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New Validation Technique for Cortical Data Smoothing

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Introduction

Over the years, various diffusion based cortical surface data smoothing techniques [1] [3] have been proposed but without any numerical validation. We present a novel validation technique that uses the analytical solution of a diffusion equation as the ground truth. The proposed framework is used in validating the performance of heat kernel smoothing [3].

Heat kernel smoothing

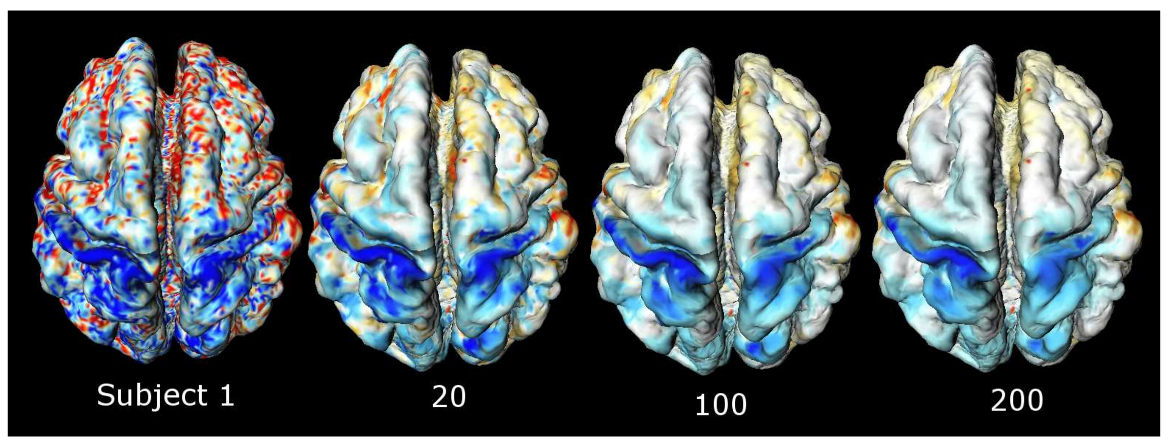

Figure 1: Heat kernel smoothing of cortical thickness with $\sigma = 1$ and $n = 20, 100, 200$ iterations. As n increases, the smoothing converges to the sample mean.

Heat kernel smoothing solves the following isotropic diffusion diffusion

$$\frac{\partial g}{\partial \sigma} = \Delta g, g(p, \sigma = 0) = f(p)$$

by iteratively performing kernel smoothing. The method has been used in many studies due to its numerical simplicity. For an arbitrary manifold $\mathcal{M}$, heat kernel is approximated using the the *parametrix expansion* [3]:

$$K_\sigma(p, q) = \frac{1}{(2\pi\sigma)^{1/2}} e^{-\frac{d^2(p,q)}{2\sigma^2}} \big[u_0(p, q) + O(\sigma^2)\big],$$

where $d(p, q)$ is the geodesic distance and $u_0(p, q) \rightarrow 1$ as $p \rightarrow q$.
Heat kernel smoothing of surface measurement f is then defined as the convolution

$$K_\sigma * f(p) = \int_{\mathcal{M}} K_\sigma(p, q) f(q)\, d\mu(q). \qquad (1)$$

For large σ, the convolution is performed iteratively with a smaller bandwidth as

$$K_\sigma * f = \underbrace{K_{\sigma/n} * \cdots * K_{\sigma/n}}_{n \text{ times}} * f.$$

Figure 1 shows the process of heat kernel smoothing of on cortical thickness. The `MATLAB` implementation can be found in `www.stat.wisc.edu/~mchung/hk`.

Weighted Fourier Series

Let λ_j and ψ_j be the eigenvalues and eigenfunctions of the Laplace-Beltrami operator Δ defined on $\mathcal{M}$, i.e.

$$\Delta \psi_j = \lambda_j \psi_j.$$

Heat kernel is defined as

$$K_\sigma(p, q) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} \psi_j(p) \psi_j(q).$$

The heat kernel smoothing is then expressed as

$$K_\sigma * f(p) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} f_j \psi_j(p), \qquad (2)$$

where f_j is the Fourier coefficient. The series expansion of heat kernel smoothing is called the *weighted Fourier series* [2], which provides the analytic reformulation of heat kernel smoothing.

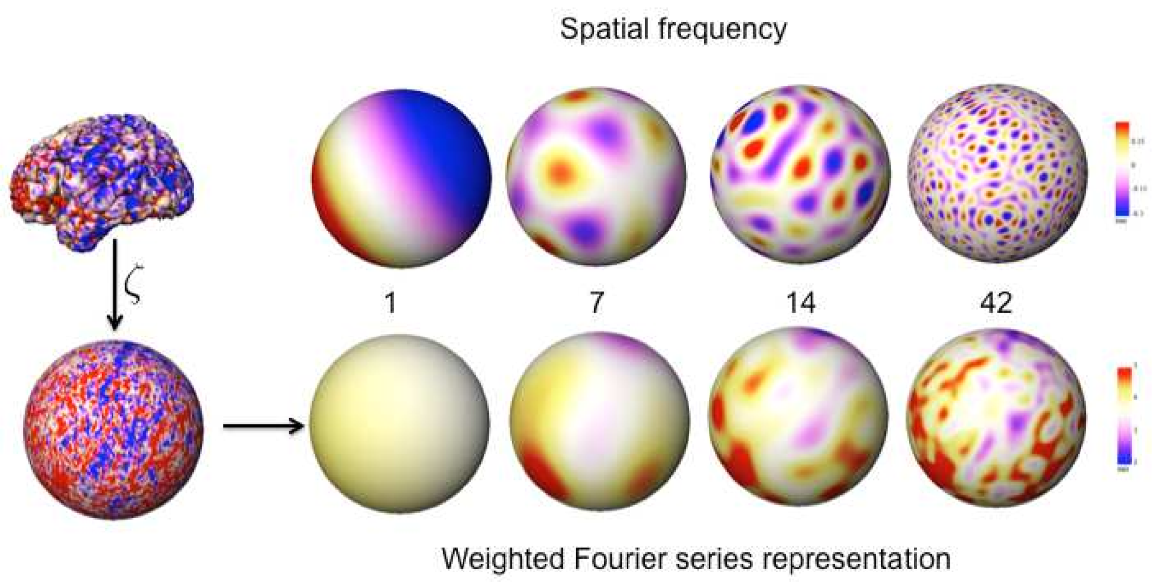

Figure 2: Weighted Fourier series representation of cortical thickness. The first row shows the spatial frequency $f_j \psi_j$ at the given degree. The second row shows the weighted Fourier series.

Validation Framework

Numerical implementation of heat kernel smoothing (1) is validated against the analytic formulation (2). Unfortunately for the lack of analytic basis in an arbitrary manifold, we validate on a unit sphere.
Cortical thickness is mapped onto a unit sphere via a cortical flattening map $\zeta : \mathcal{M} \rightarrow \mathcal{N}$ (Figure 2) and its Fourier series expansion is obtained with respect to spherical harmonics:

$$f(p) = \sum_{j=0}^{k} f_j \psi_j(p). \qquad (3)$$

Then taking the expression (3) as the input signal (simulated cortical thickness in Figure), we perform heat kernel smoothing:

$$K_\sigma * f(u) = \sum_{j=0}^{k} e^{-\lambda_j \sigma} f_j \psi_j(p). \qquad (4)$$

Equation (4) serves as the ground truth to be compared to heat kernel smoothing.

Results

We have compared the output of heat kernel smoothing and the ground truth (Figure 3). We have used $\sigma = 0.001$ and the degree 42 degree spherical harmonics. For heat kernel smoothing, varying number of iterations $1 \leq n \leq 70$ and the corresponding bandwidth $\sigma = 0.001/n$ were used. The minimum relative error is obtained when $n = 21$. The relative error is up to 0.055 at some vertex and the mean relative error is 0.0067. Hence, we conclude that heat kernel smoothing provides a sufficiently good approximation to isotropic diffusion.

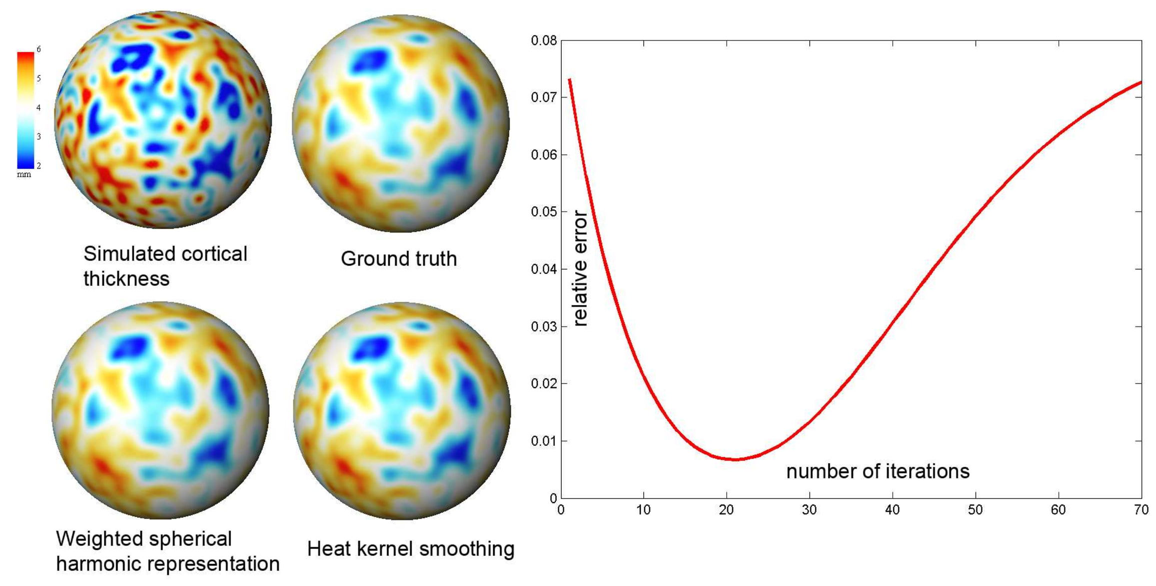

Figure 3: Cortical thickness is simulated from the real data. The ground truth is analytically constructed from the simulation. Heat kernel smoothing of the simulation is compared against the ground truth. The plot is the relative error over the number of iterations.

Discussion

Although the proposed method provides the first validation framework for surface data smoothing, the validation has been limited to a unit sphere. In order to extend the validation framework to an arbitrary manifold, we have constructed the eigenfunctions of the Laplace-Beltrami operator using the finite element method [4] (Figure 4). We are currently investing this generalization.

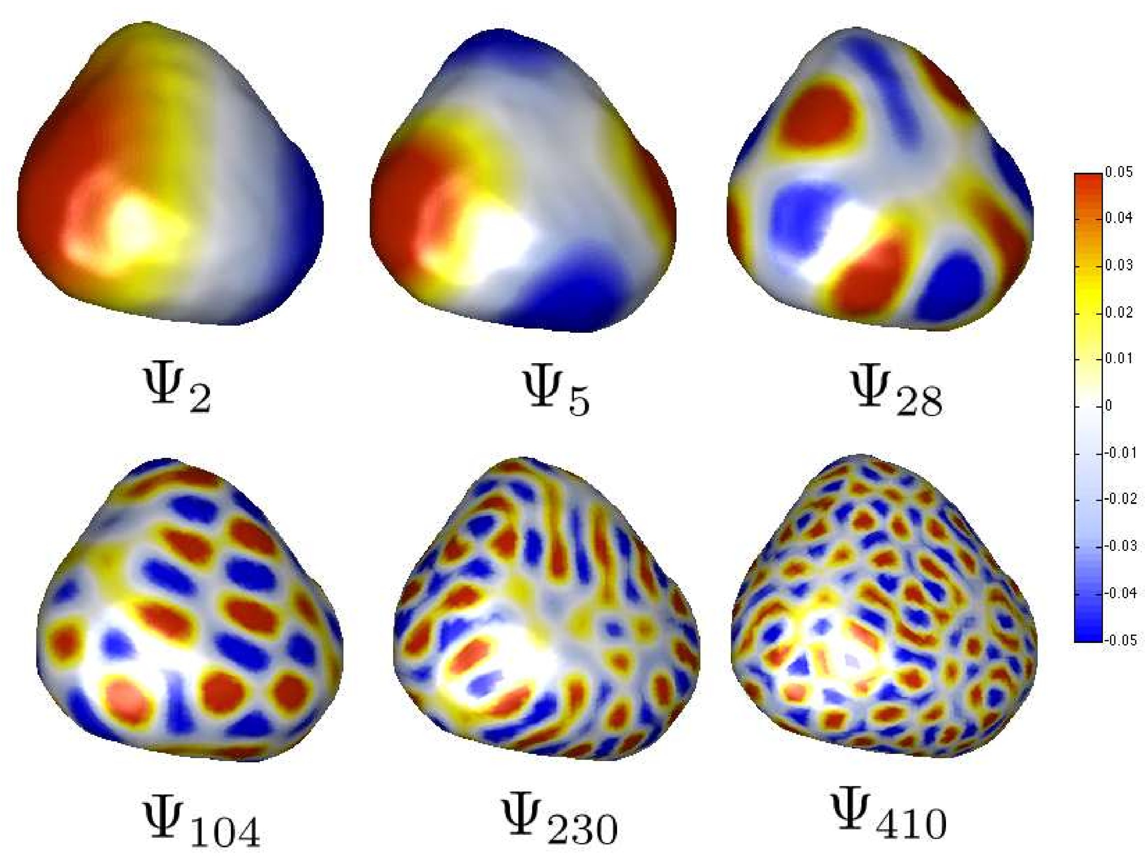

Figure 4: Eigenfunctions of the Laplace-Beltrami operator on an average amygdala surface

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