The Waisman Laboratory
for Brain Imaging and Behavior

Fourier Spectral Method for Shape Asymmetry Analysis

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Acknowledgments

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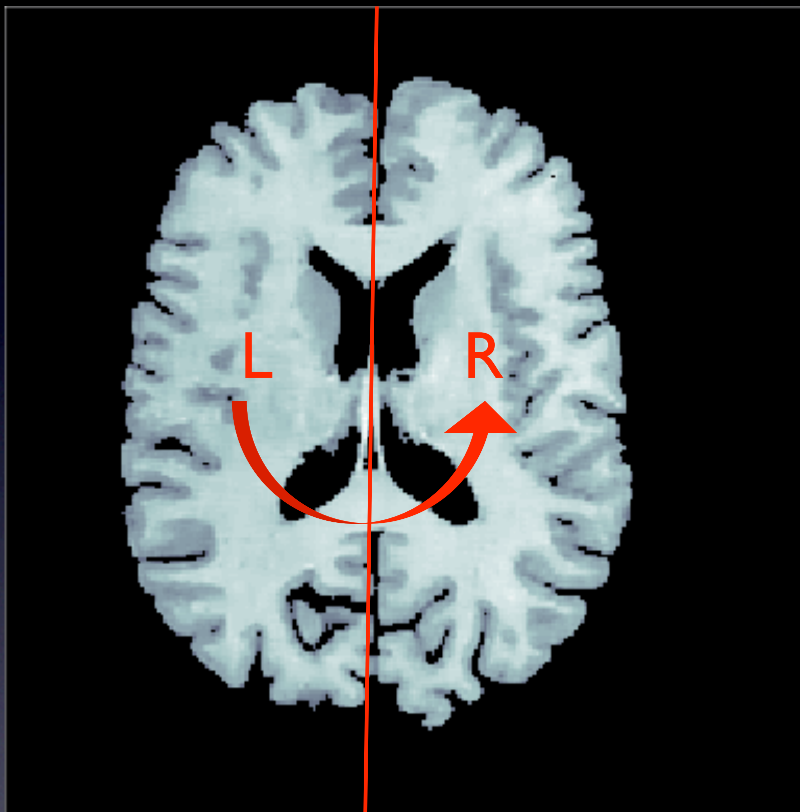
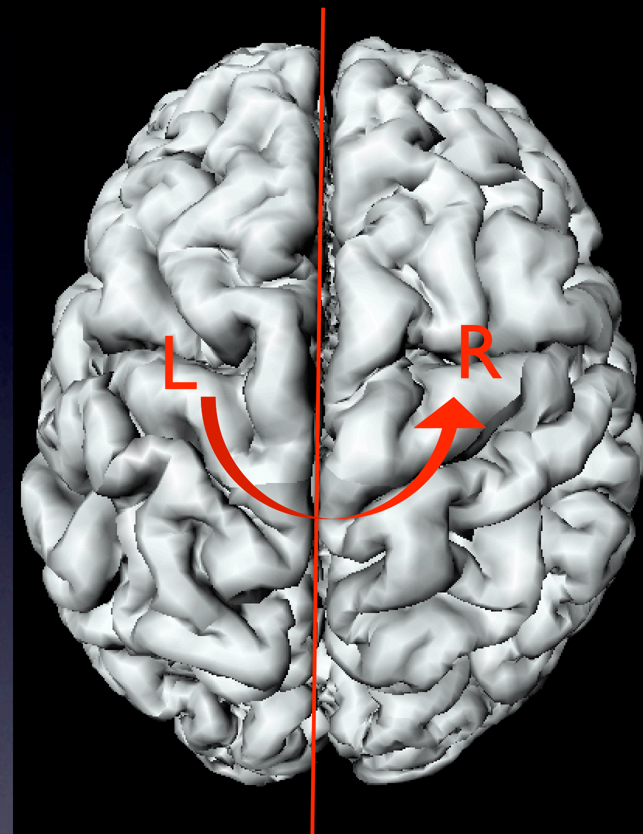
Abstracts

Although shape asymmetry has been investigated in manybranches of science, there is a lack of unified methodologicalframework for quantifying local shape asymmetry. Previous literature mainly deals with quantifying a global amount of shape asymmetry. Amore interesting question would be to ask if we could spatiallylocalize the source of asymmetry.In brain imaging, this question has been successfully addressed by using the deformable template approach of Grenander and Miller. By registering the original and its mirror reflected 3D magneticresonance image (MRI), one can establish the correspondence across brain hemispheres and, in turn, able to construct the localize asymmetry index of type (L-R)/(L+R). The additional computational burden ofestablishing deformation across hemispheres and possible mismatching of sulcal pattern across subjects are two major shortcomings of this widely used approach. In this talk, we present a different framework for shape as ymmetryanalysis that basically combines the deformable template idea and Brechbuler's 3D Fourier descriptor. Surface shape registration,surface data smoothing and surface parameterizations are all tackled in a unified framework. This is a joint work with Kim Dalton and Richard Davidson of the Waisman Laboratory for Brain Imaging and Behavior. An application of the same technique to longitudinalmandible shape modeling (in collaboration with Houri Vorperian of the Vocal Tract Development Laboratory) on 300 subjects will be alsobriefly discussed.

Brain hemisphere asymmetry





Localized asymmetry index (L-R)/(L+R)

Motivation: quantify abnormal brain structural asymmetry across hemispheres in a group of autistic subjects

Previous 3D approach

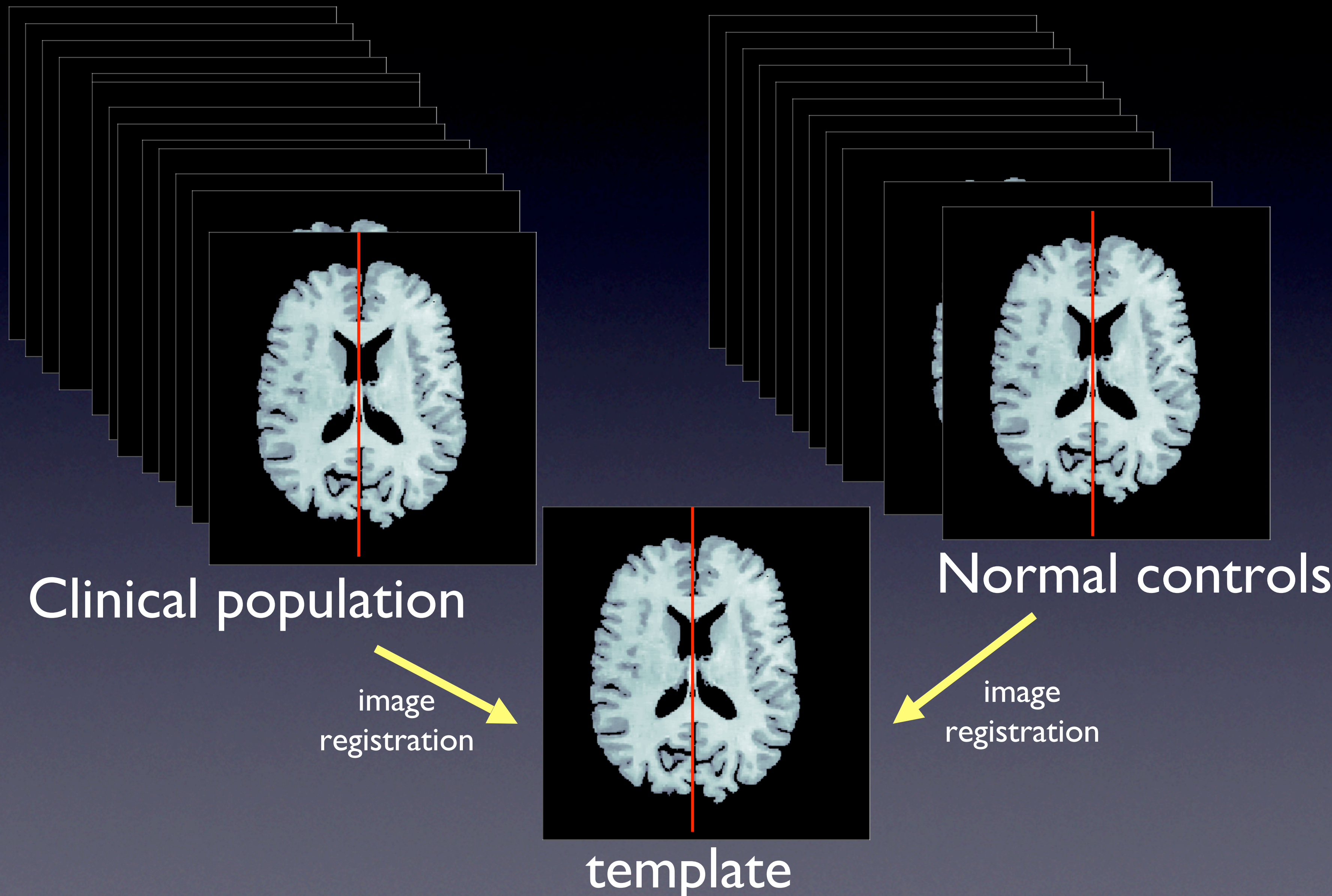
1. Image registration across subjects via a template

2. Image registration across hemispheres by registering the original MRI and its mirror reflection.

3. Construct asymmetry index at each voxel.

4. Feed the index into a statistical model.

Two population asymmetry analysis framework



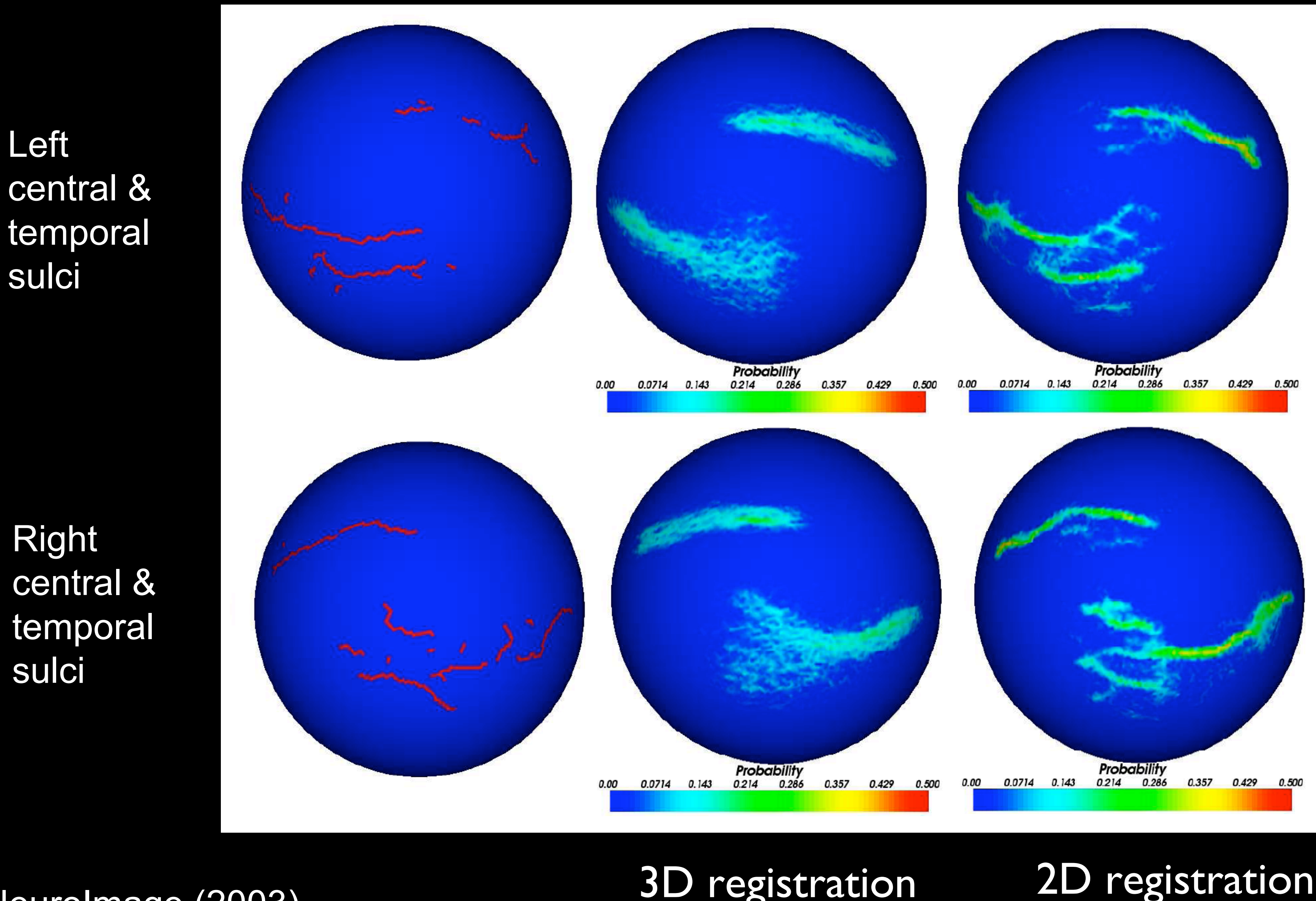
Three issues with this well established 3D approach

1. 3D image registration can easily misalign sulcal pattern.

2. Mirror reflection and doing image registration is an additional computational burden.

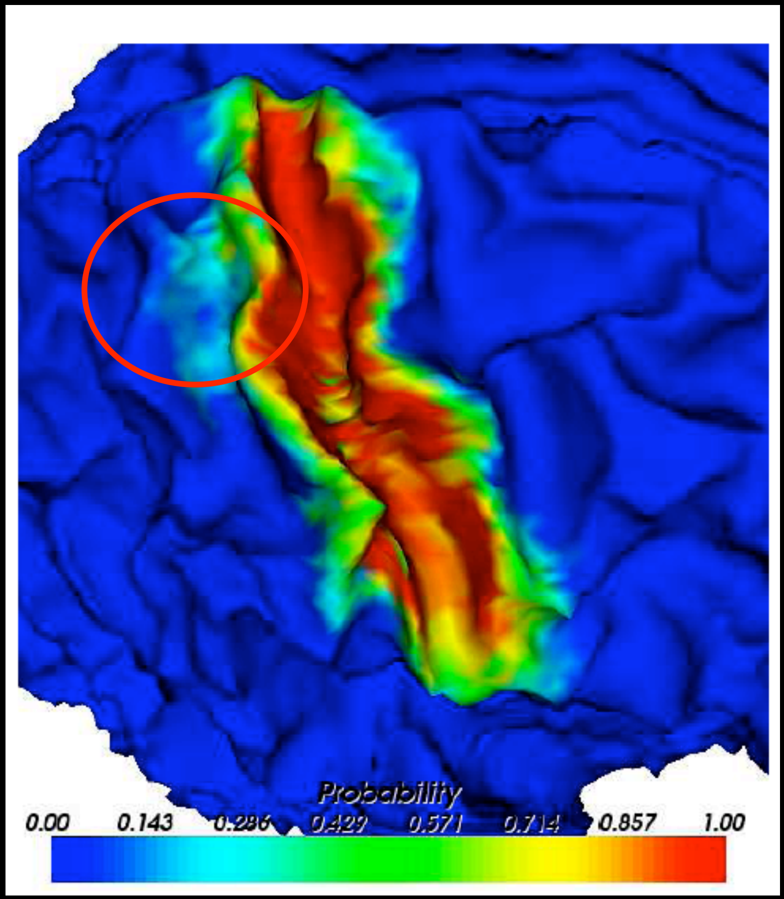
3. The 3D approach does not work for 2D cortical surface data. New 2D framework is needed.

Comparison of surface registration on 149 subjects

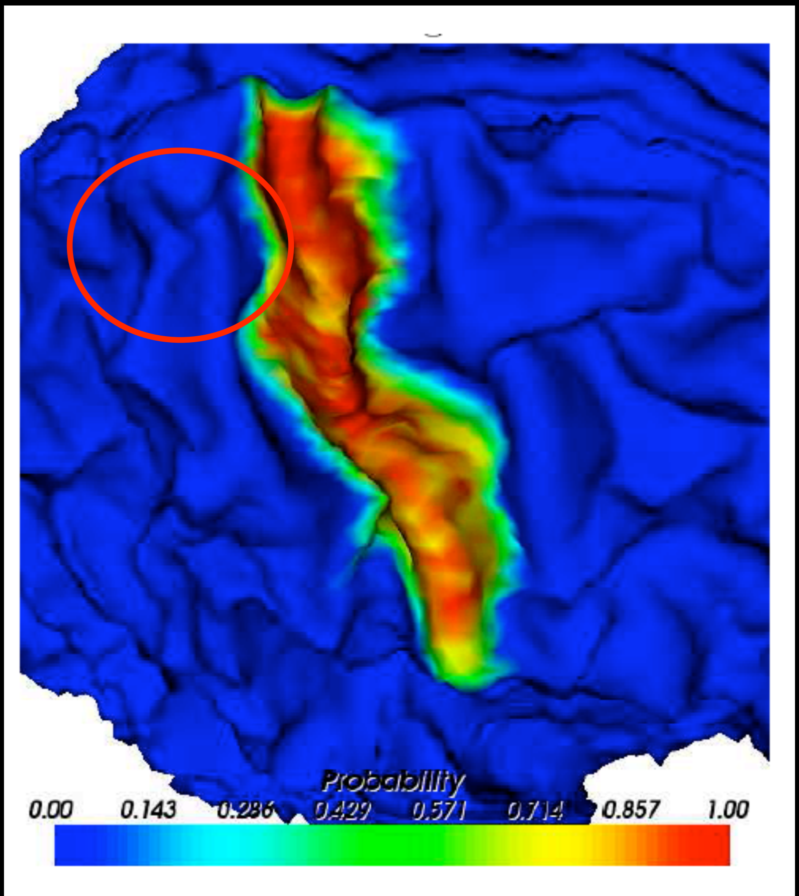


NeuroImage (2003)

Probability of matching in the right central sulcus



3D volume registration



2D surface registration

Literature vs. new framework

Surface data smoothing	Surface parameterization	Surface registration	Multiple comparison correction
diffusion smoothing (NeuroImage, 2003) heat kernel smoothing (NeuroImage, 2005)	SPHARM Guido Gerig Martin Styner Li Shen	PDE Paul Thompson Michael Miller	Random field theory Keith Worsley Jonathan Taylor

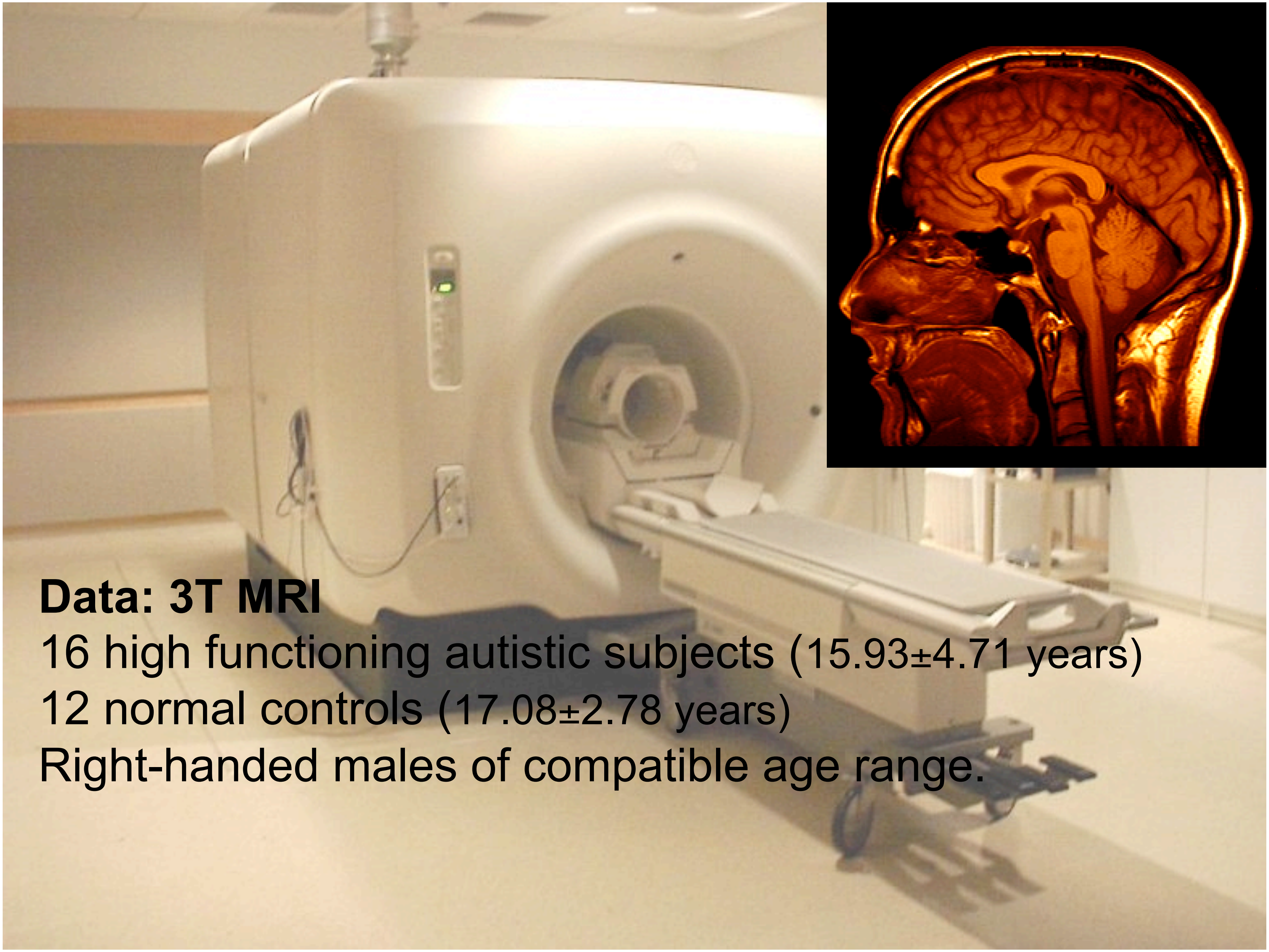
New unified approach:

Weighted spherical harmonic representation

(TMI, 2007)

Outline of talk

1. Introduction to cortical surface data
2. Weighted Fourier series representation
3. Surface registration
4. Surface asymmetry index
5. Statistical analysis
6. Future research direction



Data: 3T MRI

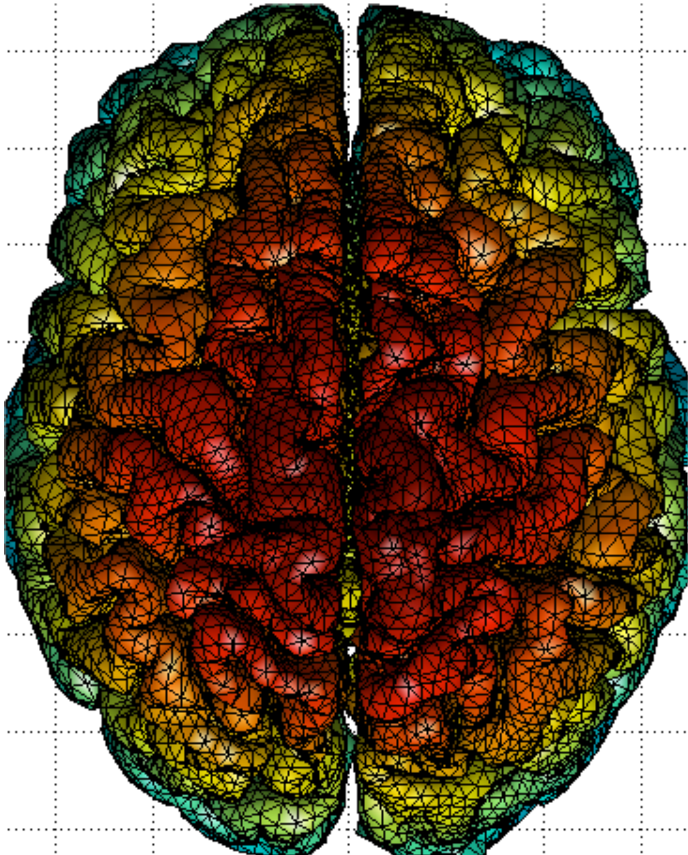
16 high functioning autistic subjects (15.93±4.71 years)

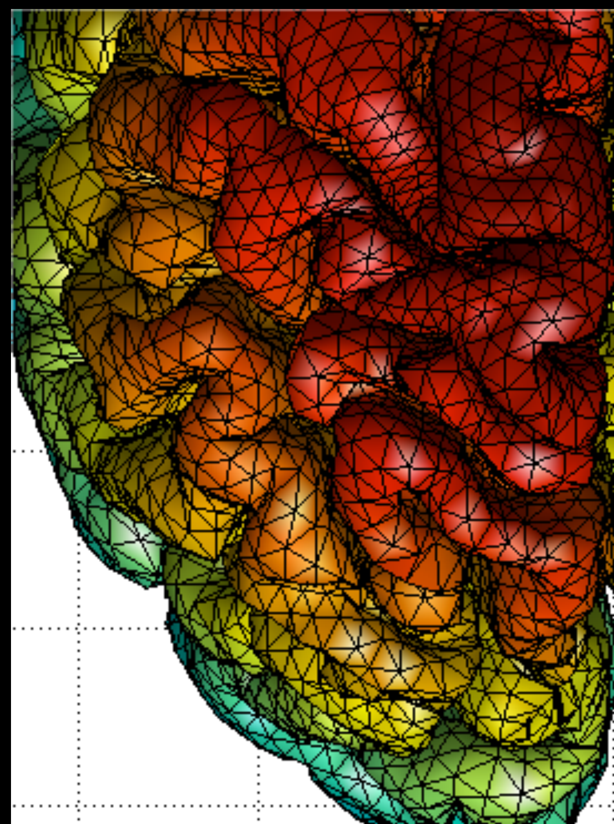
12 normal controls (17.08±2.78 years)

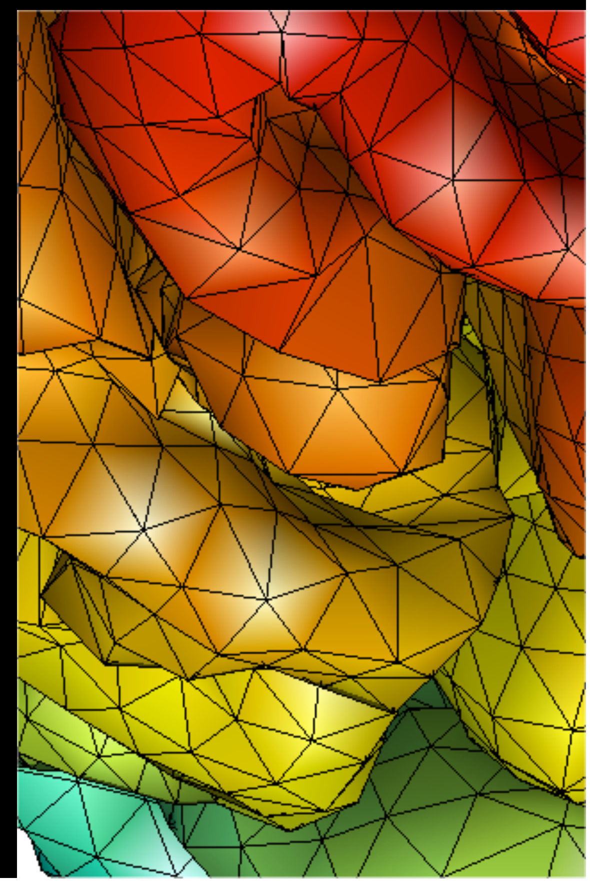
Right-handed males of compatible age range.

Polygonal mesh

Mesh resolution 3mm







82,190 triangles

40,962 vertices

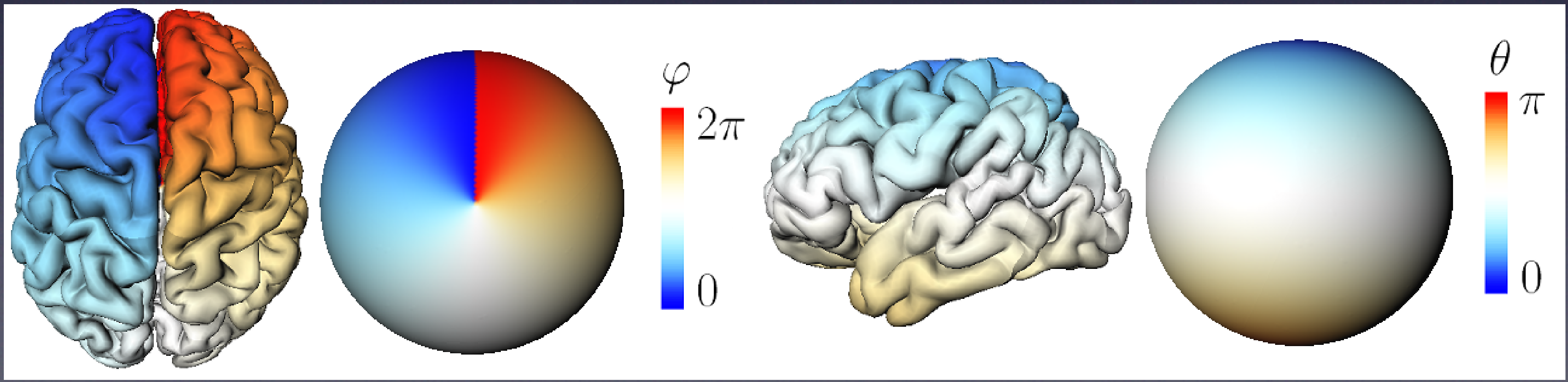
Our method ↓

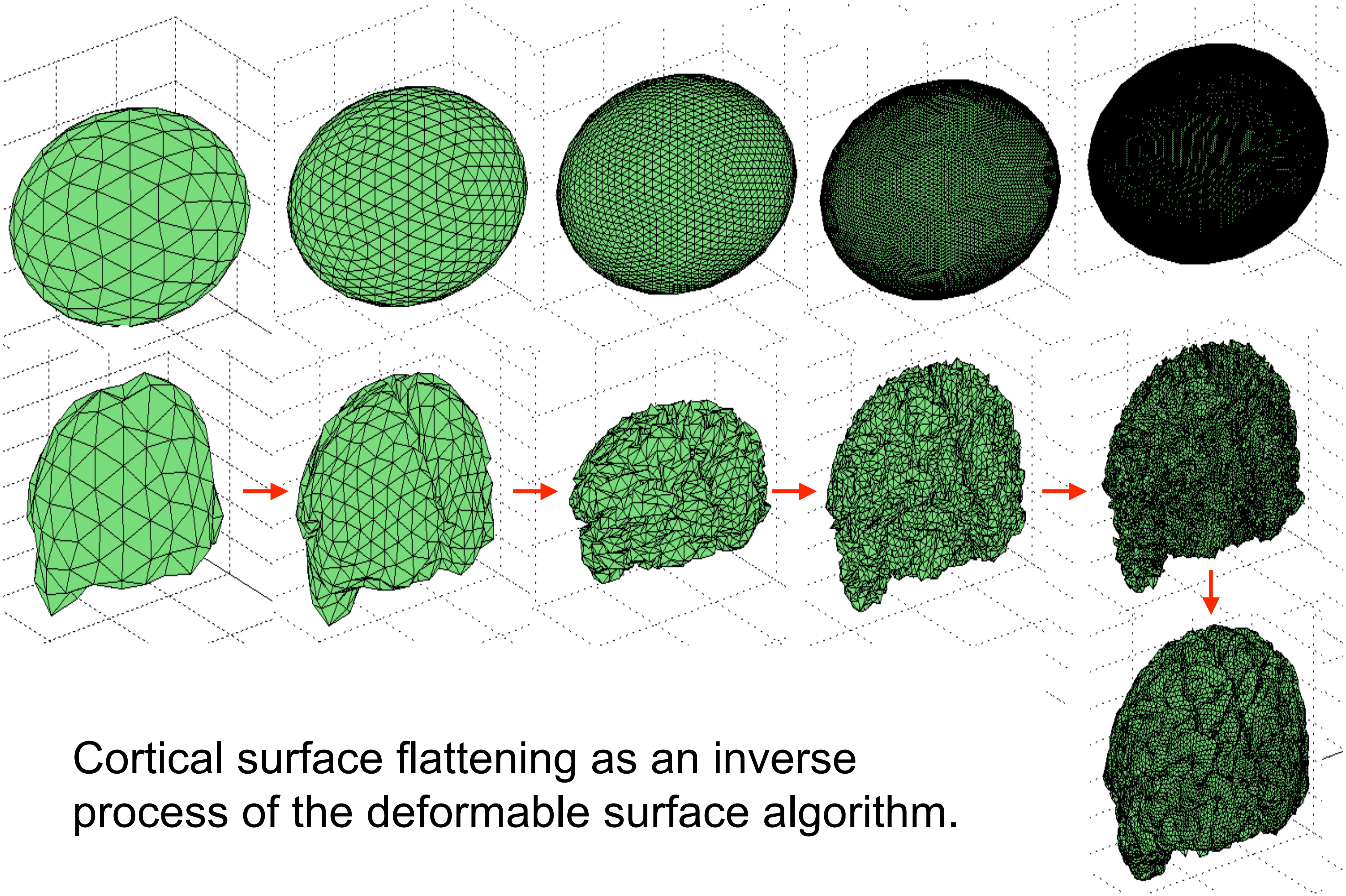
20,000 parameters per surface

Cortical surface flattening

•Deformable surface algorithm (McDonalds et al., 2001) is used to segment surfaces and obtain the mapping from a unit sphere to a cortical surface.

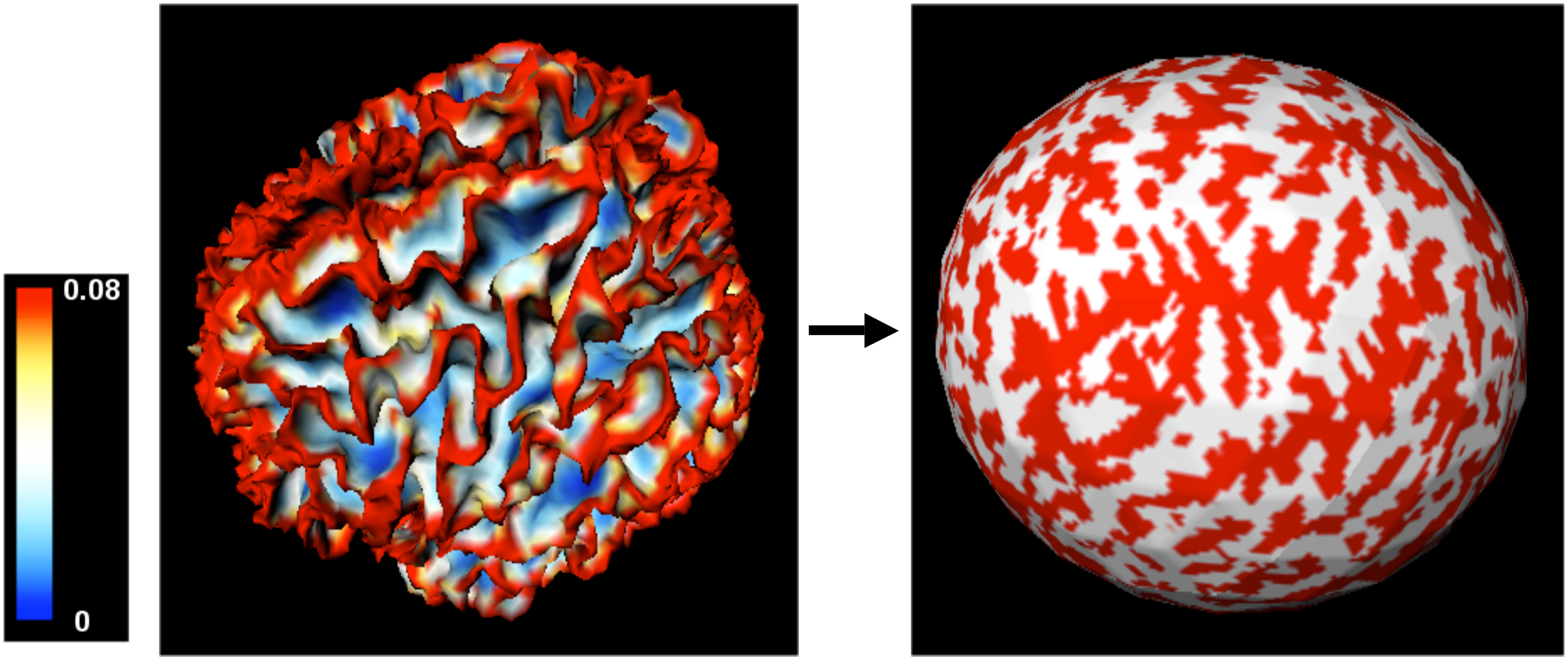
•Functional measurement defined on cortical surface will be pulled back onto the unit sphere.





Cortical surface flattening as an inverse process of the deformable surface algorithm.

Example of functional measurement pulled back onto unit sphere



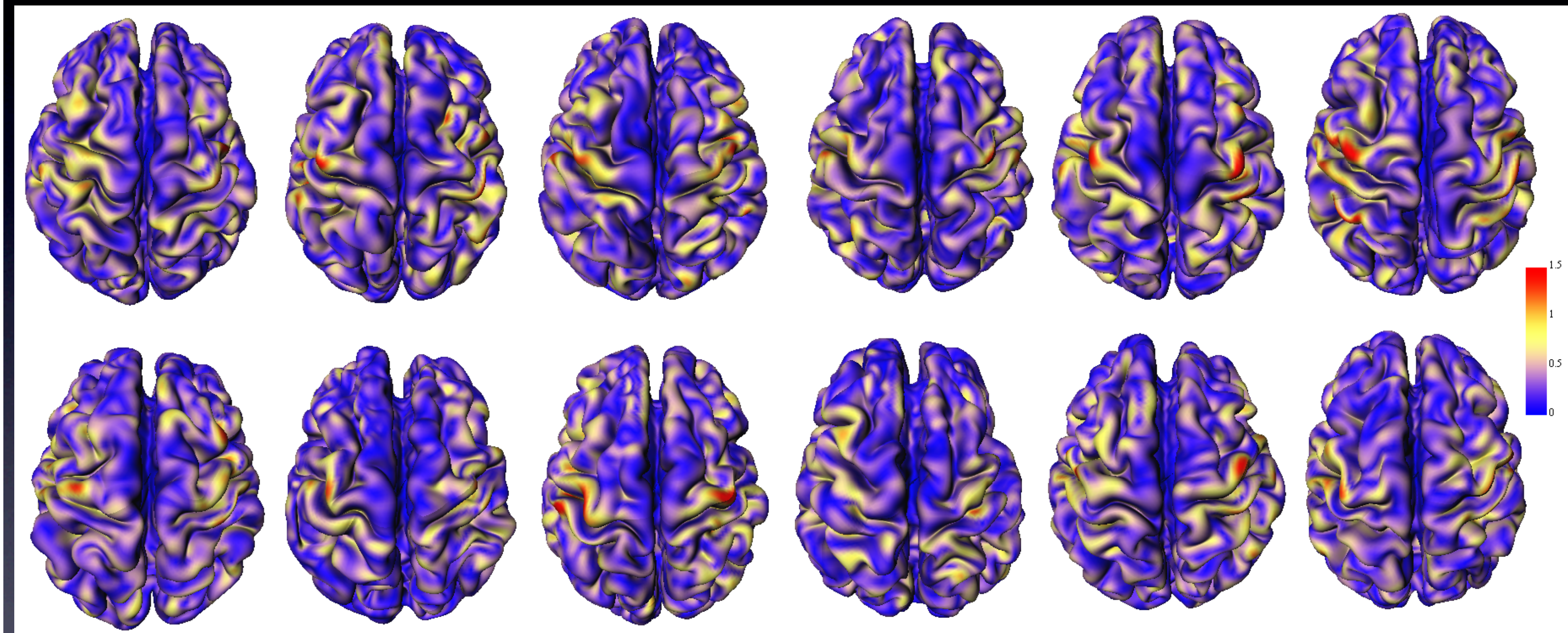
Sum of principal curvatures

Cortical flat map

Note: metric distortion might influence the final statistical analysis.

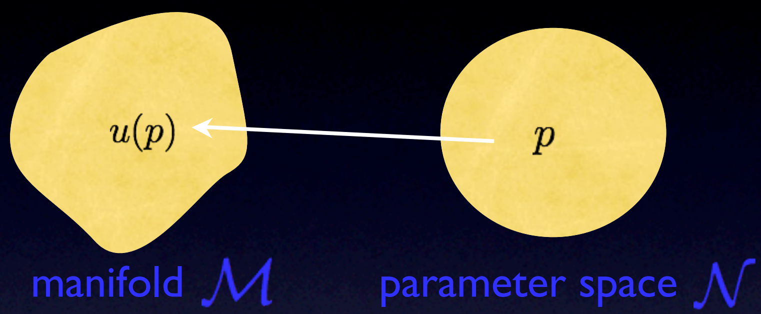
There is a way to address area distortion

Local area element can be obtained by analytically differentiating WFS and computing metric tensors.



This measures amount of area distortion associated with cortical flattening. It can be used as a nuisance covariate in a statistical analysis.

Cortical manifold and function measurement defined on the manifold



Anatomical manifold $\mathcal{M} \in \mathbb{R}^d$

Parameter space $\mathcal{N} \in \mathbb{R}^m$

Parameterization $u : \mathcal{N} \rightarrow \mathcal{M}$

Hilbert space $L^2(\mathcal{N})$ with inner product

$$\langle g_1, g_2 \rangle = \int_{\mathcal{N}} g_1(p) g_2(p) \mu(p)$$

Self-adjoint operator $\mathcal{L}$

$$\langle \mathcal{L} g_1, g_2 \rangle = \langle g_1, \mathcal{L} g_2 \rangle$$

→

Basis function

$$\mathcal{L} \psi_j = \lambda_j \psi_j$$

Weighted Fourier Series (WFS) representation

cortical thickness
(function)
+ surface coordinates
(surface)

t = scale, bandwidth

PDE

$$\partial_t g + \mathcal{L} g = 0, g(p, t = 0) = f(p)$$

solution

Spectral representation

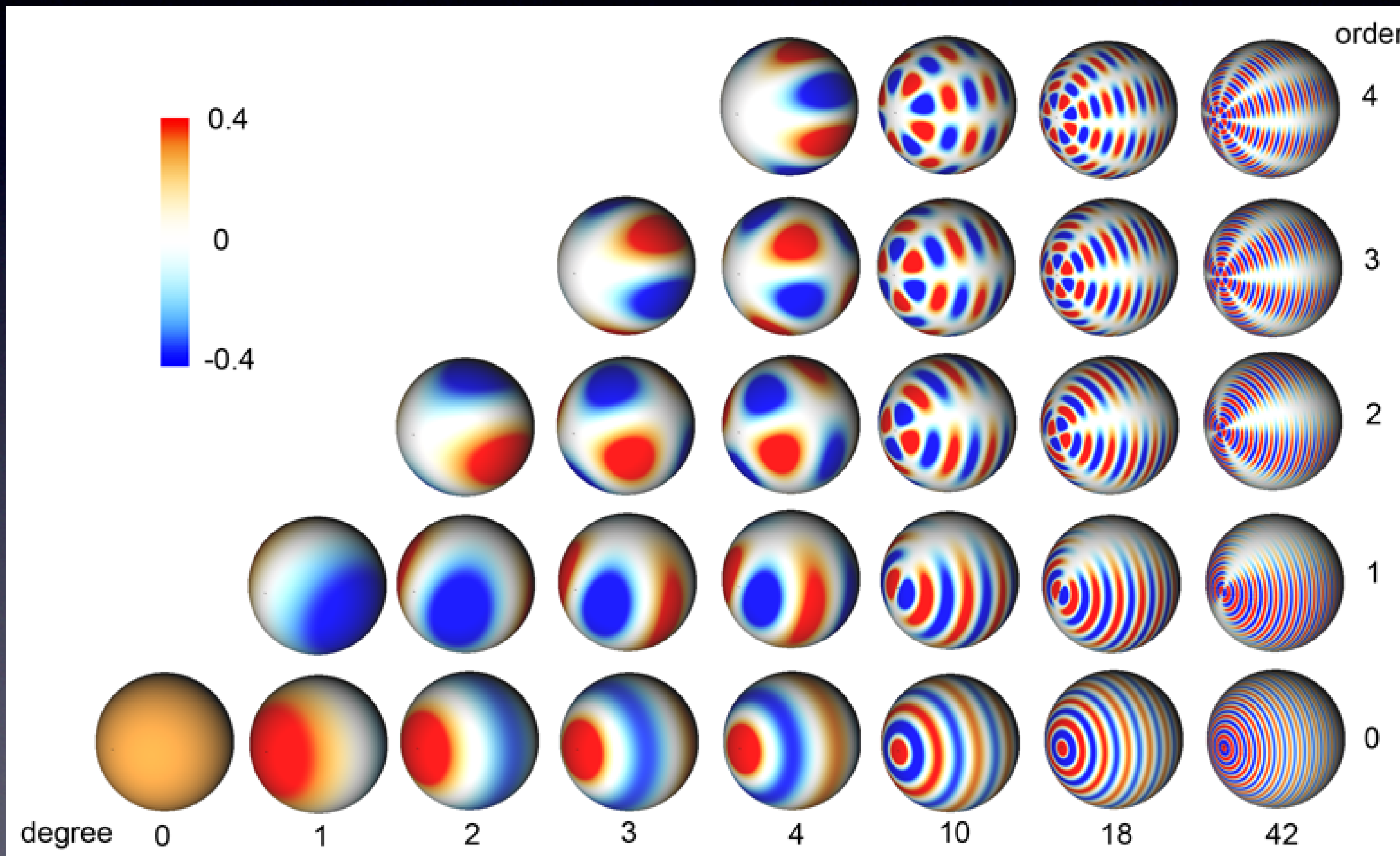
$$g(p,t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p)$$

Kernel smoothing

$$= \int_{\mathcal{N}} K_t(p,q) f(q) \; d\mu(q)$$

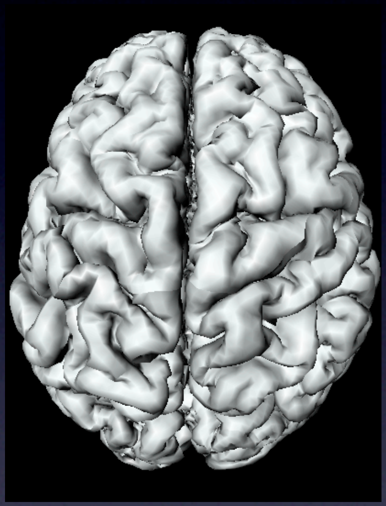
Spherical harmonic of degree *l* and order *m*

$$Y_{lm} = \begin{cases} c_{lm} P_l^{|m|}(\cos\theta)\sin(|m|\varphi), & -l \leq m \leq -1, \\ \frac{c_{lm}}{\sqrt{2}} P_l^0(\cos\theta), & m = 0, \\ c_{lm} P_l^{|m|}(\cos\theta)\cos(|m|\varphi), & 1 \leq m \leq l, \end{cases}$$

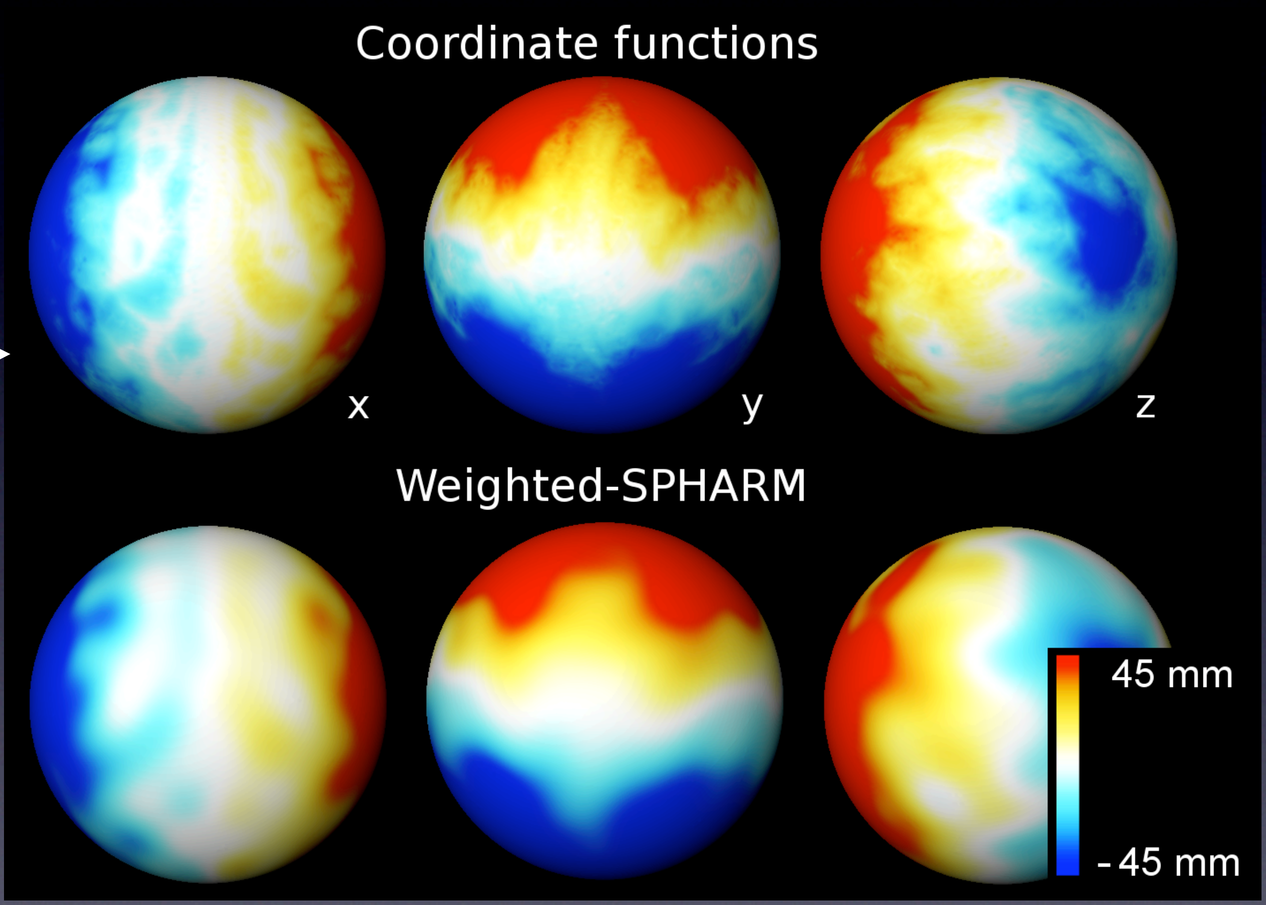


Coarse detail ⟷ Fine detail

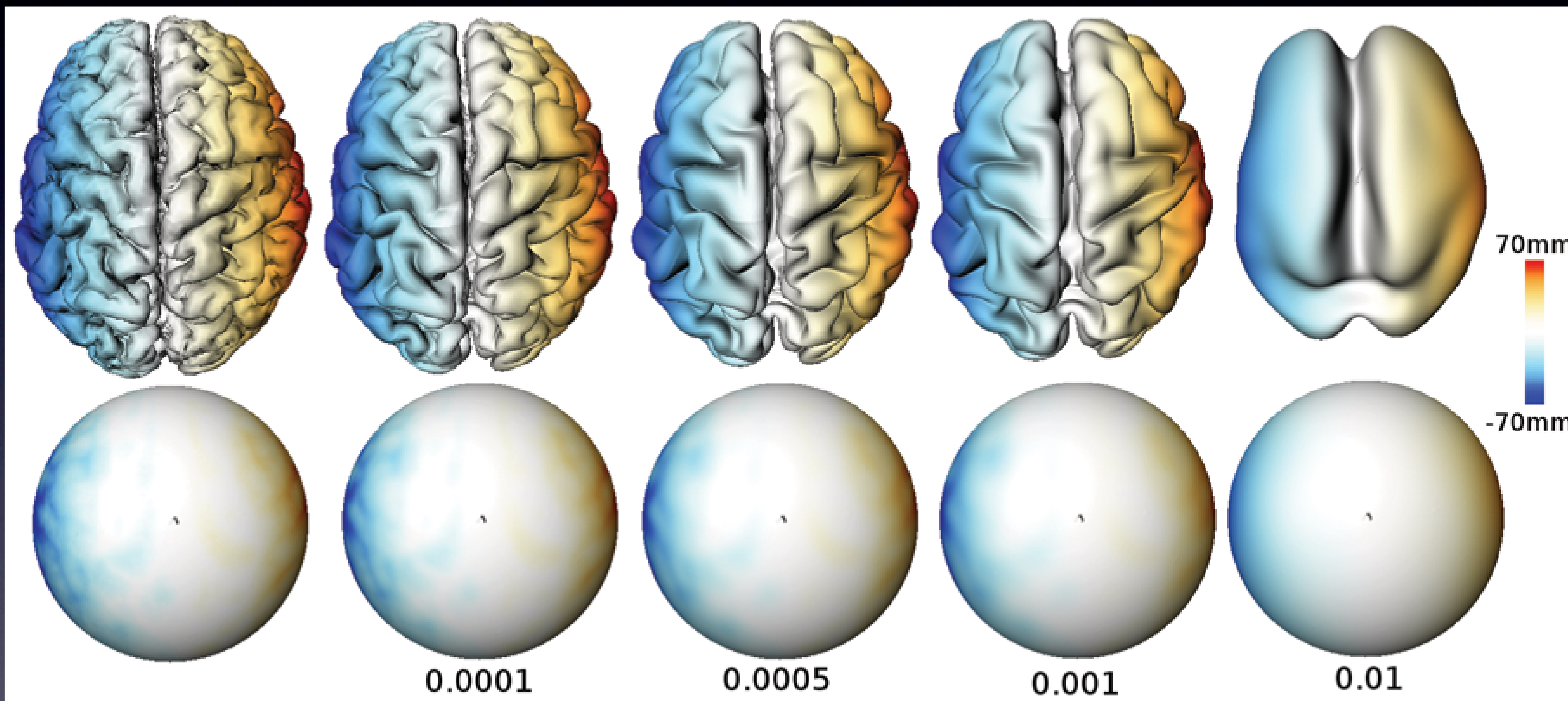
WFS = parameterization + smoothing



Original cortical surface



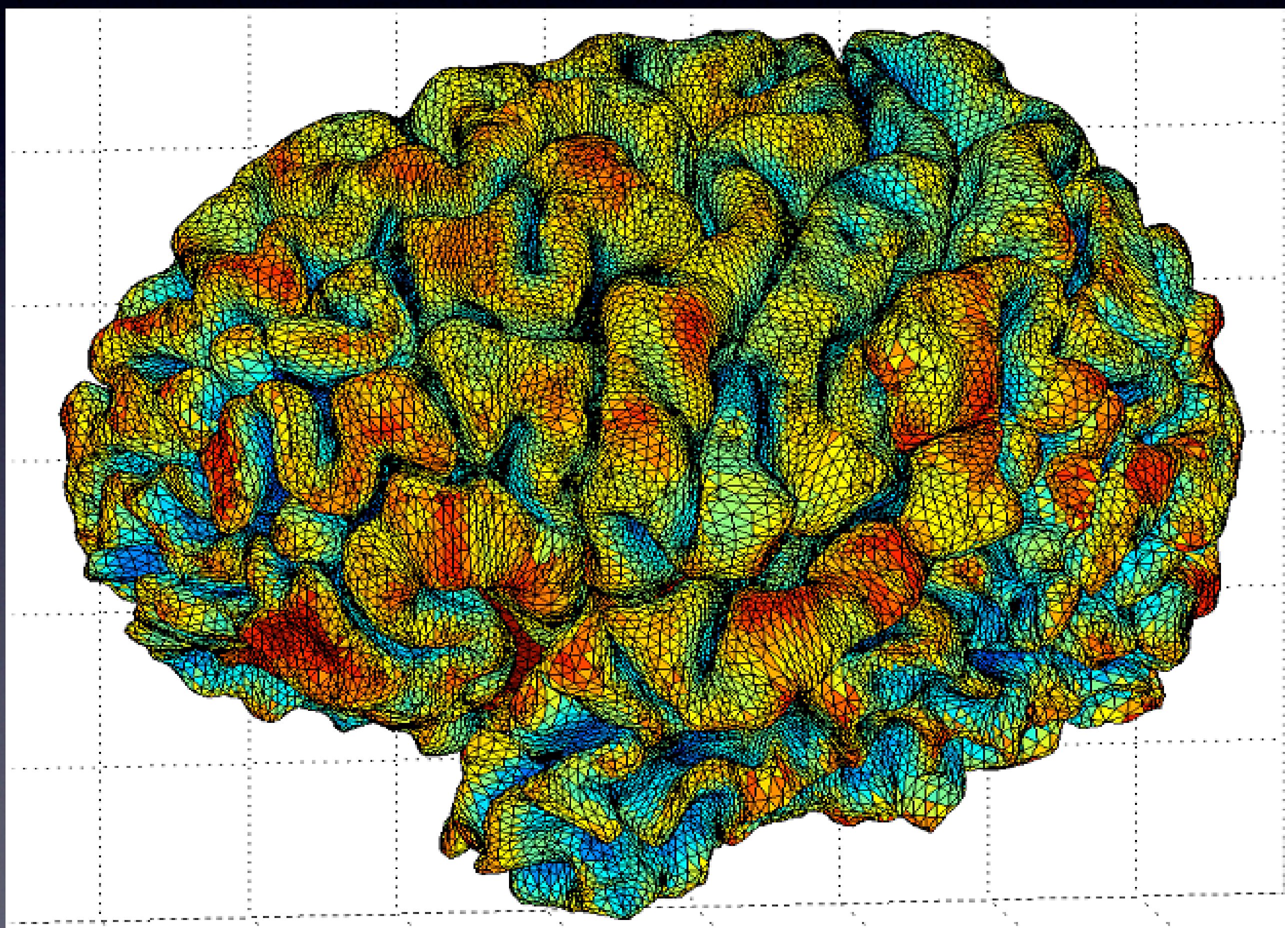
WFS = multiscale representation



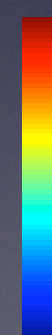
Color scale= x-coordinate

WFS can be applied to functional data like cortical thickness.

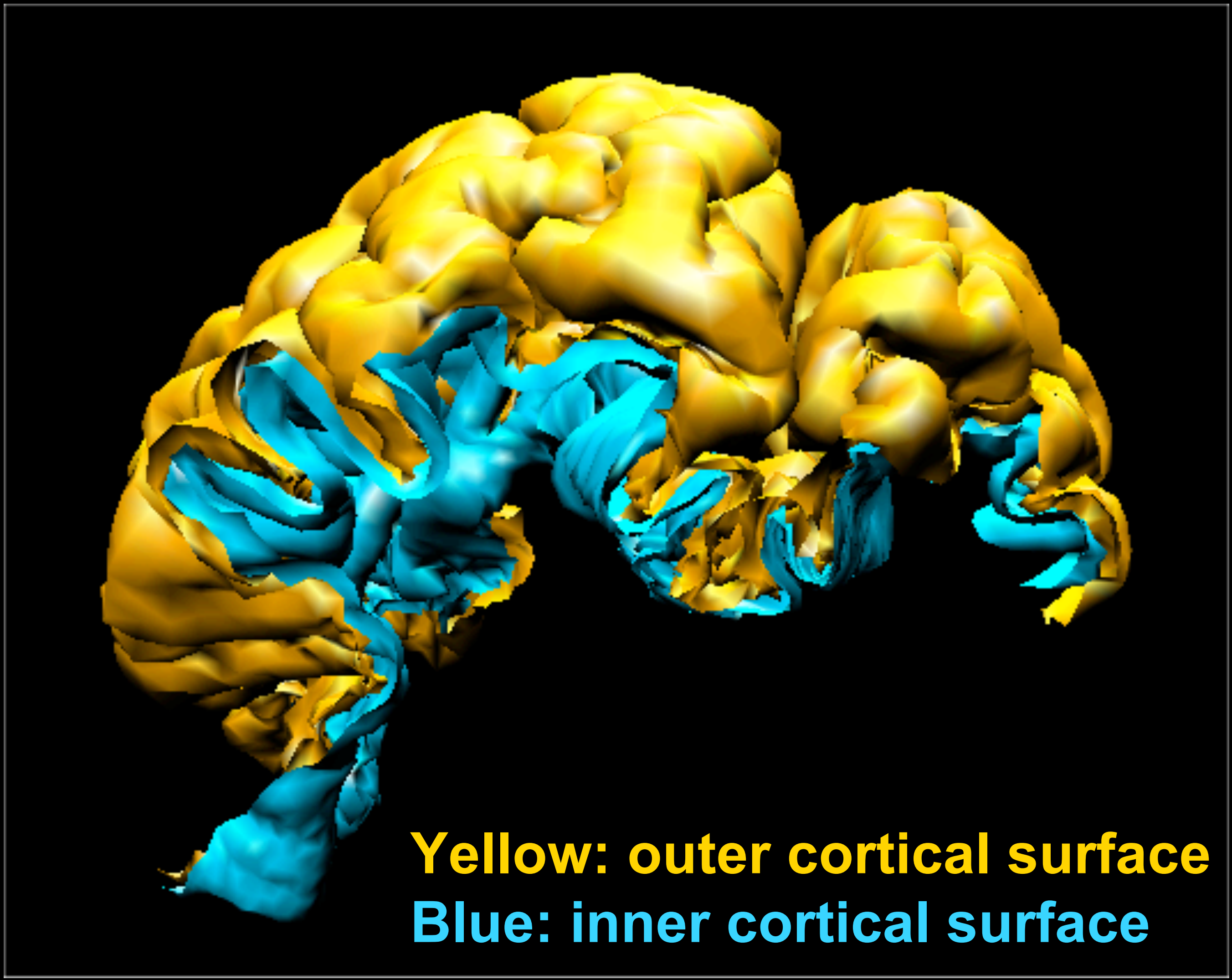
Cortical thickness = most widely used cortical structural measure



Cortical thickness

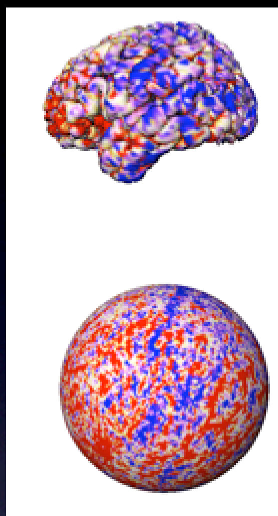


What is cortical thickness ?



- distance between surfaces
- measures amount of gray matter bounded by these two surfaces

WFS of cortical thickness

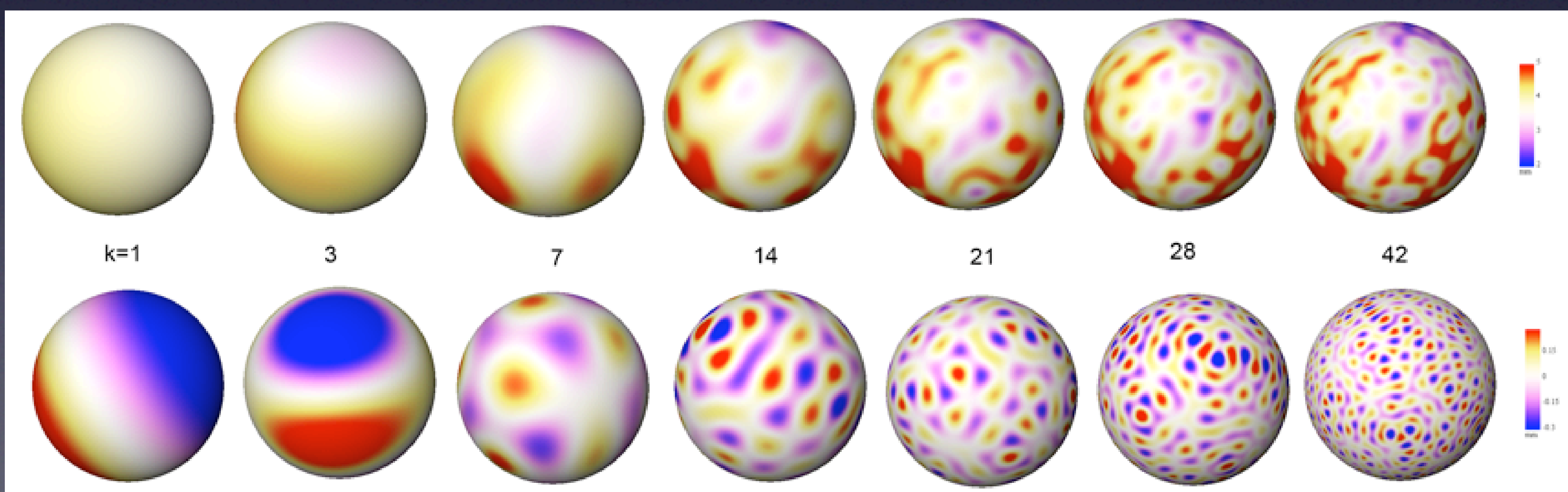


Cortical thickness

Pull back

1st row: $$\sum_{l=0}^{k}\sum_{m=-l}^{l} e^{-l(l+1)t}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$

2nd row: $$\sum_{m=-k}^{k} e^{-l(l+1)t}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$



Iterative residual fitting (IRF) algorithm (TMI, 2007)

Can estimate more than 20,000 coefficients per surface

Joint work with Li Shen

Step 1. measurements $f(p_1), \cdots, f(p_n)$

Step 2. Set initial degree=0 $k = 0$

Step 3. Solve $f(p_i) = \sum_{m=-k}^{k} \beta_{km} Y_{km}(p_i)$

Project data into a finite subspace

Iterate

Step 3.5. $f \leftarrow f - \hat{f}$

Once low frequency parts are estimated, we throw them away

Step 4. Set degree $k \leftarrow k + 1$

MATLAB code available at http://www.stat.wisc.edu/~mchung/

Similar method in literature: Matching pursuit (MP) method (Mallat and Zhang, 1993, IEEE Trans. Signal Processing)

MR is identical to IRF in principle except the methods for estimating Fourier coefficients are different.

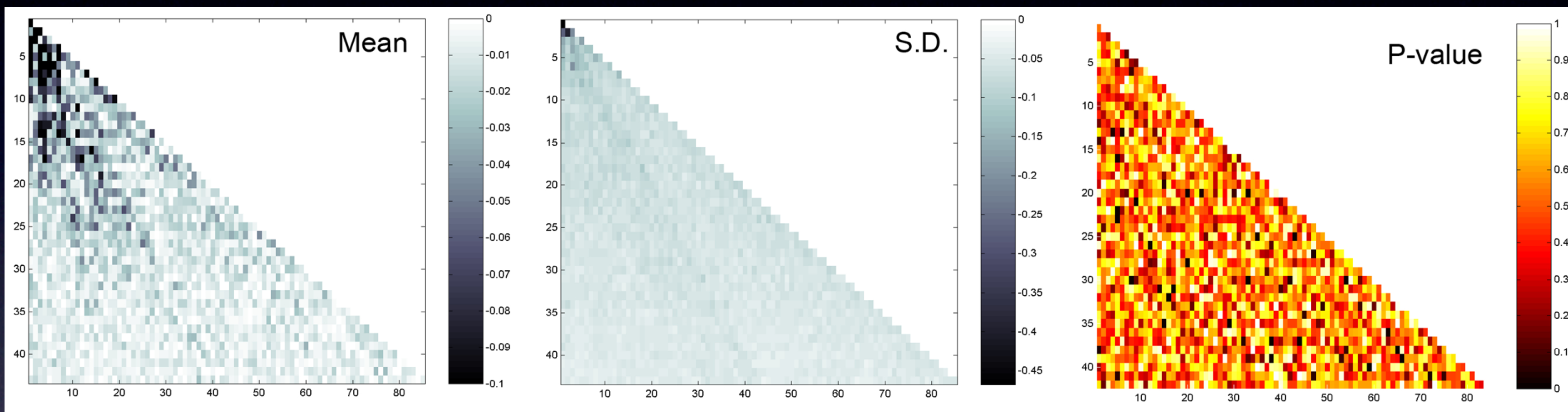
IRF was developed to solve a large linear system (with othonormality contrain) iteratively.

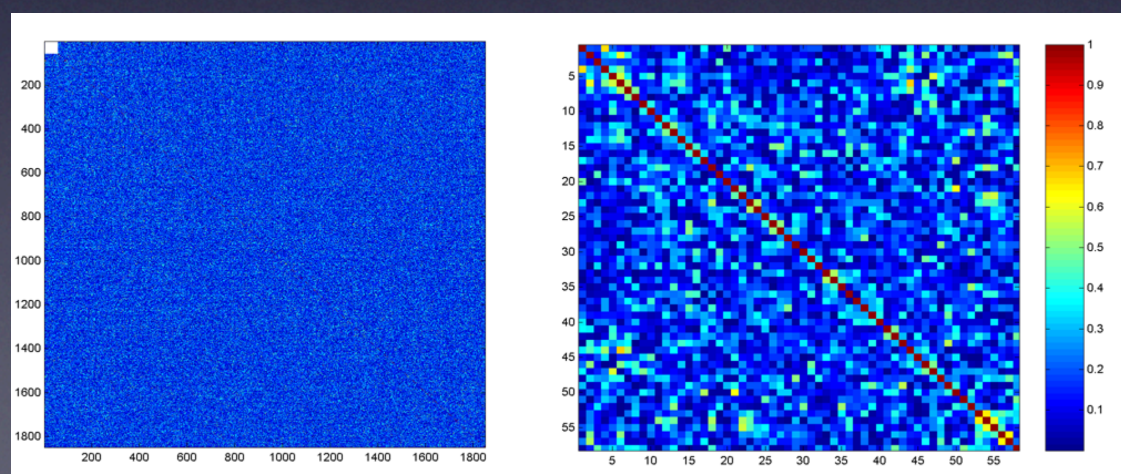
MP was developed as a way to compactly decompose time frequency signal into a linear combination of basis in a dictionary.

Statistical Model on WFS: Karhunen-Loeve expansion

$$\sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)t} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)$$

Uncorrelated normal



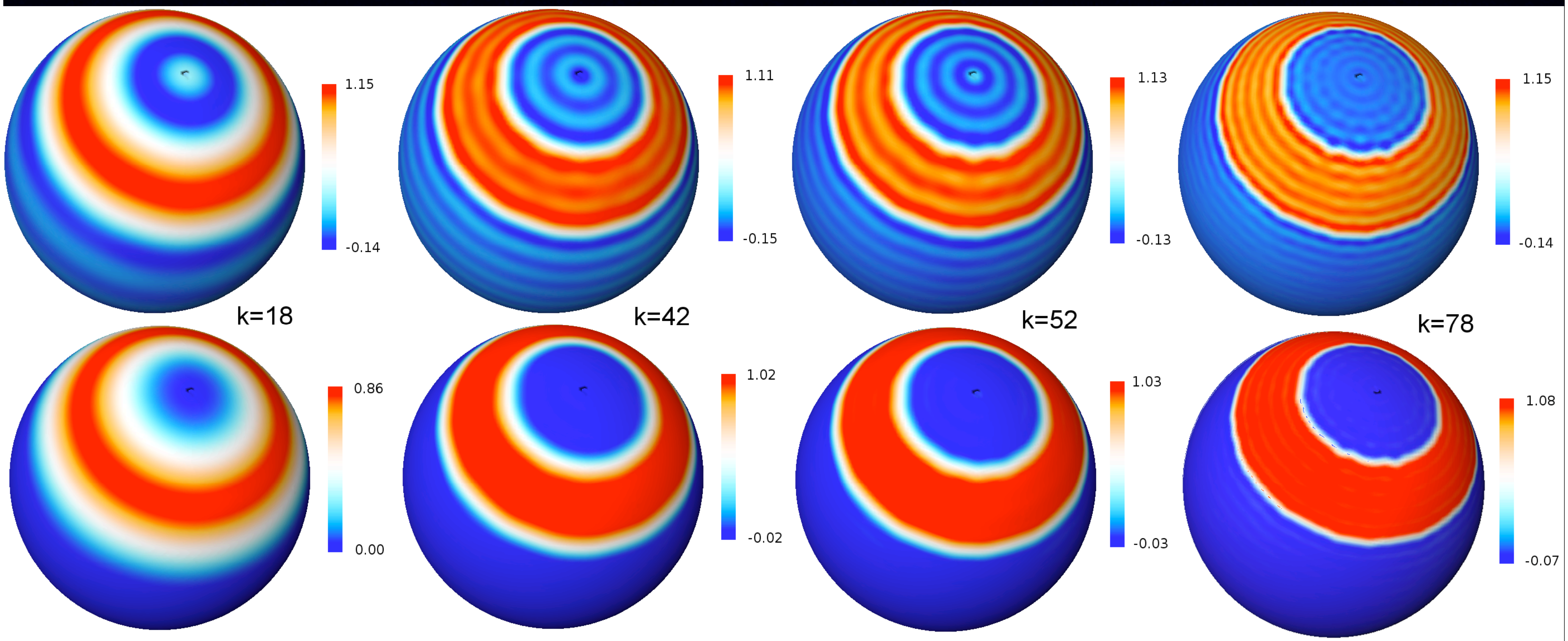


Cross correlations

Why WFS ?

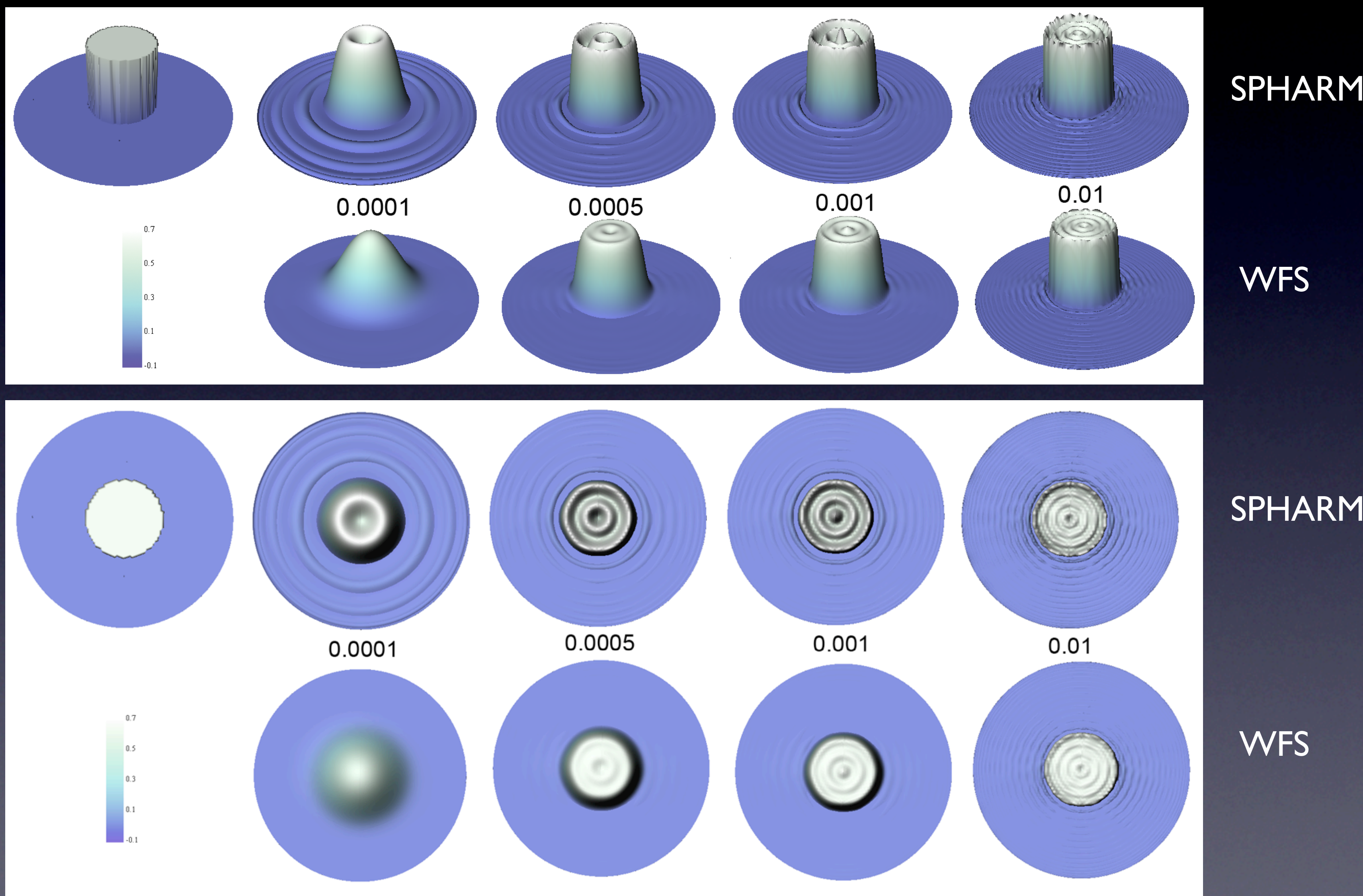
Reduction of Gibbs phenomenon (ringing artifacts)

Functional data defined on sphere



Top: Fourier series expansion (SPHARM)
Bottom: WFS

Why WFS ? Gibbs phenomenon
Anatomical data



Surface registration via WFS

Given two *l*-th degree WFS surfaces v_{i1}, v_{i2}

find the displacement d_i that minimizes the discrepancy between two surfaces:

$$v_{i2} - v_{i1} = \arg \min_{d_i \in \mathcal{H}_l} \int_{\mathcal{M}} \left[v_{i1} + d_i(v_{i1}) - v_{i2} \right]^2 d\mu(p).$$

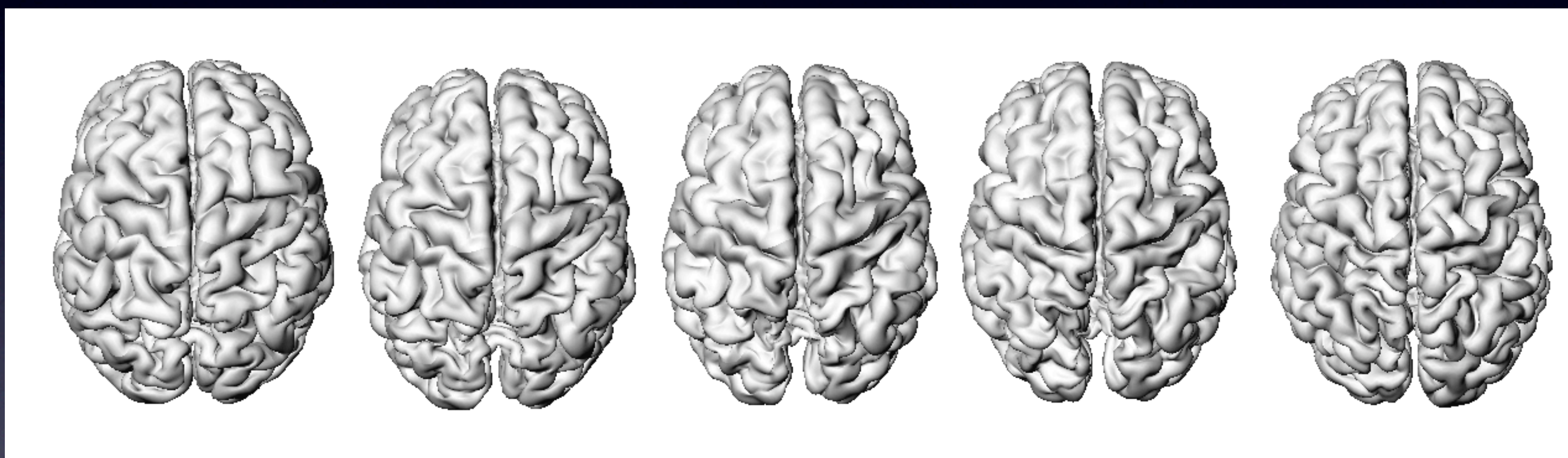
$\mathcal{H}_l$: subspace spanned by up to *l*-th degree spherical harmonics

$v_{i1} + d_i(v_{i1})$: deformation of coordinates v_{i1}

Consequence: For fixed (θ, φ),

$v_{i1}(\theta, \varphi)$ corresponds to $v_{i2}(\theta, \varphi)$.

Example of surface registration



subject 1

subject 2

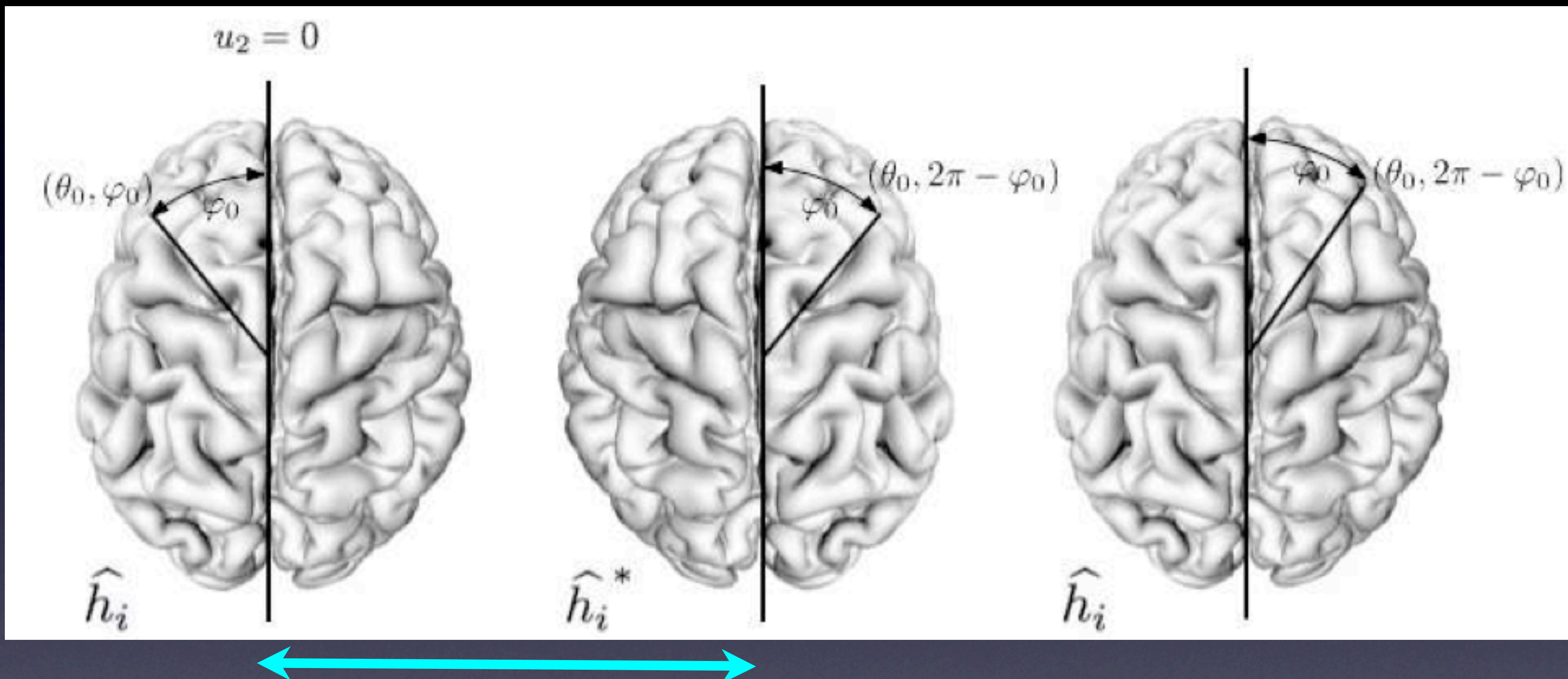
$$\alpha = 0$$

$$v_{i1} + \alpha d_i(v_{i1})$$

$$\alpha = 1$$

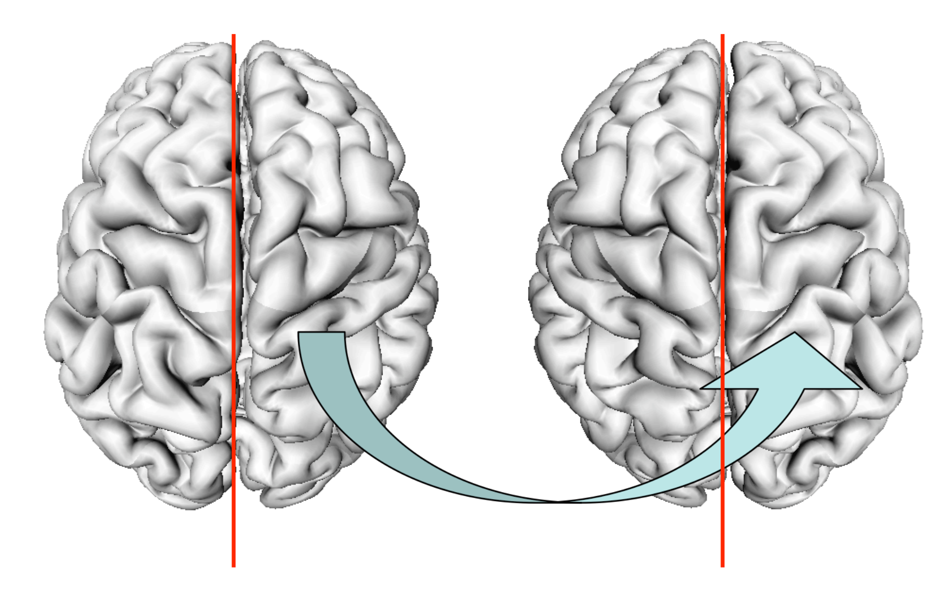
Cortical asymmetry analysis

Establishing hemispheric correspondence algebraically



Mirror reflection: It is done algebraically on WFS

Surface registration



Invariance under mirror reflection

$$\widehat{g}(\theta,\varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi).$$

Mirror reflection

$$\widehat{g}(\theta,2\pi-\varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{-1} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi) - \sum_{l=0}^{k} \sum_{m=0}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi)$$

Shape decomposition into symmetric and asymmetric parts

Symmetric part

$$S(\theta,\varphi) = \frac{1}{2}\Big[\widehat{g}(\theta,\varphi) + \widehat{g}(\theta, 2\pi-\varphi)\Big] = \sum_{l=0}^{k}\sum_{m=-l}^{-1} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$

Asymmetric part

$$A(\theta,\varphi) = \frac{1}{2}\Big[\widehat{g}(\theta,\varphi) - \widehat{g}(\theta, 2\pi-\varphi)\Big] = \sum_{l=0}^{k}\sum_{m=0}^{l} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$

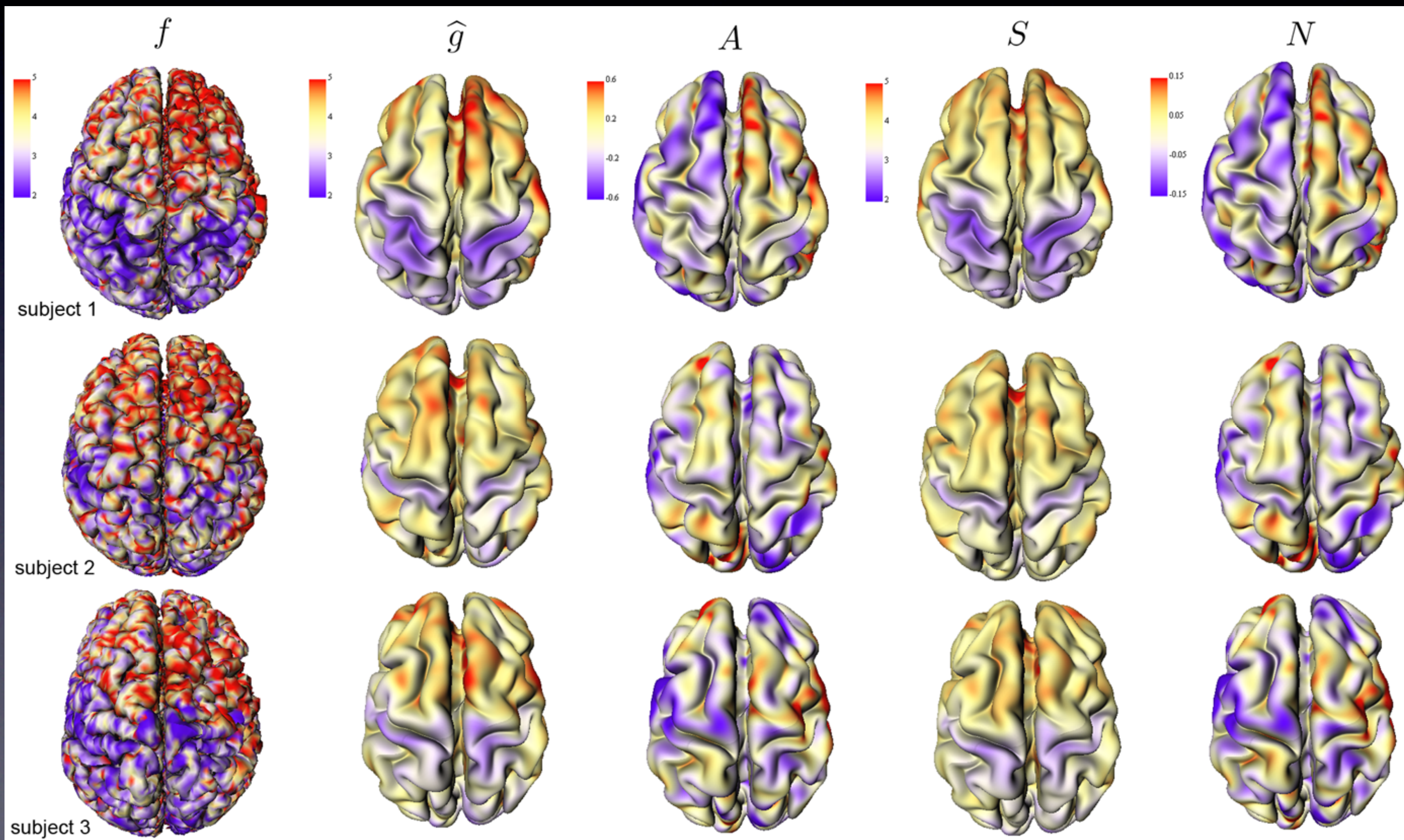
Normalized asymmetry index

$$N(\theta,\varphi) = \frac{\widehat{g}(\theta,\varphi) - \widehat{g}(\theta,2\pi-\varphi)}{\widehat{g}(\theta,\varphi) + \widehat{g}(\theta,2\pi-\varphi)} = \frac{\sum_{l=1}^{k}\sum_{m=-l}^{-1} e^{-1(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)}{\sum_{l=0}^{k}\sum_{m=0}^{l} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)}$$

Ratio of negative and positive degree expansions

Asymmetry index

Cortical thickness	Weighted SPHARM	Asymmetry index	Symmetry index	Normalized asymmetry index



Multiple comparisons

$$H_0 : \theta_1(p) = \theta_2(p) \text{ for all } p \in \partial\Omega$$

v.s.

$$H_1 : \theta_1(p) > \theta_2(p) \text{ for some } p \in \partial\Omega.$$

The above hull hypothesis is the intersection of collection of hypothesis

$$H_0 = \bigcap_{p \in \partial\Omega} H_0(p)$$

Type I error

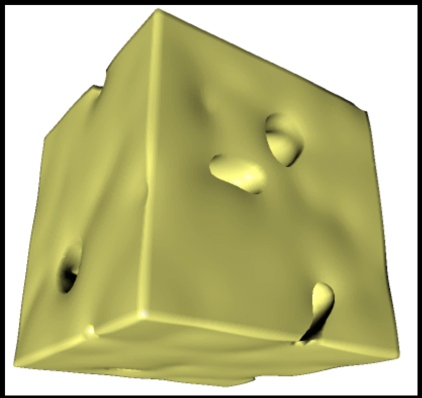
$$\begin{aligned}
\alpha &= P(\text{ reject at least one } H_0(p) | H_0 \text{ true }) \\
&= P\Big(\bigcup_{p \in \partial\Omega} \{T(p) > h\}\Big) \\
&= 1 - P\Big(\bigcap_{p \in \partial\Omega} T(p) \le h\}\Big) \\
&= 1 - P(\sup_{p \in \partial\Omega} T(p) \le h) \\
&= P(\sup_{p \in \partial\Omega} T(p) > h).
\end{aligned}$$

***t* random field**

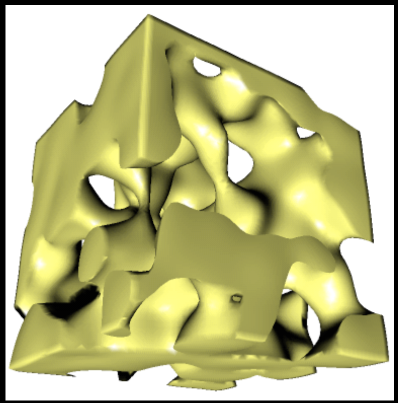
$Z(x)$: Stationary isotropic random field in $x \in \Omega \subset \mathbb{R}^N$

$A_z = \{x : Z(x) > z\}$ excursion set

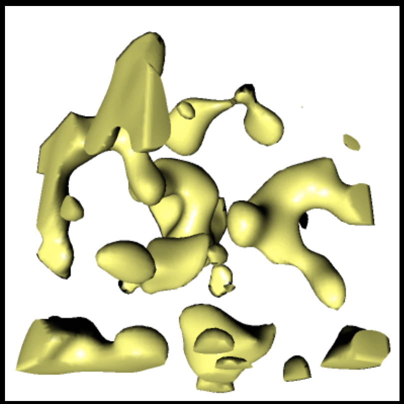
$\chi(A_z)$: Euler characteristic



z = -10



z = 0



z = 10

$$P\left(\max_{x\in\Omega} Z(x) > z\right) \approx \mathbb{E}\left(\chi(A_z)\right)$$

(Adler, 1984)

T random field on manifolds

$$P\Big(\max_{\mathbf{x}\in\partial\Omega_{atlas}} T(\mathbf{x}) \geq y\Big) \approx 2\rho_0(y) + \|\partial\Omega_{atlas}\|\rho_2(y)$$

Euler characteristic density

$$\rho_0(y) = \int_y^\infty \frac{\Gamma(\frac{n}{2})}{((n-1)\pi)^{1/2}\Gamma(\frac{n-1}{2})}\Big(1+\frac{y^2}{n-1}\Big)^{-n/2}\,dy,$$

$$\rho_2(y) = \frac{1}{\mathrm{FWHM}^2}\frac{4\ln 2}{(2\pi)^{3/2}}\frac{\Gamma(\frac{n}{2})}{(\frac{n-1}{2})^{1/2}\Gamma(\frac{n-1}{2})}y\Big(1+\frac{y^2}{n-1}\Big)^{-(n-2)/2}$$

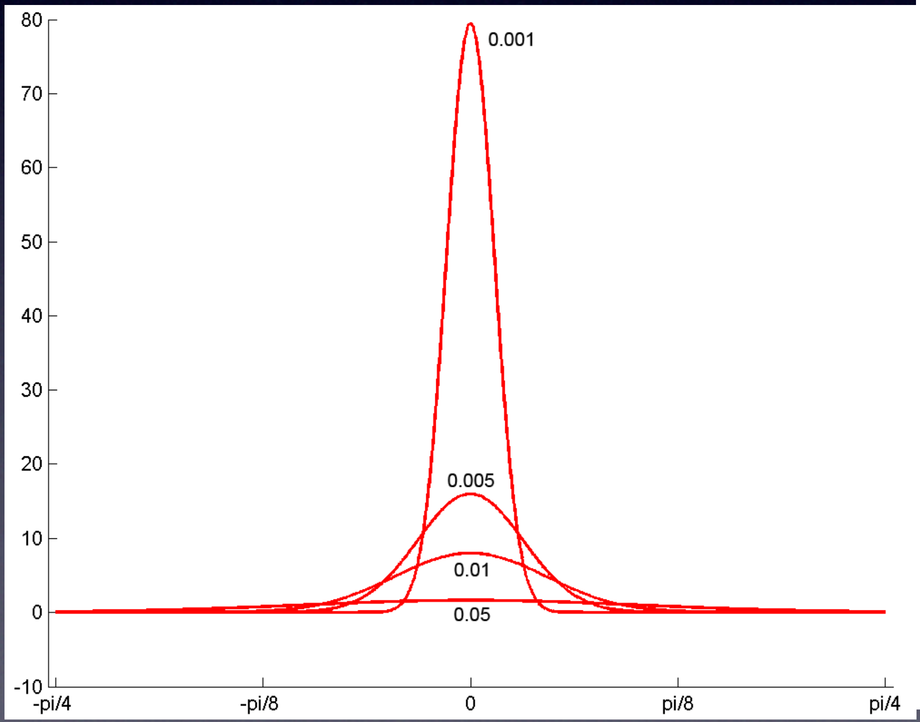
Worsley (1995, NeuroImage)

FWHM of smoothing kernel or residual field

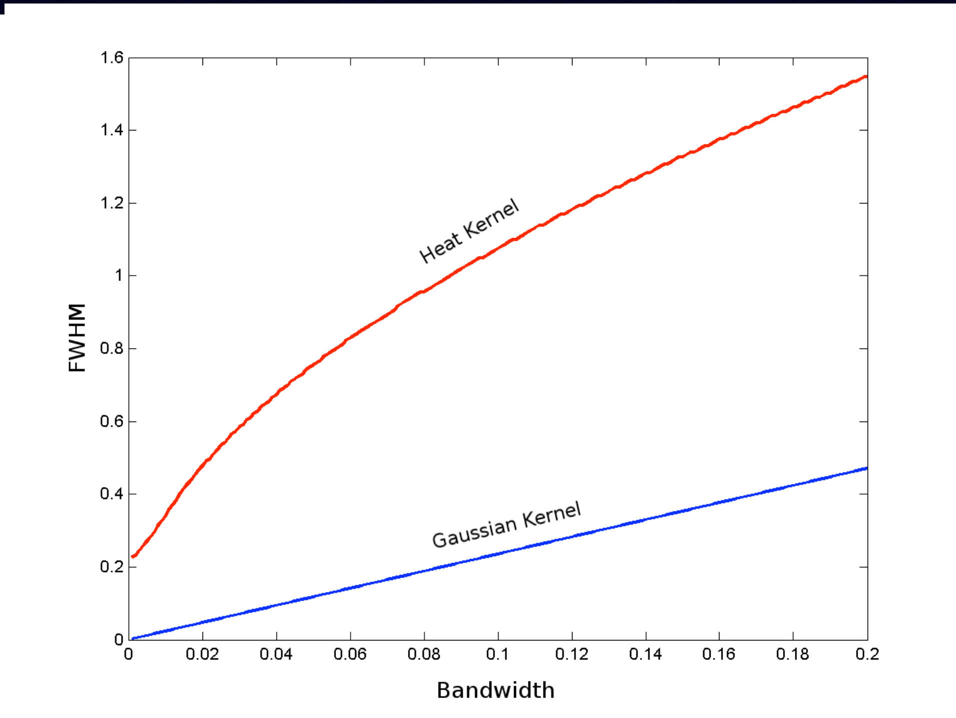
WFS is related to heat kernel smoothing

WFS $$g(p,t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p)$$

Heat kernel smoothing $$= \int_{\mathcal{N}} K_t(p,q) f(q) \, d\mu(q)$$



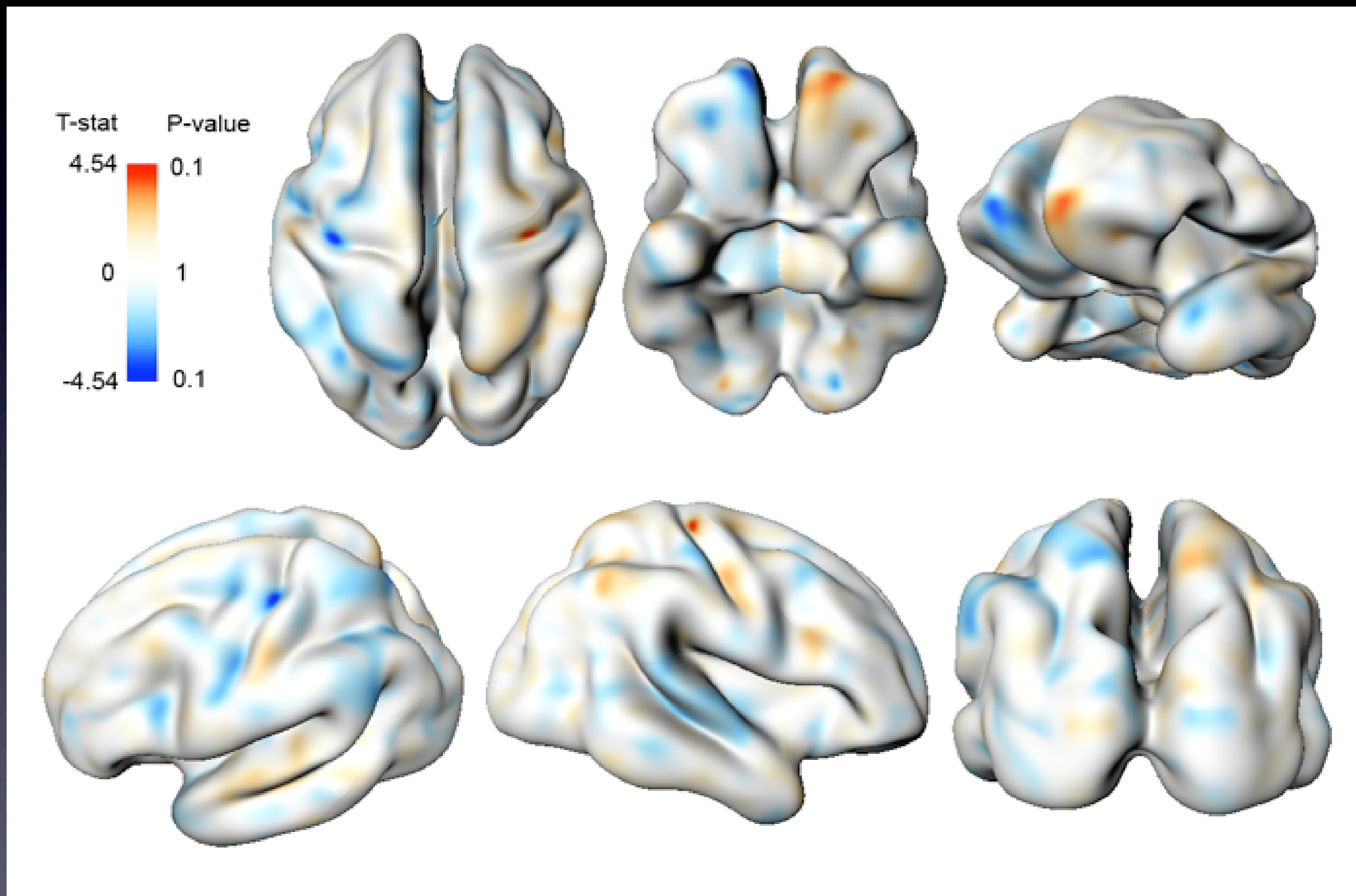
Shape of heat kernel



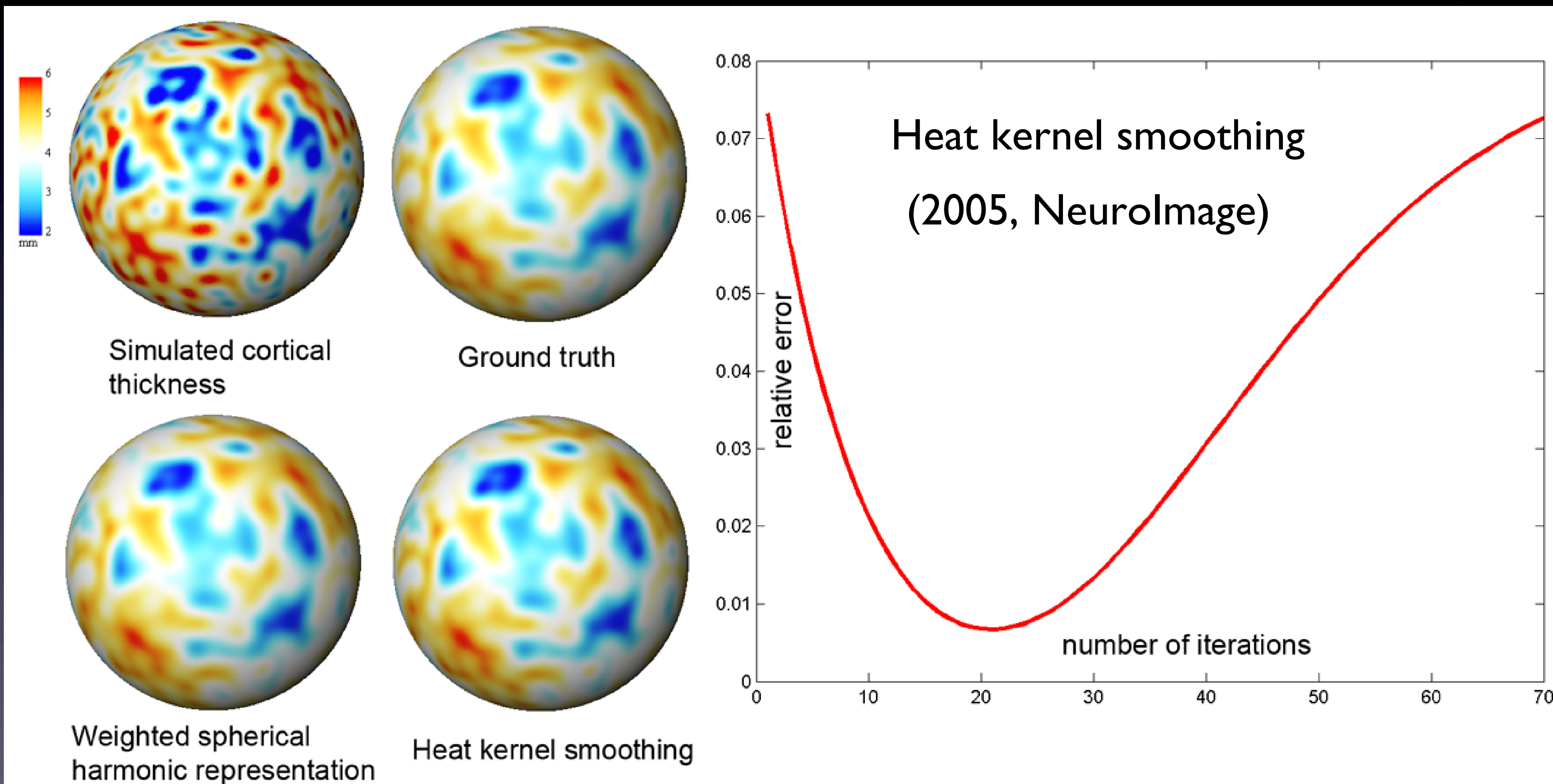
Numerical computation

Statistical parametric map

multiple comparison correction via
the random field theory



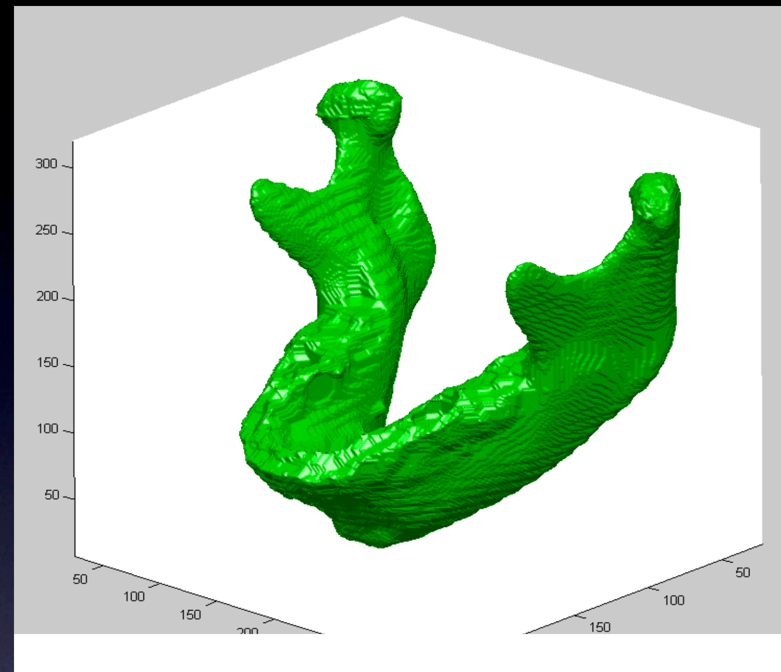
Validation of WFS against analytical ground truth

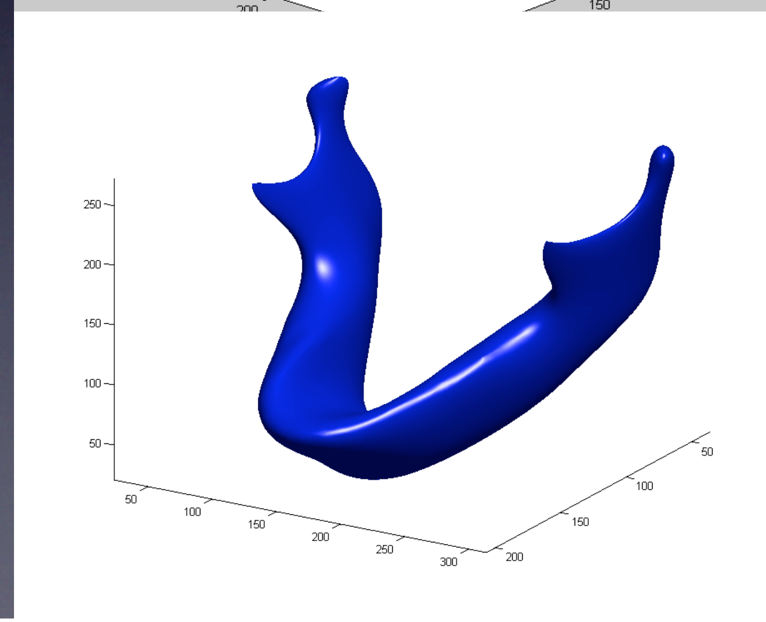


Next project? Mandible surface modeling

Histogram thresholding

Hole patching

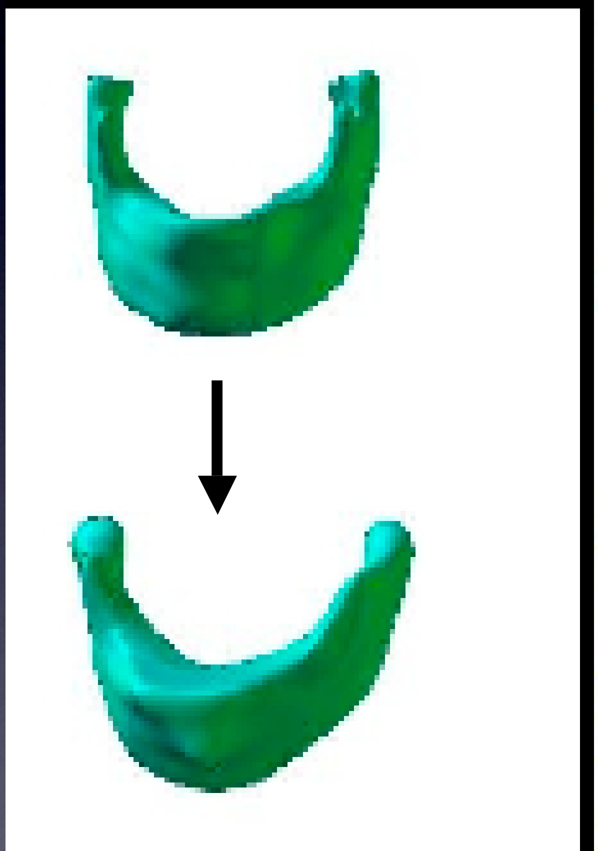


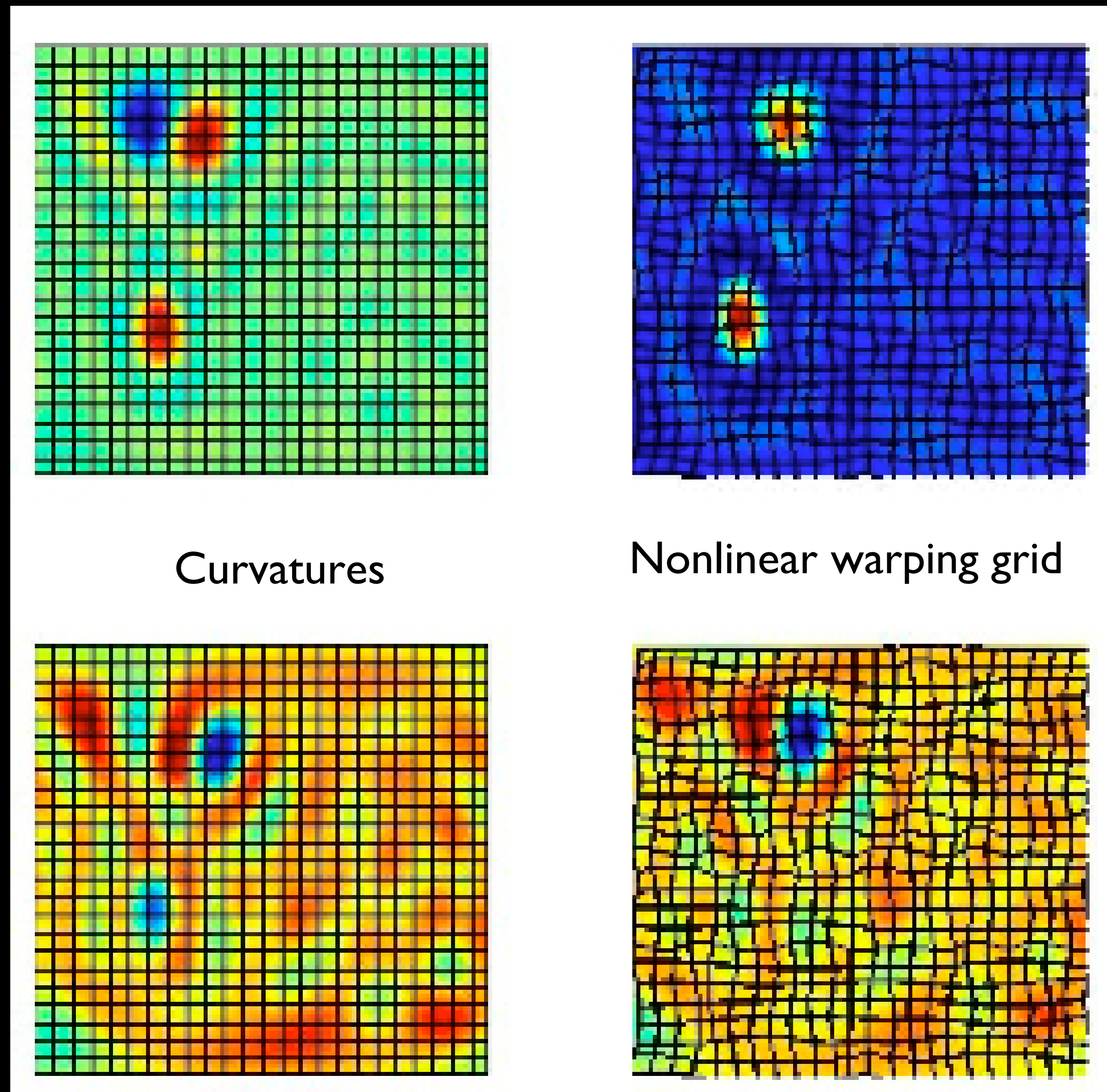


Automatic hole patching is necessary to construct surface topologically equivalent to sphere.

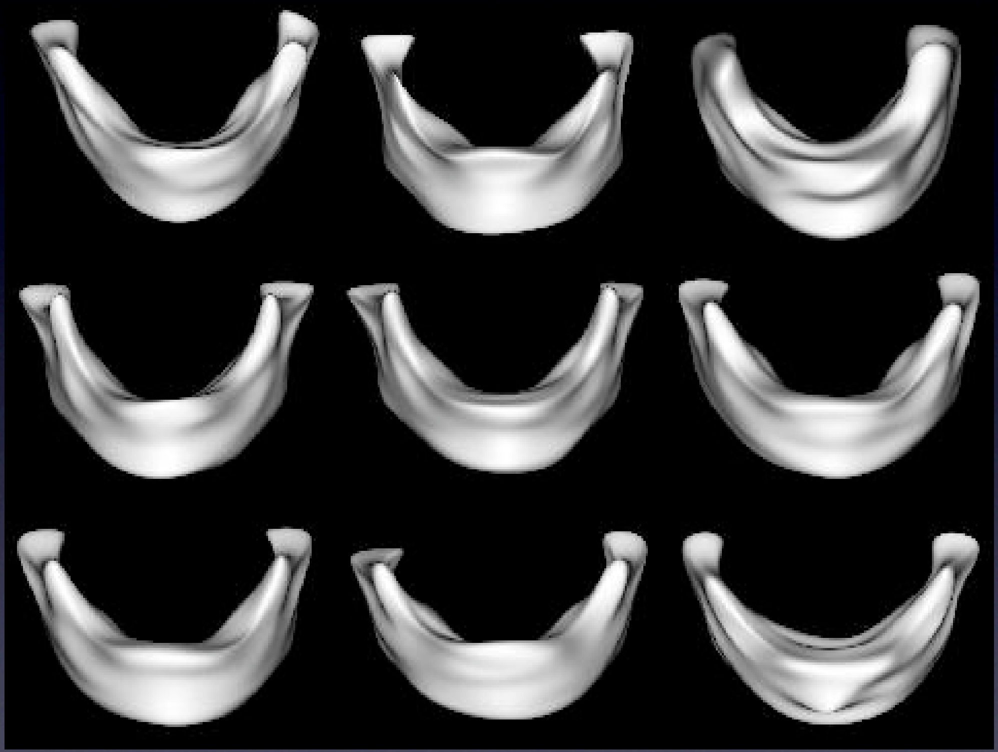
Approximately 20,000 triangle elements

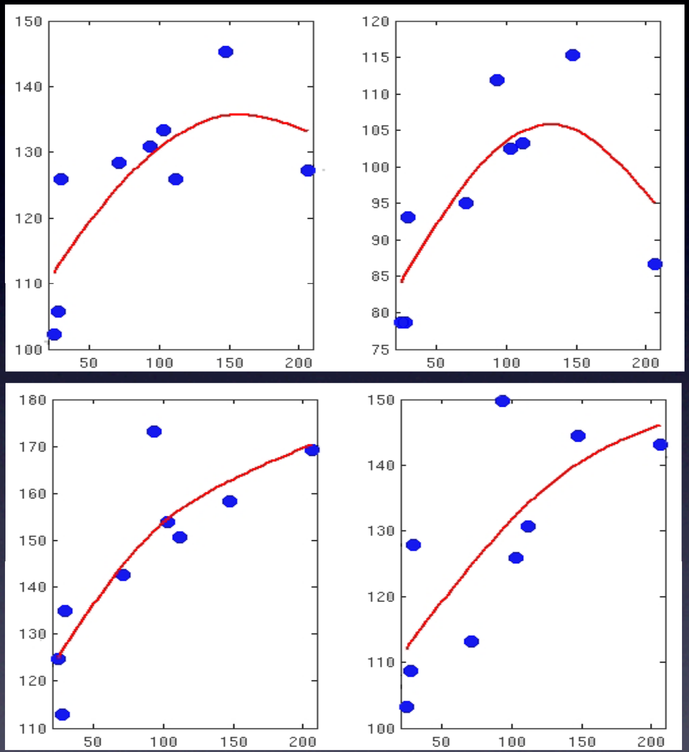
Nonlinear surface registration via curvature matching





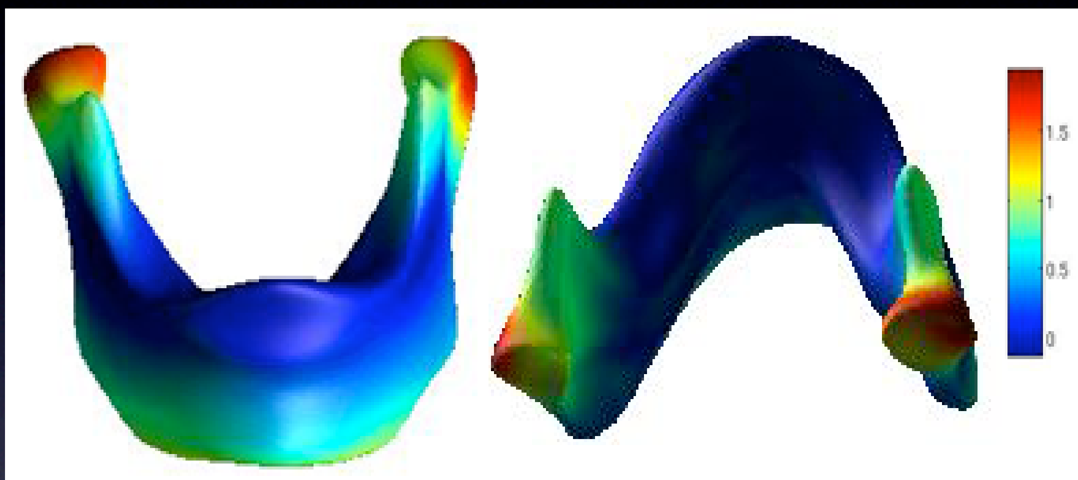
Mandible surface modeling

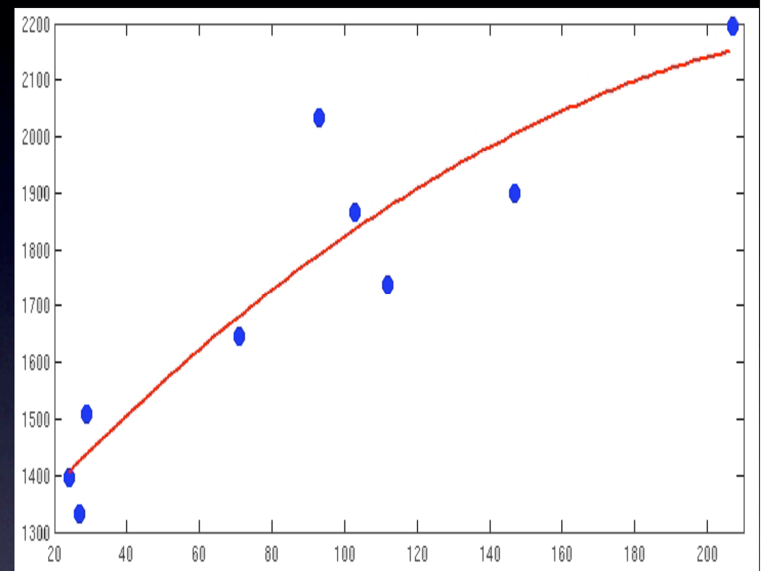




Quadratic fit of 9 male subjects over time in one particular point on the mandible surface Plan: do this on 300 subjects

Locally varying growth rate modeling





Growth rate (obtained directly from the regression model) projected on average mandible surface

Total surface area growth

Plan: incorporate gender and other variables into analysis.