

# Probabilistic Connectivity Using Kullback-Leibler Distance

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## INTRODUCTION

**White matter tractography (WMT)** is a promising method for characterizing the white matter pathways that connect brain regions. **Probabilistic WMT** is one approach to describe the white matter connectivity. Propagated diffusion profiles based on the measured diffusion tensor from diffusion tensor imaging may be used for the probabilistic WMT. In this study we use two continuous propagators: isotropic Gaussian kernel smoothing and anisotropic Gaussian kernel smoothing. We aim to show the utilization of the Kullback-Leibler (KL) distance as incorporated with isotropic Gaussian kernel smoothing and measured diffusion tensor based anisotropic Gaussian kernel smoothing, produces robust probabilistic connectivity maps.

## THEORY

The **KL distance** is a statistical measure for measuring the **dissimilarity** between two probability distributions --  $p$  and  $q$ .  $p$  is a distribution of interest and  $q$  usually represents a null distribution in the KL distance formula,

$$KL(\bar{r}) = \sum_{\bar{s}} p_{\bar{r}}(\bar{s}) \cdot \log\left(\frac{p_{\bar{r}}(\bar{s})}{q_{\bar{r}}(\bar{s})}\right)$$

The distribution  $p$  may be generated using the anisotropic Gaussian kernel that is based on tensor in DTI WMT.

The anisotropic Gaussian kernel [2] is formulated as,

$$K_{t, \text{Anisotropic}}(r) = \frac{\exp(-r\bar{D}^{-1}r / 4t)}{(4\pi t)^{n/2} (\det \bar{D})^{1/2}}$$

Where  $\bar{D}$  is diffusion tensor and  $t$  is a parameter that governs the extent of the diffusion, i.e. smoothing. The anisotropic Gaussian kernel is convolved with a volume of matrix that has a seeding point (intensity 1) to generate the distribution of the interest ( $p$ ). The null distribution  $q$  is obtained using isotropic Gaussian diffusion propagation. The isotropic kernel is formed simply by inserting the identity matrix for the in the anisotropic Gaussian kernel above.

## CONCLUSIONS and DISCUSSION

The KL distance is a promising method for comparing the connectivity maps using isotropic and anisotropic kernel smoothing. It is similar to a continuous version of probabilistic tractography testing methods proposed by Miller et al.[1]. The approach appears to display the prominent connection pathways from the selected seed. However, some regions displayed connectivity drop-outs where the distribution was low from rapid 'washout'. Future implementations need to address this shortcoming.

## METHODS

**DATA Acquisition** : A single-shot spin echo EPI sequence with diffusion-tensor encoding (12 directions,  $b=1000\text{s/mm}^2$ ), was used to get a DTI data from a subject (identical slice locations, voxels =  $0.84 \times 0.84 \times 1.8\text{mm}$ , 52 slices, acquisition matrix  $128 \times 128$ , 10 NEX, 23 cm FOV).

**Gaussian smoothing (diffusion propagation)** : For the distribution  $p$ , anisotropic diffusion was propagated using the anisotropic Gaussian kernel that was constructed as described in [2] with a seed point at midline of the splenium (Fig 1) and internal capsule (Fig 2). The corresponding null distribution was acquired using isotropic Gaussian kernel smoothing. Isotropic Gaussian kernel that was constructed by using the MD (mean diffusivity) for each eigenvalue with the same  $t$  as in the anisotropic Gaussian kernel construction in its formula.

## RESULTS

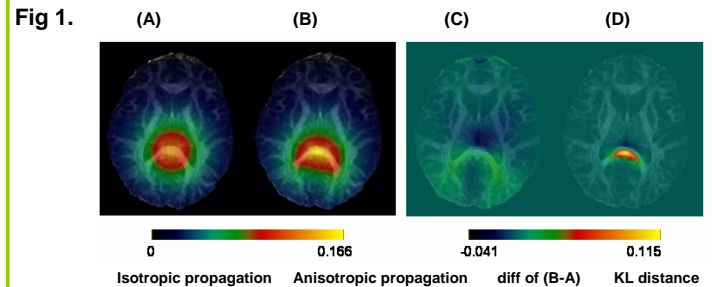


Fig 1: Isotropic and anisotropic diffusion propagation starting from the midline of the corpus callosum splenium. Both propagation maps were acquired at the iteration 150 with the same  $t=1e-8$  and tensor was normalized with the determinant. It is clear that anisotropic distribution extends further in the R/L direction. The map (C) is simply the difference map of B and A. The K-L distance map (D) revealed greatest apparent connectivity in the CC with more dynamic detailed ranges.

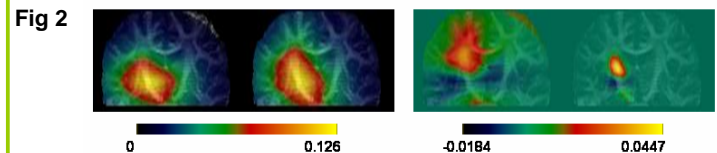


Fig 2: Diffusion propagation maps in the right internal capsule. The KL distance map shows dominant connectivity patterns along the cortico-spinal tract although 'gaps' in the connectivity map are apparent. In these regions, the lower distribution following anisotropic smoothing appears to be from the 'washout' effect.

## REFERENCES

- [1] Morris et al., Proc.intl. Soc. Mag. Reson. Med., 2006
- [2] Lee et al., Proc. Intl. Soc. Mag. Reson. Med 2005