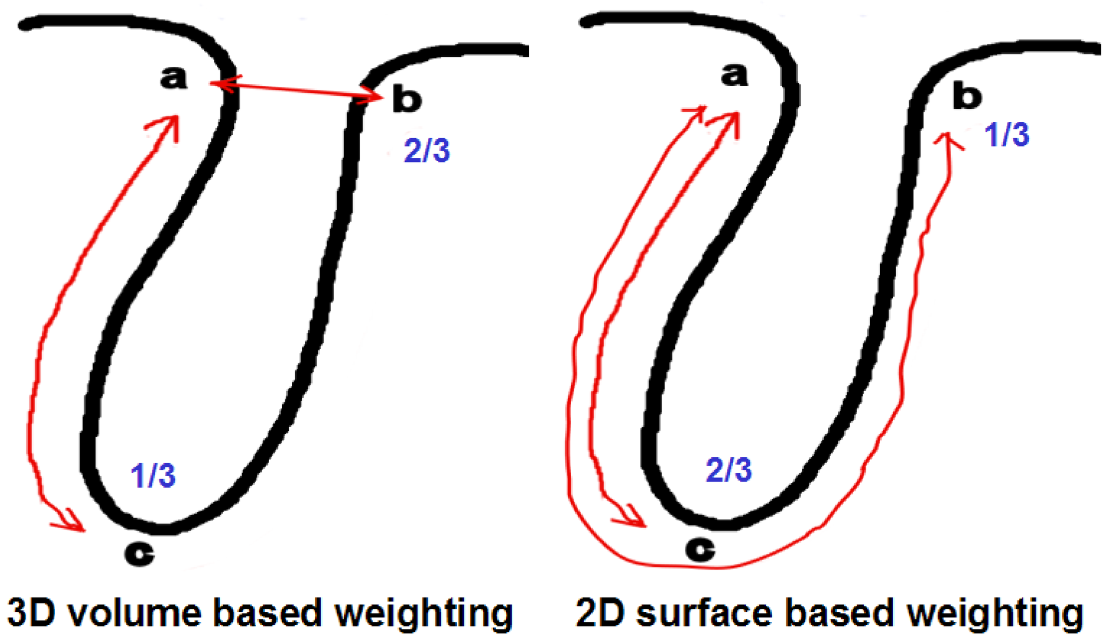

Figure 1: Comparison of different kernel weights between 3D Gaussian kernel smoothing and 2D heat kernel smoothing.

Heat kernel smoothing

Observations Y measured on the cortex $\partial\Omega$ is assumed to follow the additive model of true signal θ plus noise ϵ: $Y(p) = \theta(p) + \epsilon(p), p \in \partial\Omega$. Then we estimate θ using *heat kernel smoothing*:

$$\widehat{\theta}(p) = K_\sigma * Y(p) = \int_{\partial\Omega} K_\sigma(p,q)Y(q)\, d\mu(q),$$

where $\mu(q)$ is the surface measure and K_σ is the heat kernel with bandwidth σ [2]. For small σ, it can be shown that $K_\sigma(p,q) \approx \frac{1}{(2\pi\sigma)^{1/2}}\Big[\exp -\frac{d^2(p,q)}{2\sigma^2}\Big]$, where $d(p,q)$ is the geodesic distance between p and q. For large σ, the smoothing is performed by iteratively: $K_{\sqrt{k}\sigma} * f = \underbrace{K_\sigma * \cdots * K_\sigma}_{k \text{ times}} * f$. The MATLAB code can be downloaded freely from `http://www.stat.wisc.edu/~mchung/hk/hk.html`.

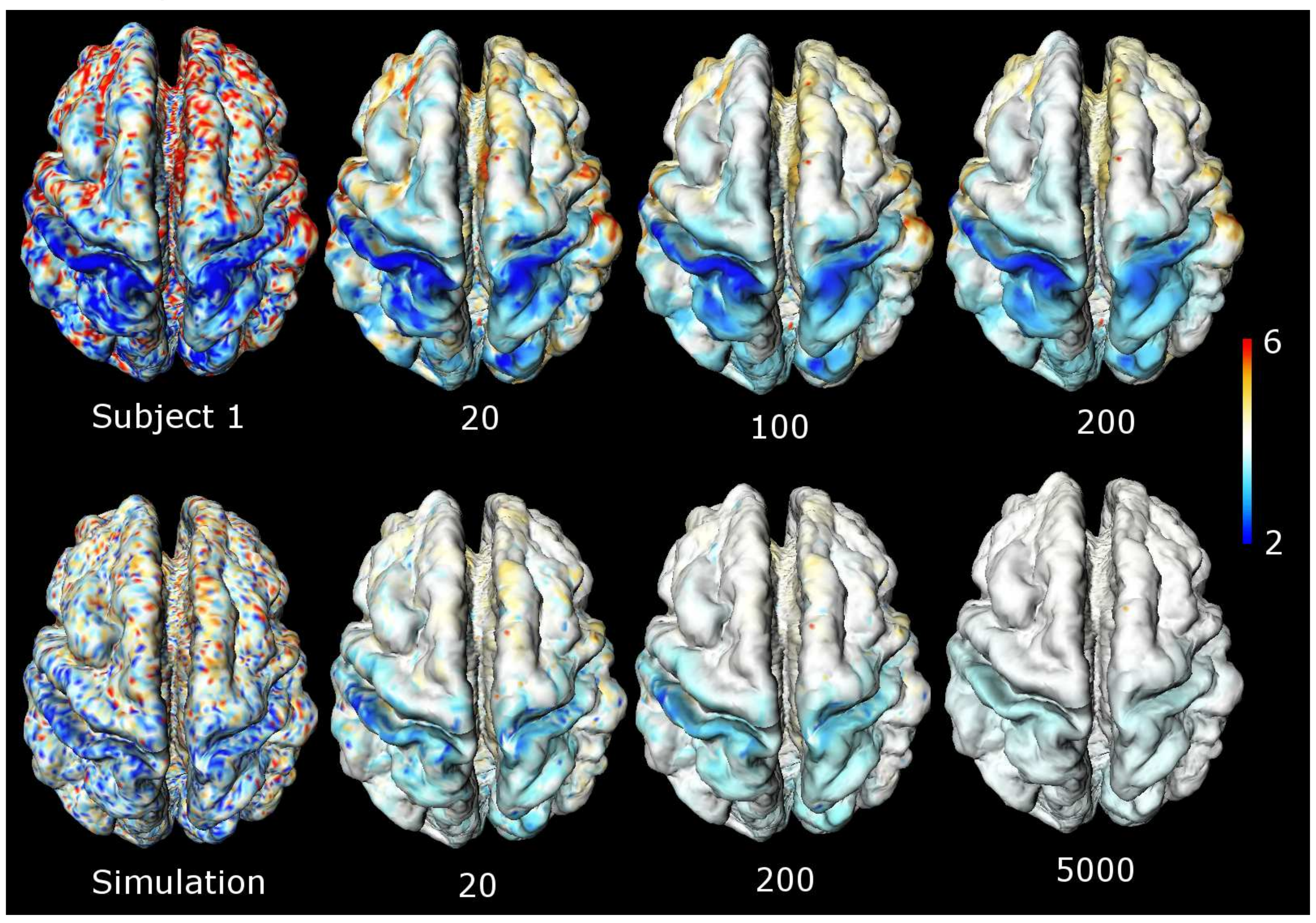

Figure 2: Heat kernel smoothing on real and simulated data with $\sigma = 1$. The numbers under the images are the numbers of iterations.

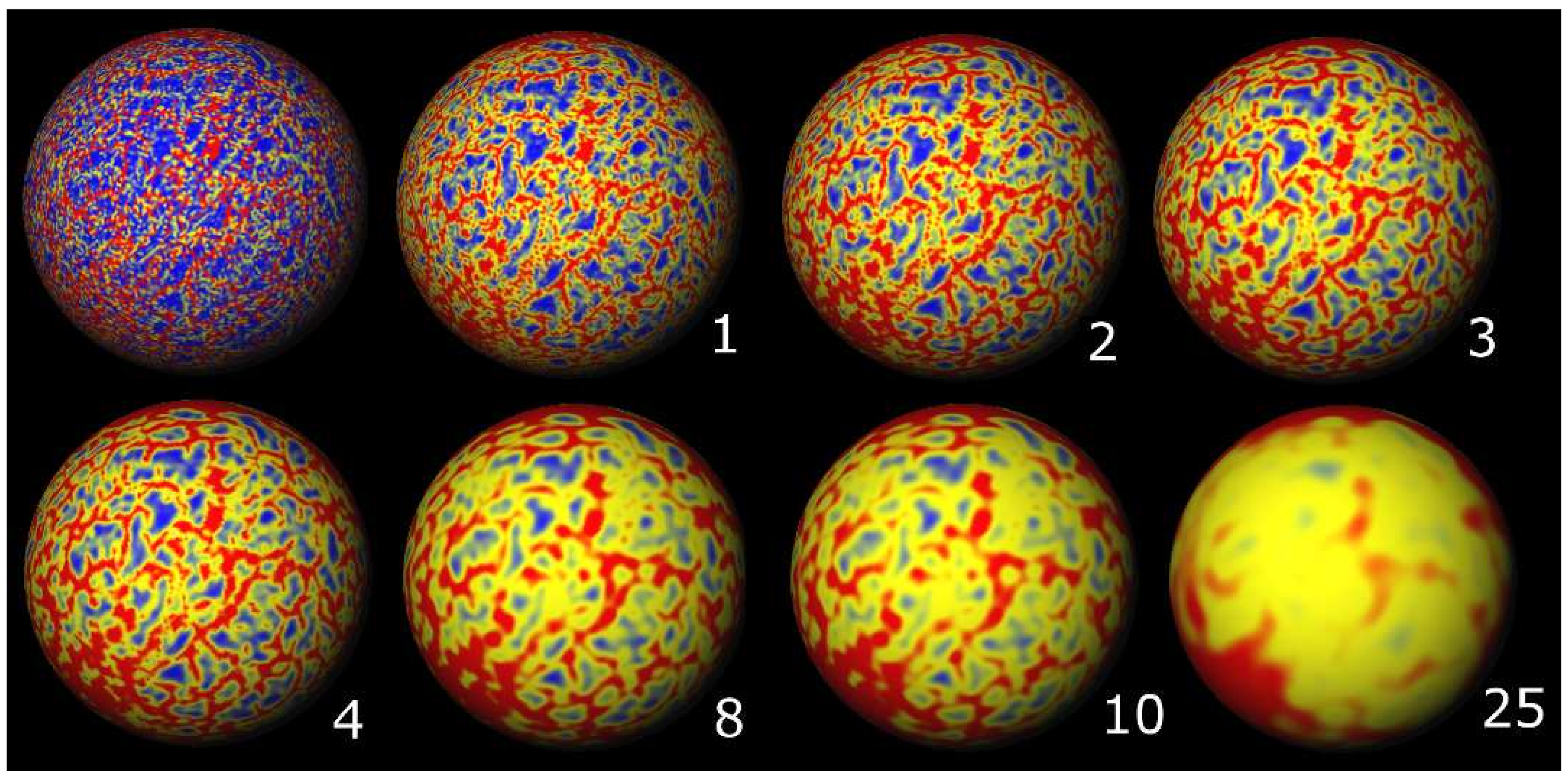

Figure 3: Sulcal pattern is extracted from the cortex using the sum of the principal curvatures onto a unit sphere [1]. Heat kernel smoothing with $\sigma = 0.01$ and $k = 1, 2, 3, 4, 8, 10, 25$ shows the varying smoothness of the sulcal pattern.

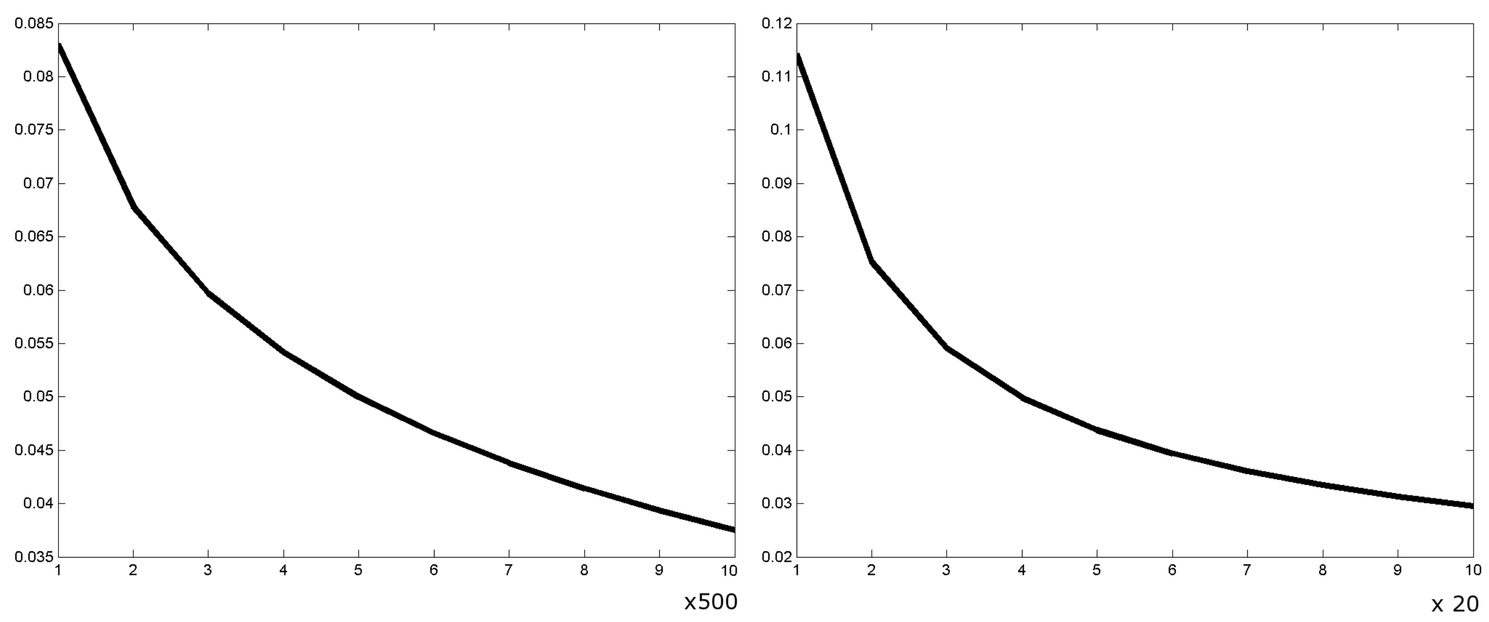

Figure 4: Variance reduction property of heat kernel smoothing [2]. Left: Within subject variance over the number of iterations with $\sigma = 1$ and $0 \le k \le 5000$. Right: Between subject variance over the number of iterations with $\sigma = 1$ and $0 \le k \le 200$.

Geodesic kernel smoothing

In the *geodesic kernel smoothing* formulation, we compute the heat kernel explicitly and assign the kernel weights accordingly. We compute the geodesic distance on the cortex using the dynamic programming (Dijktra's algorithm). Then the heat kernel is constructed by assigning weights as a function of the geodesic distance and appropriately normalizing the kernel. This approach avoid the necessity of choosing a small bandwidth for convergence.

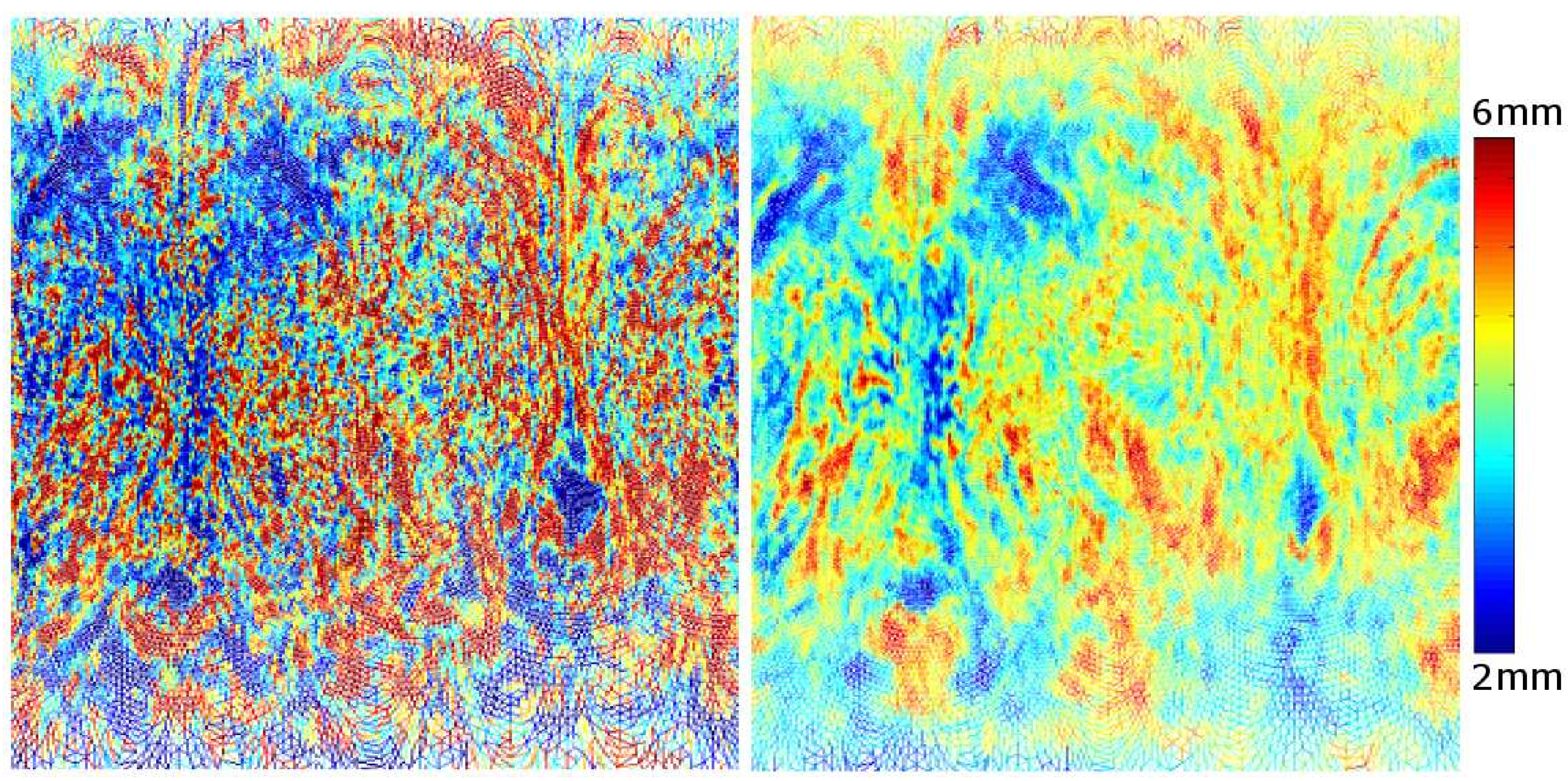

Figure 5: Thickness maps are projected onto a unit square before and after smoothing. Left: original noisy thickness map. Right: Heat kernel smoothing with $\sigma = 1$ and $k = 200$ iterations.

References

1. Chung, M.K., Worsley, K.J., Robbins, S., Paus, P., Taylor, J., Giedd, J.N., Rapoport, J.L., Evans, A.C. 2003. Deformation-Based Surface Morphometry with an Application to Gray Matter Deformation, *NeuroImage*. **18**:198-213.
2. Chung, M.K., Robbins,S., Dalton, K.M., Davidson, R.J., Alexander, A.L., Evans, A.C. 2005. Cortical Thickness Analysis in Autism via Heat Kernel Smoothing. *NeuroImage* **25**:1256-1265.