Persistent Network Homology from the perspective of Dendrograms

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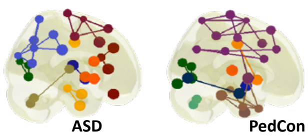
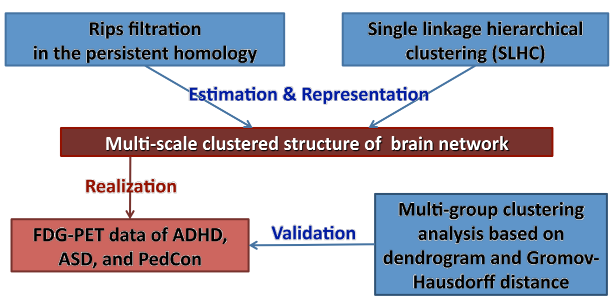
Introduction



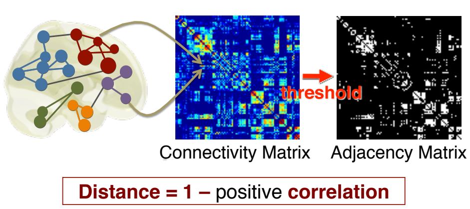
Modular structure with the fixed scale

- The ***modular structure*** of brain connectivity helps to understand from the local module information to their global relationships of brain network [1,2].
- Usually, we select only one optimal modular network by maximizing the predefined metric such as a modularity. But the optimal network can be changed depending on the metric and it is not yet known which network is truly modular.
- In this study, we seek the evolutionary changes of modular structures when the threshold in correlation matrix increases, instead of choosing a fixed modular structure. It can be directly related with the **hierarchical clustering** with the persistent property and visualized by a **dendrogram**.
- As an application, we constructed the brain networks using the FDG-PET data of 24 attention deficit hyperactivity disorder (ADHD), 26 autism spectrum disorder (ASD) children and 11 pediatric control (PedCon) subjects.
- The difference between the changes of the modular structures was compared by **Gromov-Hausdorff distance** [3,4], which measures the distance between dendrograms.

Outline



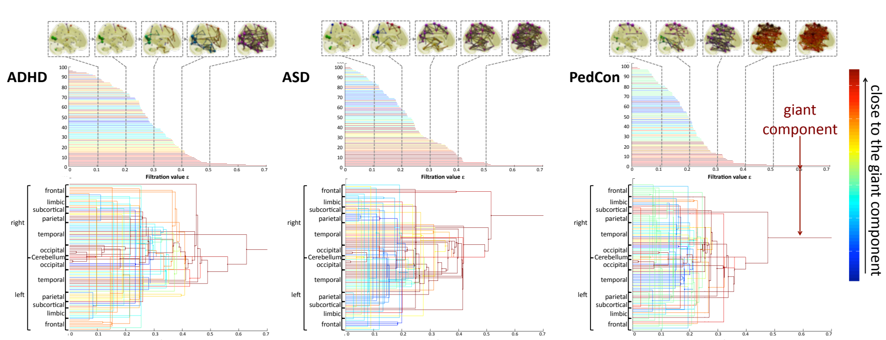
Network Construction



Different adjacency matrices are constructed depending on the thresholds.

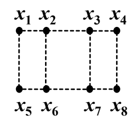
Which one do you prefer?

Multi-scale modular structure of brain networks



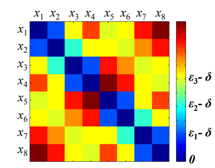
Persistent Homology

Given point cloud data X & their metric $d(x_i, x_j)$,



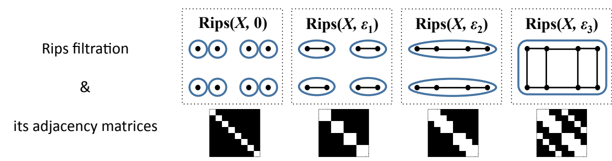
$X = \{x_1, x_2, \ldots, x_8\}$

$$\begin{cases} d(x_1, x_2) = d(x_3, x_4) = \ldots = \varepsilon_1 - \delta, \\ d(x_2, x_3) = d(x_6, x_7) = \varepsilon_2 - \delta, \\ d(x_1, x_5) = d(x_2, x_6) = \ldots = \varepsilon_3 - \delta \end{cases}$$

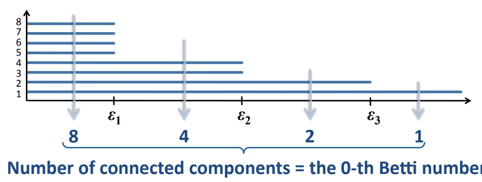


Connectivity matrix $d(x_i, x_j)$

- **Rips complex, Rips(X, ε)**
  approximate the topology of the point cloud data by connecting two point cloud data, x_i and x_j, if $d(x_i, x_j) < \varepsilon$
- **Rips filtration**
  the sequence of Rips complexes satisfying the persistent property such as Rips(X, 0) ⊆ Rips(X, ε_1) ⊆ Rips(X, ε_2) ⊆ … ⊆ Rips(X, ε_n) for $0 \le \varepsilon_1 \le \varepsilon_2 \le \ldots \le \varepsilon_n$



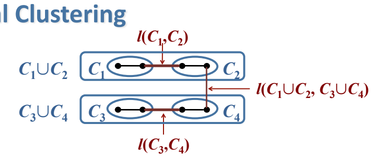
- **Barcode**
  visualize the changes of the connected components during the filtration



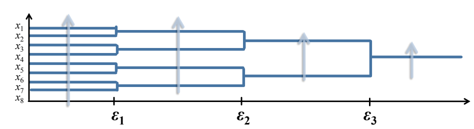
Single Linkage Hierarchical Clustering

- **Single Linkage Distance**

$$l(C_i, C_j) = \min_{x_i \in C_i} \min_{x_j \in C_j} d(x_i, x_j)$$

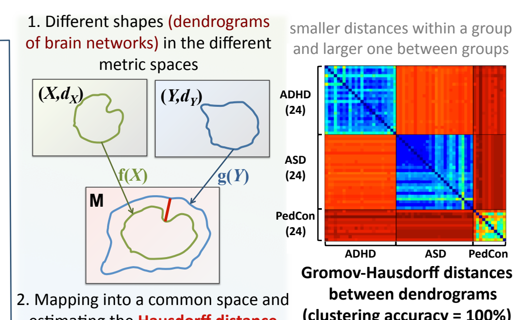


- **Dendrogram**
  rearrange the barcode according to the index of nodes



Gromov-Hausdorff Distance

We constructed 61 dendrograms using 24 ADHD, 26 ASD and 11 PedCon jackknifed resampled datasets and estimated their pairwise differences based on Gromov-Hausdorff distance.



Gromov-Hausdorff distances between dendrograms (clustering accuracy = 100%)

References

[1] Bassett, D. (2006), The Neuroscientist, vol. 12, pp. 512-523.
[2] Chen, Z.J., et. al. (2008), Cerebral Cortex, vol. 18, pp. 2374-2381.
[3] Calrlsson, G. and Memoli, F. (2010), JMLR, vol. 11, pp. 1425-1470.
[4] Lee, H., et. al. (2011), MICCAI2011.