

Electrical Circuit Model for White Matter Fiber Tracts in Diffusion Tensor Imaging

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Motivation

The strength of connection from one gray matter region to another is often measured by counting the number of fiber tracts connecting the two regions in predefined parcellations. However, the problems with this approach are the use of arbitrary parcellation and the negligence of the distance between the regions.

We present a new parcellation-free data driven network construction method. We model fiber tracts as wires in an electrical circuit. The strength of connection corresponds to the resistance of the wire. As an application, we apply the proposed technique in characterizing the structural network of autistic and normal control (NC) subjects.

DTI Data

DTI were acquired on a 3-Tesla scanner for age-matched 36 NC and 41 autistic subjects. Age range is between 6 and 30 years. Spatial normalization of DTI data was done via a diffeomorphic registration strategy similar to ANTS. A population specific tensor template was constructed. Tractography was done in the normalized space using TEND (Figure 1).

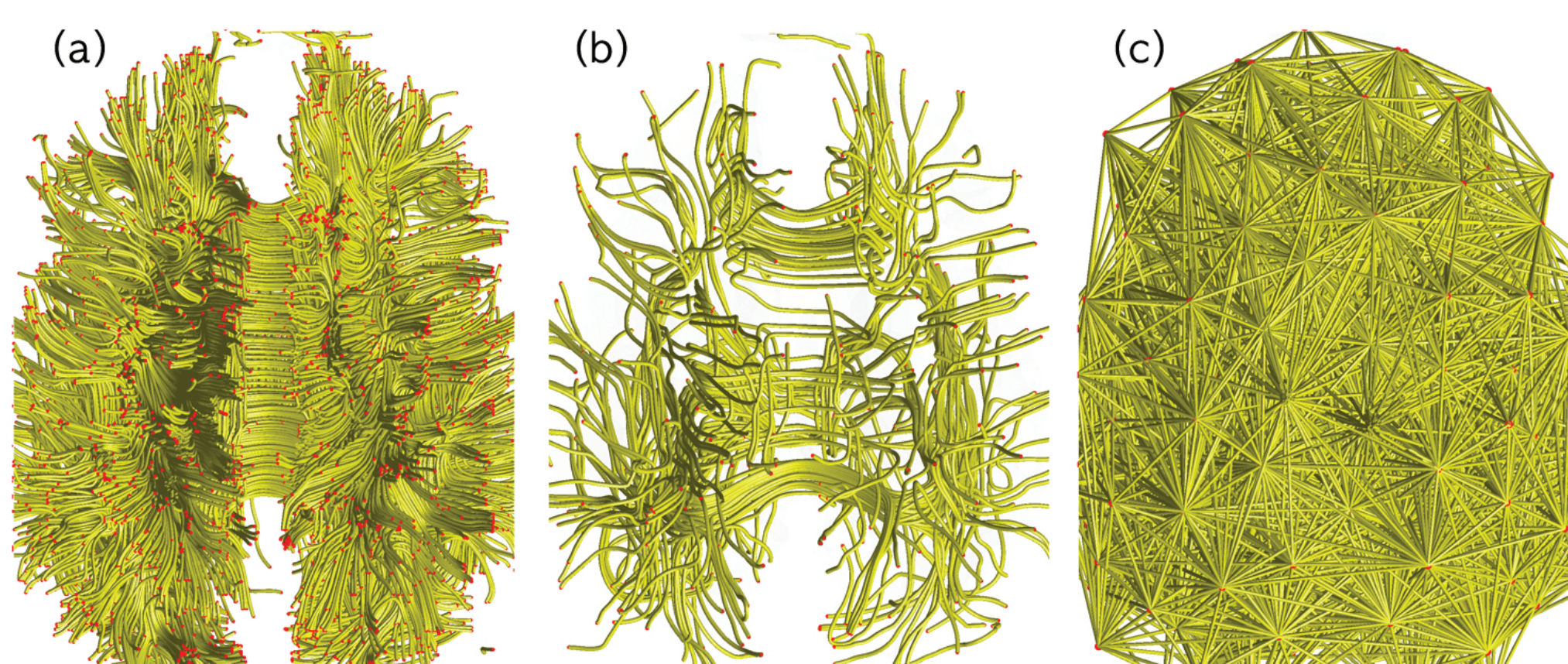


Figure 1: The end points of tracts are identified. Tracts whose end points (red dots) are within the ball of radius $\epsilon = 10\text{mm}$ are considered as connected (a). The fiber tracts and the balls constitute a complex electronic circuit. Most tracts form parallel circuits within ϵ radius but less than 1% of tracts are not parallel circuits (b). Each parallel circuit is replaced by a single tract with equivalent resistance. (c) The equivalent simplified circuit then forms a graph where the edge weights are given by the resistance. We reduce the number of edges by less than 10% per subjects.

Electronic Circuit Model for Fiber Tracts

The brain network can be modeled as an electrical system consisting of series and parallel circuits. Each fiber tract is viewed as a wire with resistance R proportional to the length of the wire.

Series Circuit. If two regions are connected through an intermediate region, it forms a series circuit. In the series circuit, the total resistance is additive, i.e.

$$R = R_1 + R_2 + \dots + R_k,$$

where R_k is the resistance of the k -th tract.

Parallel Circuit. If multiple fiber tracts connect two regions, it forms a parallel circuit, where the total resistance is

$$\frac{1}{R} = \frac{1}{R_1} + \dots + \frac{1}{R_k}.$$

Any parallel circuits in an electrical system can be simplified using a single wire with the equivalent resistance (Figure 4).

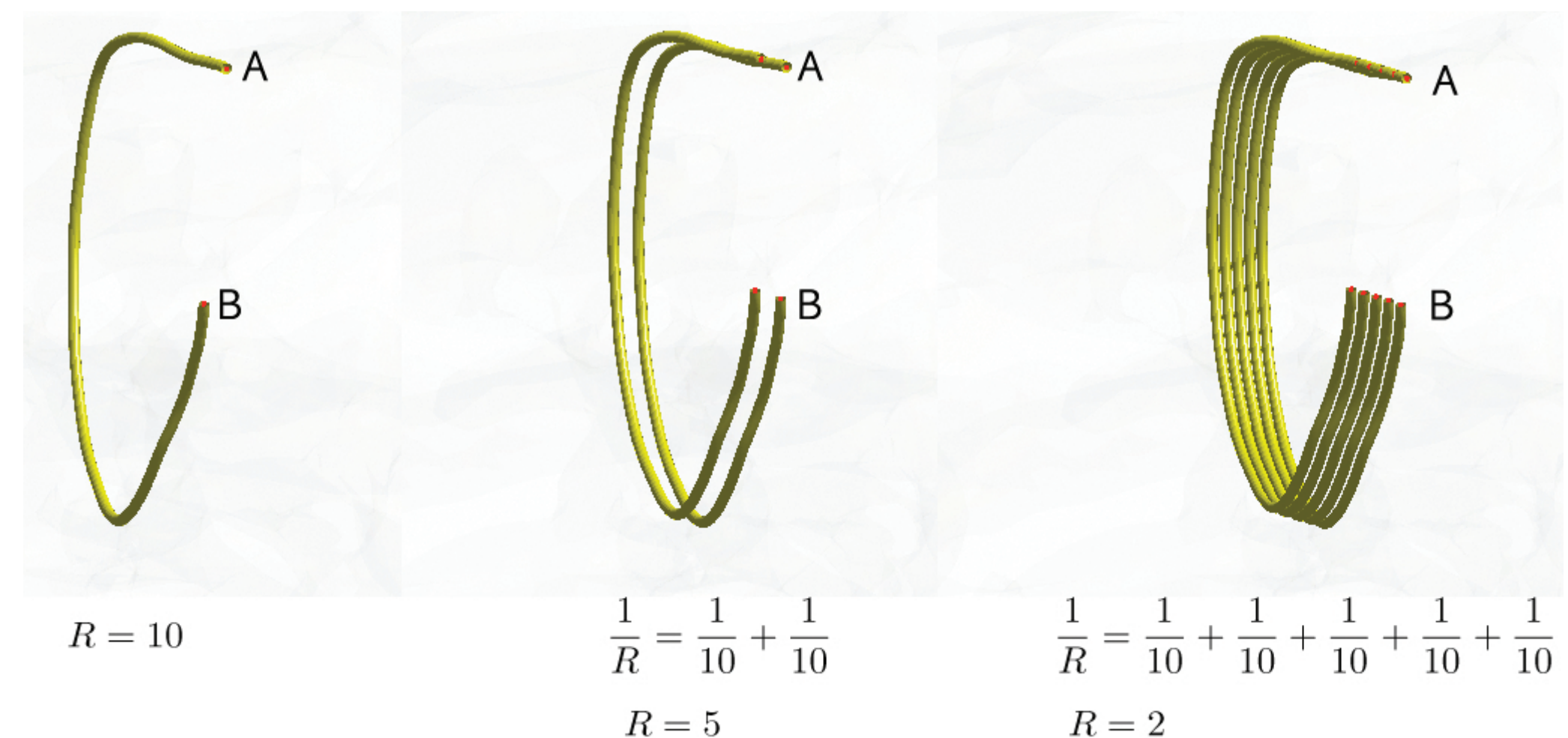


Figure 2: Multiple fiber tracts connecting regions A and B are modeled as a parallel circuit. The resistance in a wire is proportional to the length of the wire. So the resistance of a tract is simply defined as the length of the tract. As more tracts connect the regions in parallel, the strength of connection increases and the resistance decreases. If all the tracts are 10 cm in length, the total resistances become $R=10$, 5 and 2 as the number of parallel tracts increases to 1, 2 and 5.

Resistance Matrix of Whole Brain

The proposed electronic circuit model is used in constructing the brain network without a predefined parcellation. All the tracts are sorted in terms of length and the two end points are identified. Every tract whose end points are within the ball of 10mm radius is considered as connected. The collection of tracts and 10mm-radius balls form a complex circuit, which is iteratively simplified by replacing a parallel circuit with a single equivalent tract (Figure 1). This iterative network simplification produces the resistance matrix $R = (R_{ij})$. Then the total resistance is defined as $\frac{1}{2} \sum_{i,j} R_{ij}$ for finite entries. Smaller resistance corresponds to stronger connection with smaller length and redundant number of parallel wires. See (Figure 3) for toy networks.

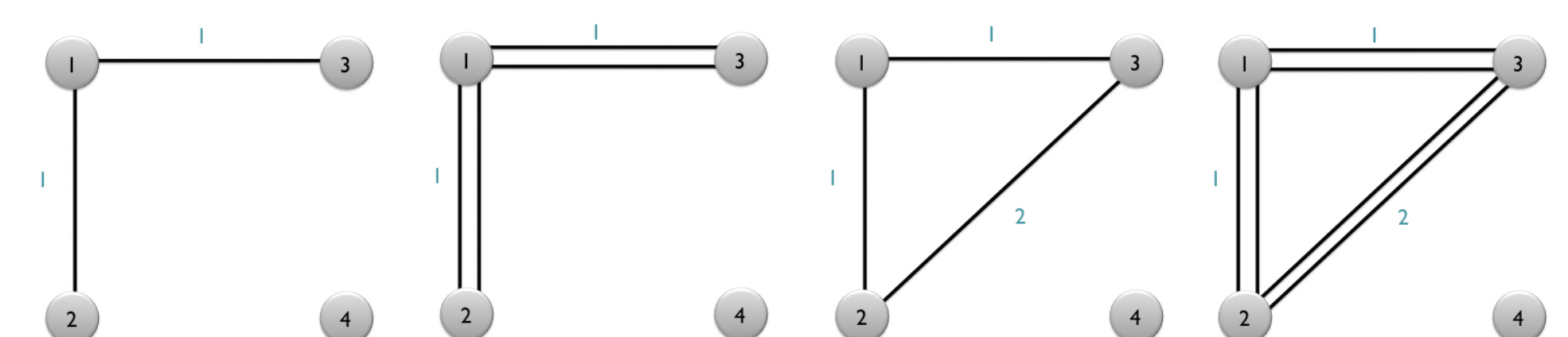


Figure 3: Toy networks with the corresponding resistance matrices:

$$\begin{pmatrix} 0 & 1 & 1 & \infty \\ 1 & 0 & 2 & \infty \\ 1 & 2 & 0 & \infty \\ \infty & \infty & \infty & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1/2 & 1/2 & \infty \\ 1/2 & 0 & 1 & \infty \\ 1/2 & 1 & 0 & \infty \\ \infty & \infty & \infty & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 3/4 & 3/4 & \infty \\ 3/4 & 0 & 1 & \infty \\ 3/4 & 1 & 0 & \infty \\ \infty & \infty & \infty & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 3/8 & 3/8 & \infty \\ 3/8 & 0 & 1/2 & \infty \\ 3/8 & 1/2 & 0 & \infty \\ \infty & \infty & \infty & 0 \end{pmatrix}.$$

Resistance between disjoint network is ∞ . The total resistances are 4, 2, $10/4$ and $10/8$. The most redundant network has the smallest total resistance.

Results

The resistance matrix is normalized by the maximum resistance. The mean resistance is 225 for NC and 212 for autistic subjects. The difference is found to be statistically significant using the nonparametric rank-sum test ($p=0.07$). Higher resistance implies that NC have relatively more longer tracts and less redundant parallel circuits.

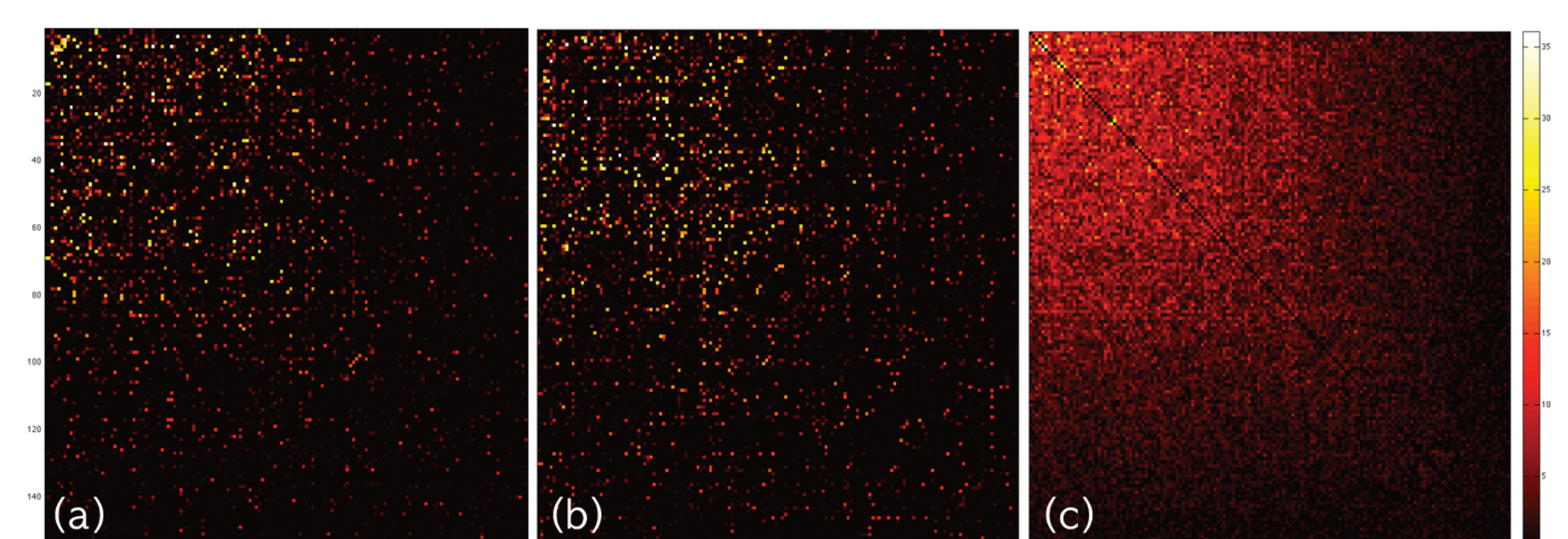


Figure 4: The resistance maps for the first (a) and second (b) NC subjects. The mean resistance map for all 36 NC subjects. The nodes are indexed in such a way that long tracts have smaller indexing. Obviously long tracts tend to have higher resistance.