Multivariate Cortical Shape Modeling Based on Sparse Representation

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Introduction

We present a new sparse shape representation method using the eigenfunctions of Laplace-Beltrami (LB) operator. Cortical surfaces can be intrinsically represented as a Fourier series expansion using the LB-eigenfunctions as a basis. However, some coefficients may not necessarily contribute significantly in reconstructing the surfaces. We sparsely filter those significant coefficients by imposing the ℓ_1 -penalty.

We apply this sparse representation in detecting abnormal local cortical shape variations in autism via general multivariate linear modeling (GMLM) [2].

Laplace-Beltrami Eigenfunctions

The eigenfunctions ψ_i of the LB operator Δ on a cortical surface are given by so

General Multivariate Linear Model

We applied the method in discriminating the cortical shape of 16 autistic and 11 control subjects (17.18 \pm 2.89 and 16.13 \pm 4.51 years respectively). The details on MRI acquisition and image processing can be found in [1].

To localize shape difference between the groups, we used the GMLM [2]:

 $[\mathbf{p}_1 \, \mathbf{p}_2 \, \mathbf{p}_3] = b_0 + \text{age } b_1 + \text{group } b_2 + \text{age} \cdot \text{group } b_3 + \epsilon.$

We have tested the group effect (b_2) while accounting for age but could not detect any shape difference between autistic and control groups even at $\alpha = 0.1$ level (corrected). However, we detected the significant growth rate difference (b₃) in the left prefrontal cortical regions (Figure 3).



$$\Delta \psi_j = -\Lambda_j \psi_j,$$

form an orthonormal basis on the surface. The LB-eigenfunctions are computed on the template surface using the FEM discretization [3, 4] (Figure 1).

The i-th surface coordinate \mathbf{p}_i can be represented as a linear combination of the LB eigenfunctions:

$$\mathbf{p}_{i} = \underbrace{[\psi_{0}, \psi_{1}, \cdots, \psi_{k}]}_{\Psi} \beta,$$

where $\beta = (\beta_0, \dots, \beta_k)$ is the unknown coefficients to be estimated.



Figure 1: Few eigenfunctions of Laplace-Beltrami operator on the template surface.

Sparse Representation

The coefficients β are estimated sparsely by minimizing ℓ_1 -norm penality

$$\widehat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \|\mathbf{p}_{i} - \boldsymbol{\Psi}\boldsymbol{\beta}\|_{2}^{2} + \tau \|\boldsymbol{\beta}\|$$

Figure 3: F-statistic map of significance of the interaction term (b_3) . (a) Least squares estimation (b) Sparse estimation. (c) The linear fit showing growth rate difference at the most significant vertex (max F = 67.31, corrected p = 0.006) in left prefrontal cortical region.

Conclusion

The parameter τ controls the sparsity and it was empirically selected $\tau = 10$. Figure 2 shows the results.



Figure 2: The plot of $|\beta_j|$ vs. degree j for the x-coordinate. The left plot is the enlargement of the black box on the right. The least squares estimation is colored in blue while the sparse estimation is colored in red. In average, only 1100 out of 7396 coefficients remain significant.

The proposed sparse shape representation demonstrates its potential for modeling cortical shape variations without using often used surfacebased smoothing that reduces statistical power unnecessarily.

Reference

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