The Waisman Laboratory
for Brain Imaging and Behavior

University of Wisconsin
SCHOOL OF MEDICINE
AND PUBLIC HEALTH

Kernel Regression on Irregular Image Domains

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Abstract

We present the discrete version of heat kernel smoothing on graph data structure. The method is used to smooth data in an irregularly shaped domains in 3D images. New statistical properties of heat kernel smoothing are derived. As an application, we show how to filter out noisy data in the lung blood vessel trees obtained from computed tomography. The method can be further used in representing the complex vessel trees parametrically as a linear combination of basis functions and extracting the skeleton representation of the trees. This talk is based on *Chung et al. 2018. EMBC*.

Acknowledgement

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University of North Carolina-Chapel Hill

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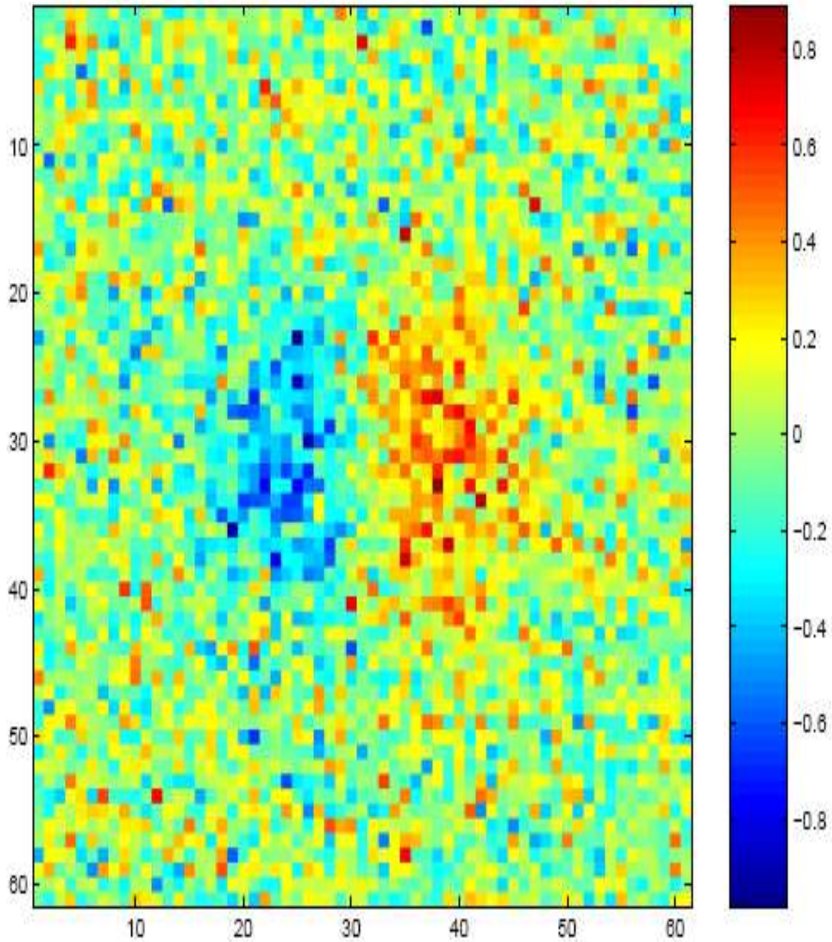
Institute of Applied Physics and Computational Mathematics, Beijing

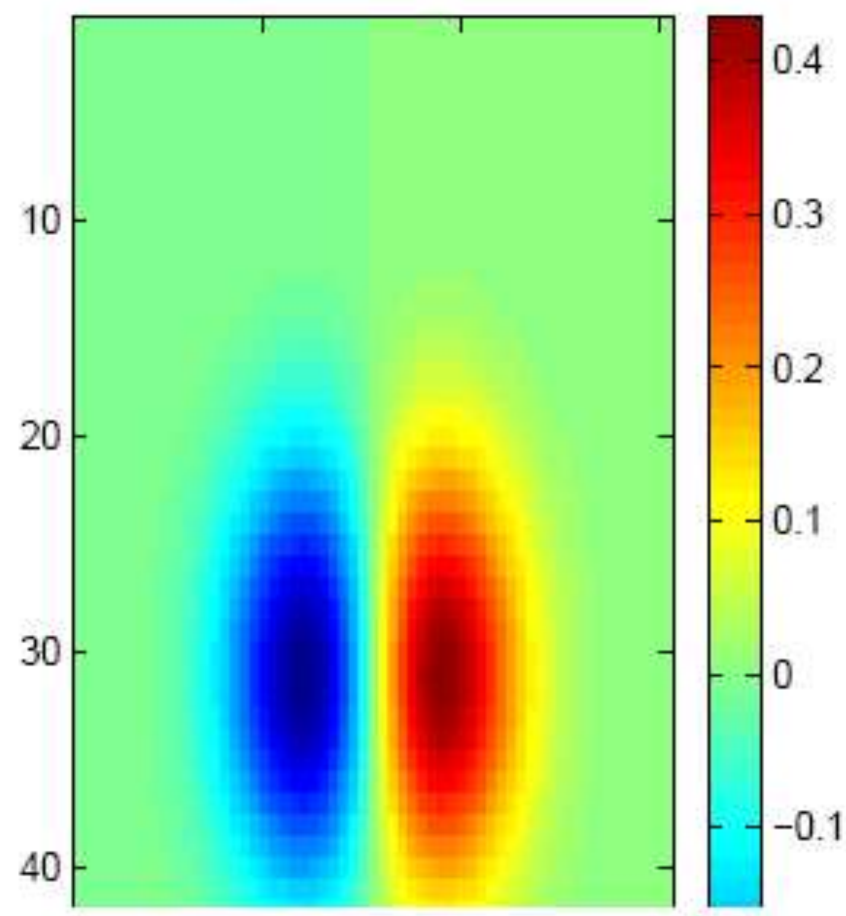
Gaussian kernel smoothing

$$Y(p) = \mu(p) + e(p)$$

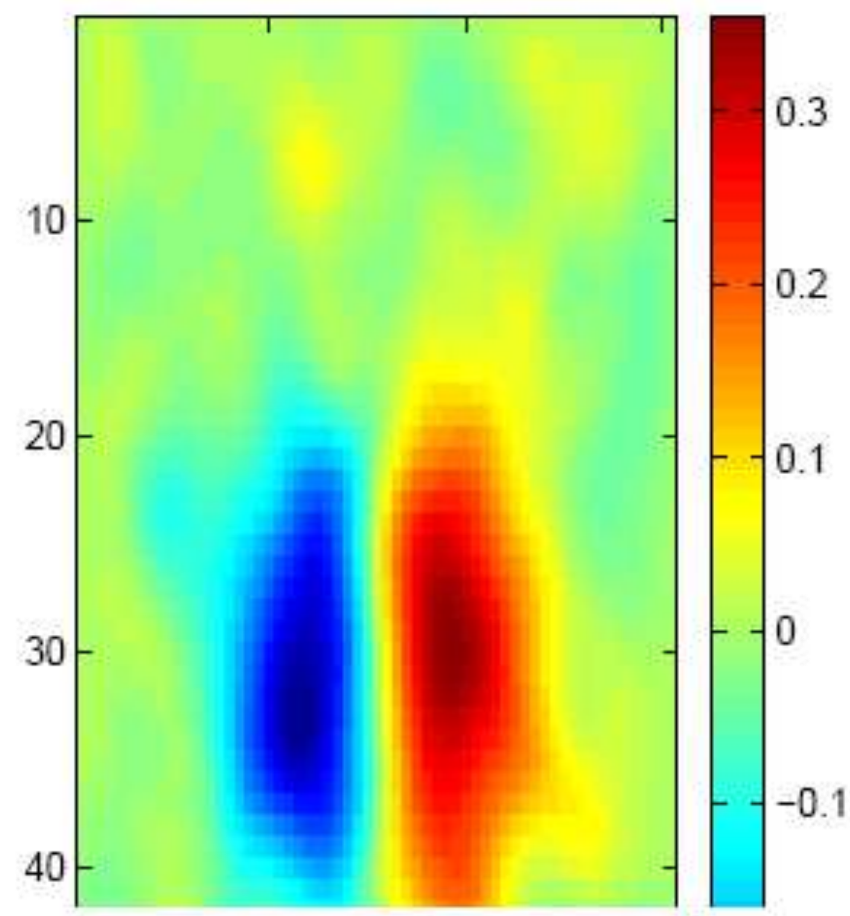
$$\widehat{\mu}(p) = \int K(p,q)Y(p)\, dp$$



Observation $Y(p)$

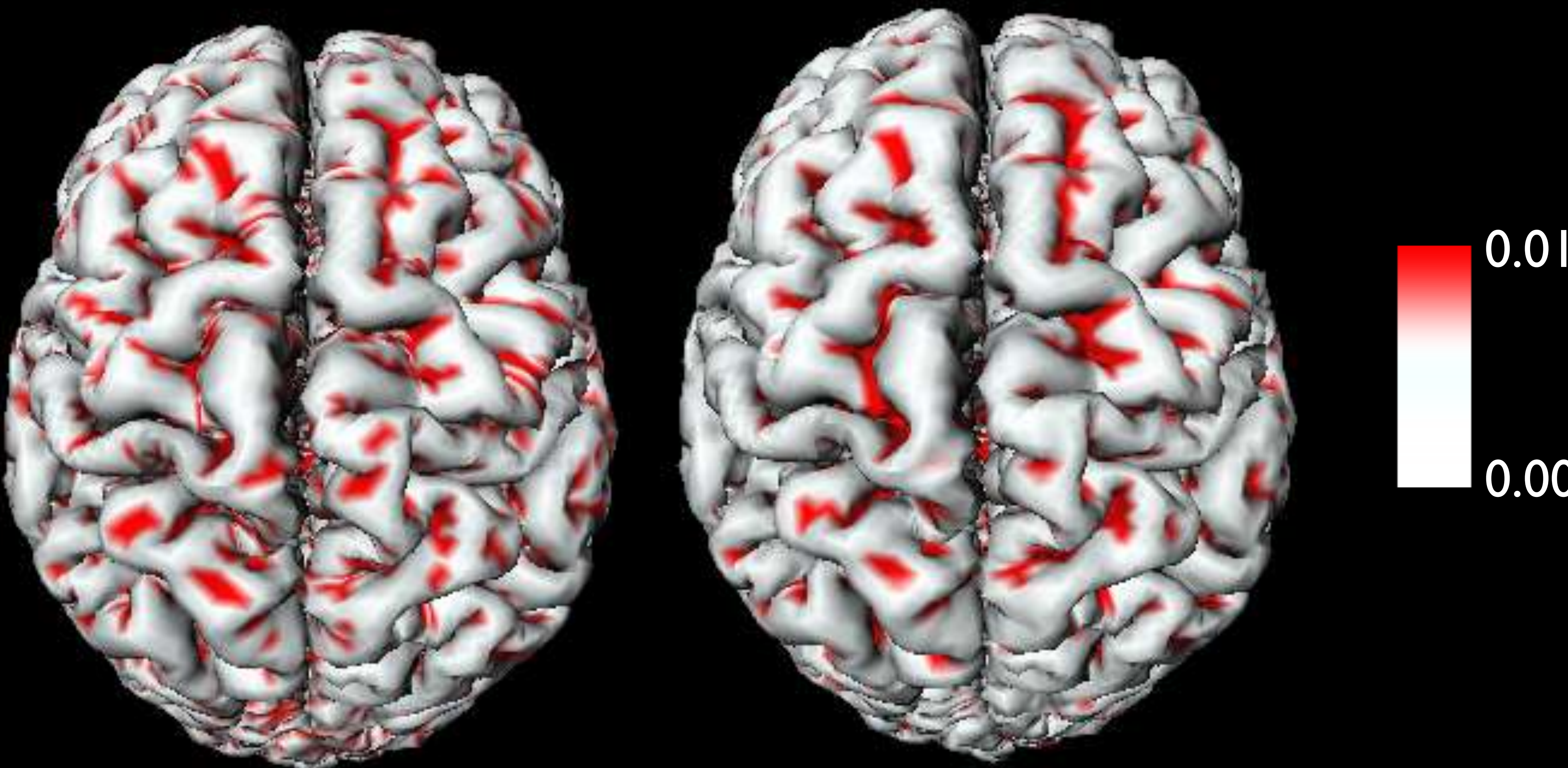


Unknown Signal $\mu(p)$



Prediction $\widehat{\mu}(p)$

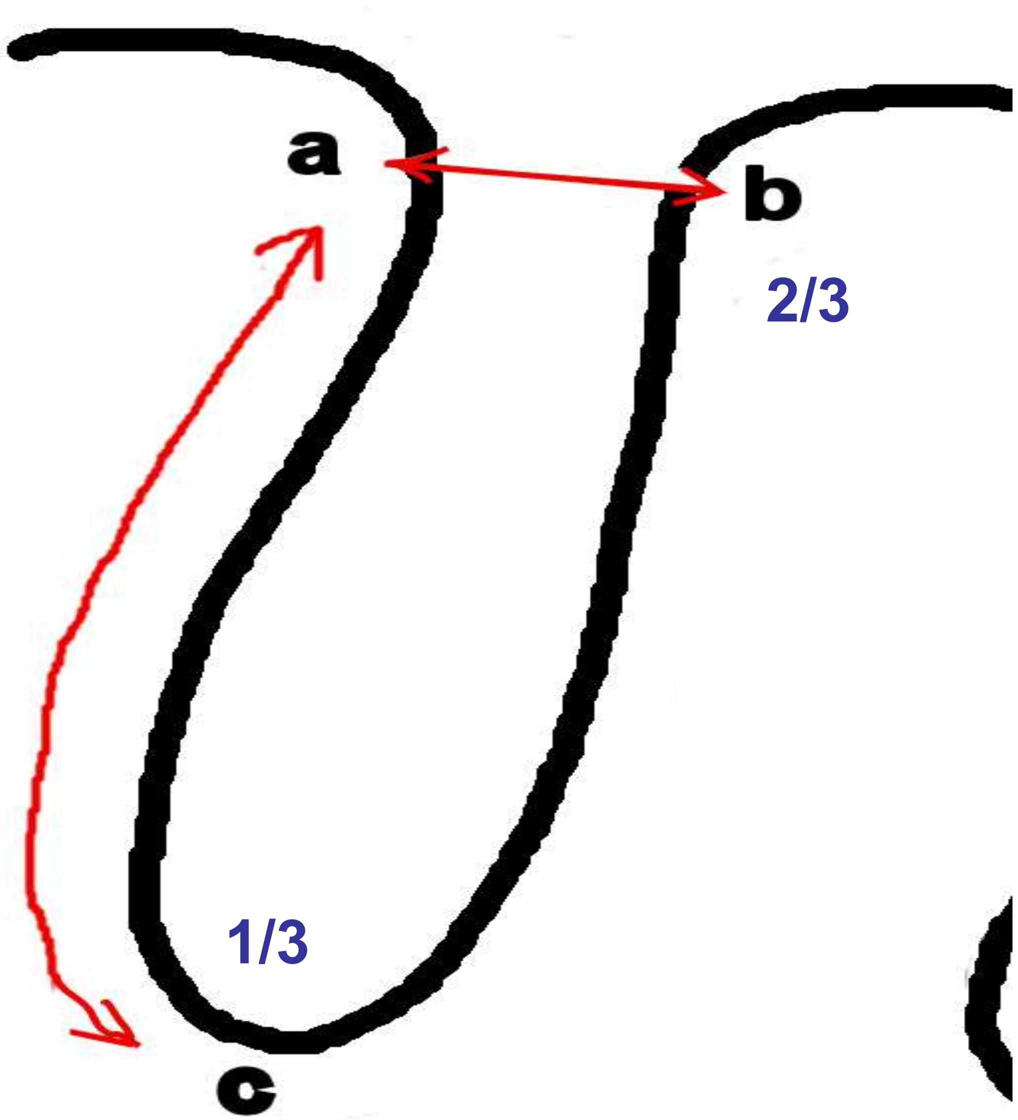
Gaussian kernel smoothing on surface curvature?



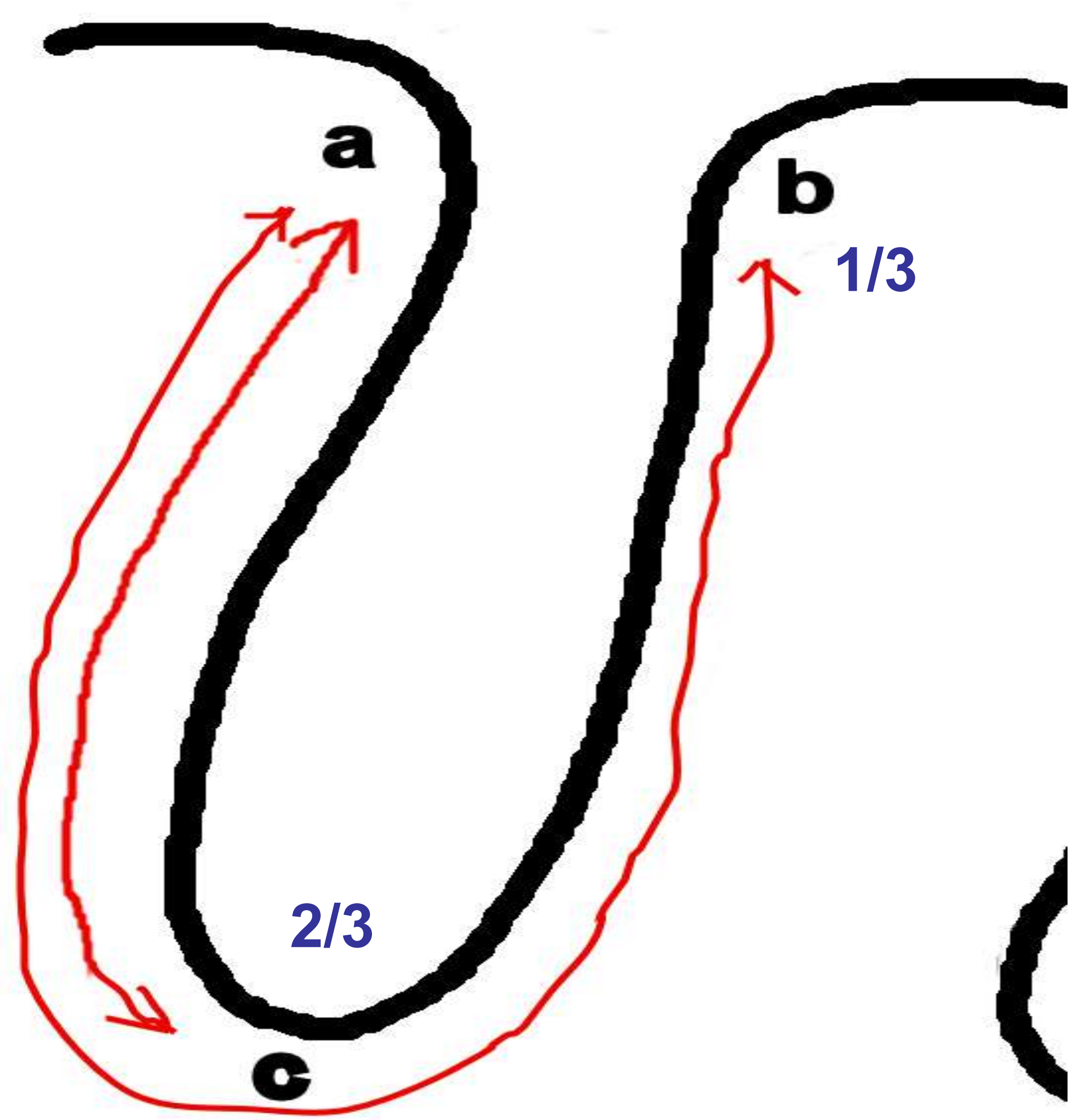
mean curvature

Noise removed curvature

Gaussian kernel does not work for surface data



Improper kernel weighting



Proper kernel weighting

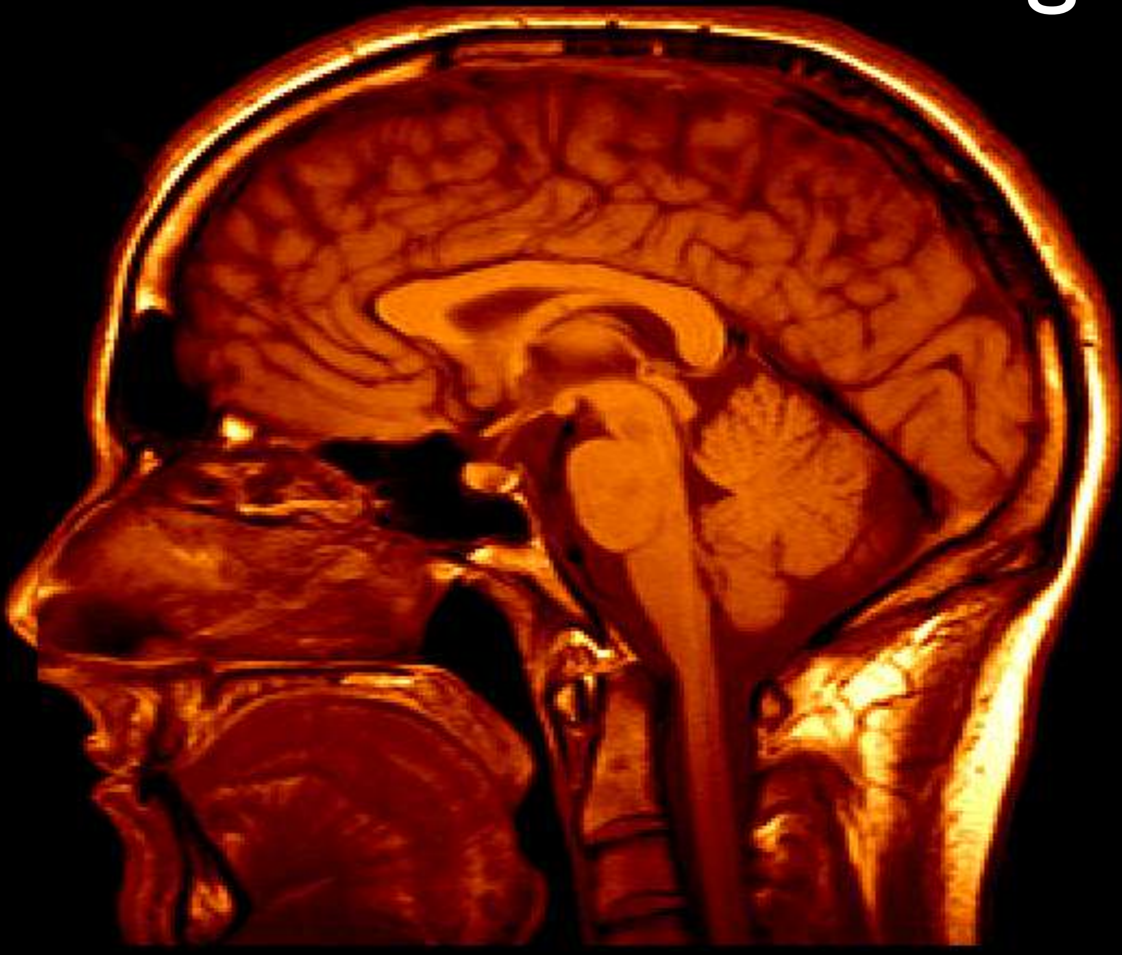
Kernel smoothing on sphere

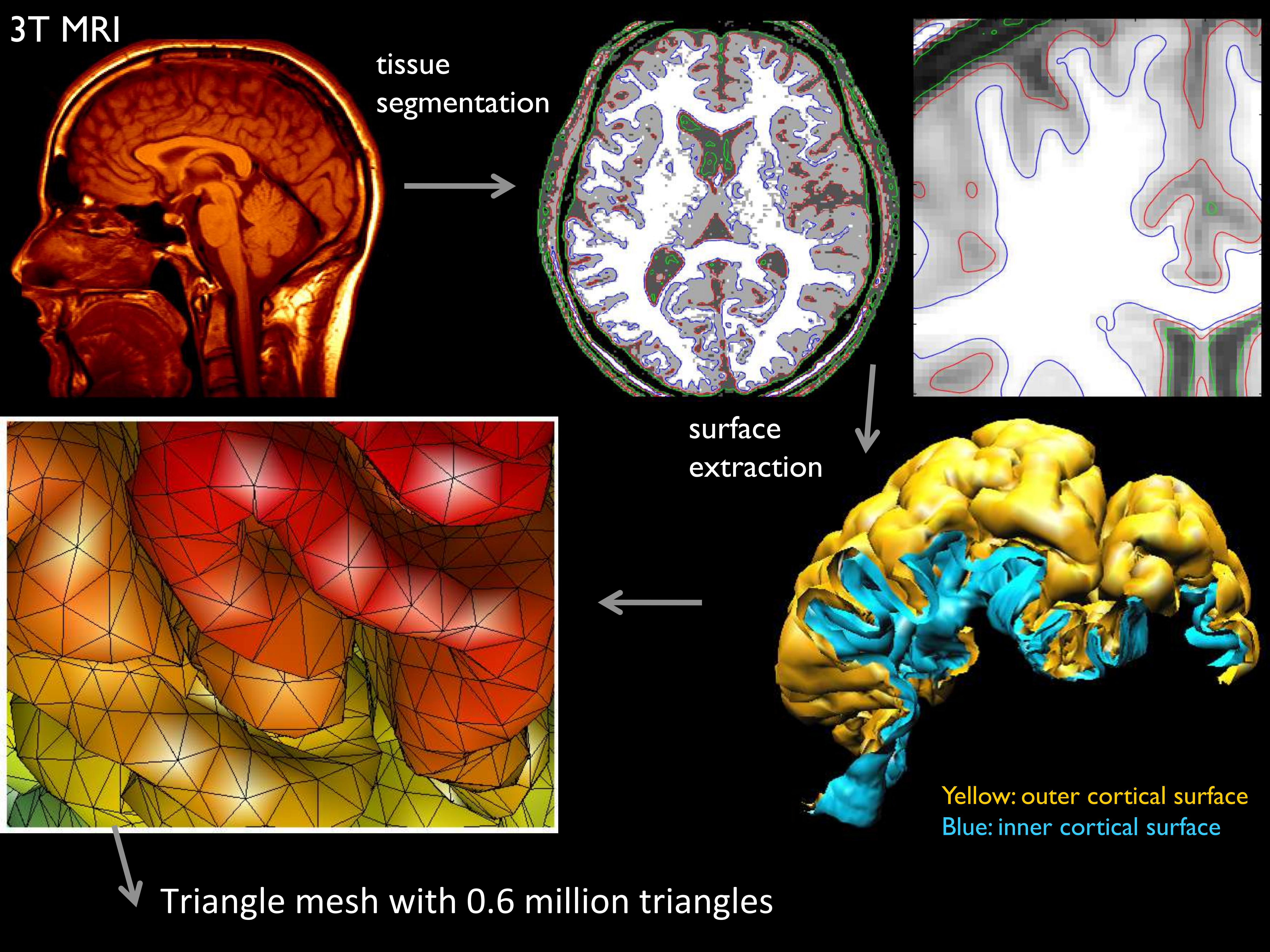
Magnetic Resonance Imaging (MRI)

3.0 Tesla GE Scanner

Soft tissues

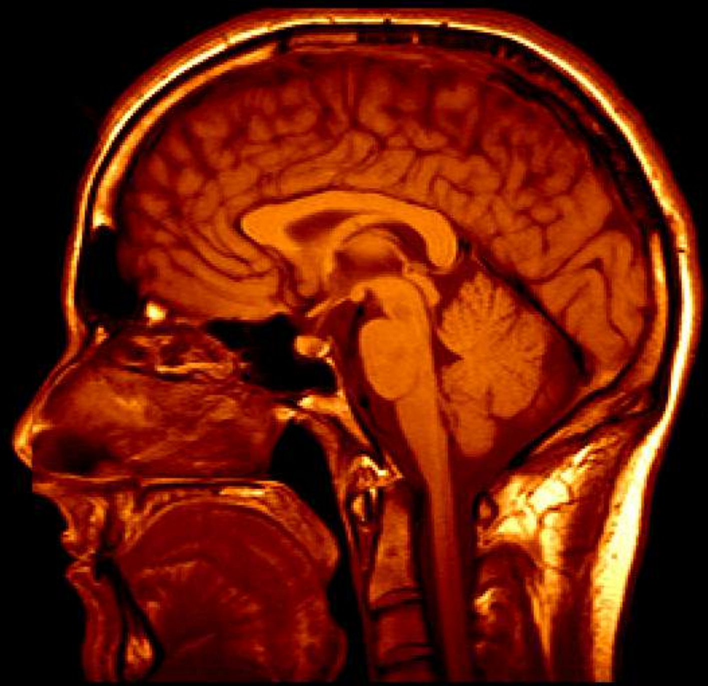
3D image



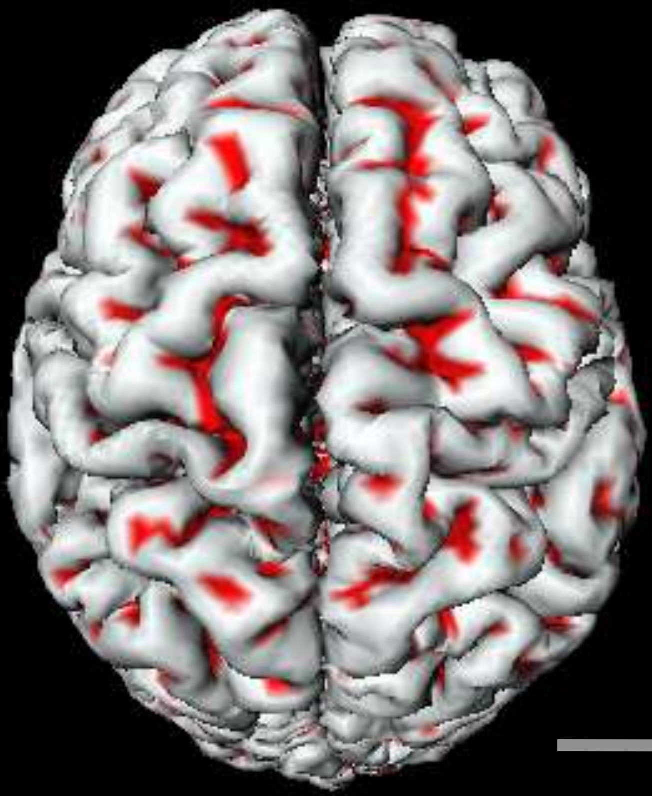

3T MRI
tissue segmentation
surface extraction
Yellow: outer cortical surface
Blue: inner cortical surface
Triangle mesh with 0.6 million triangles

Surface parameterization

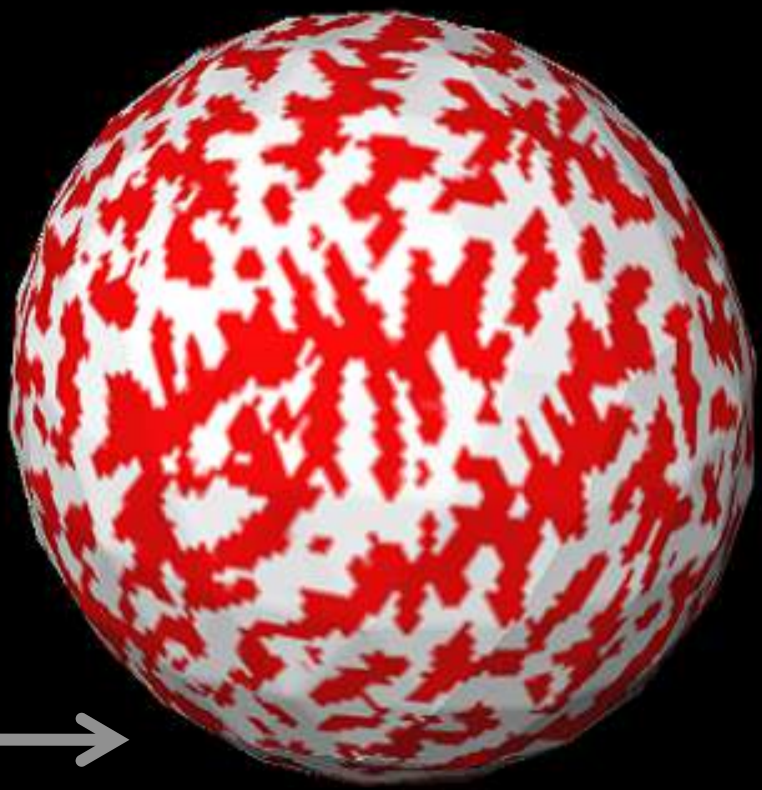
3T MRI

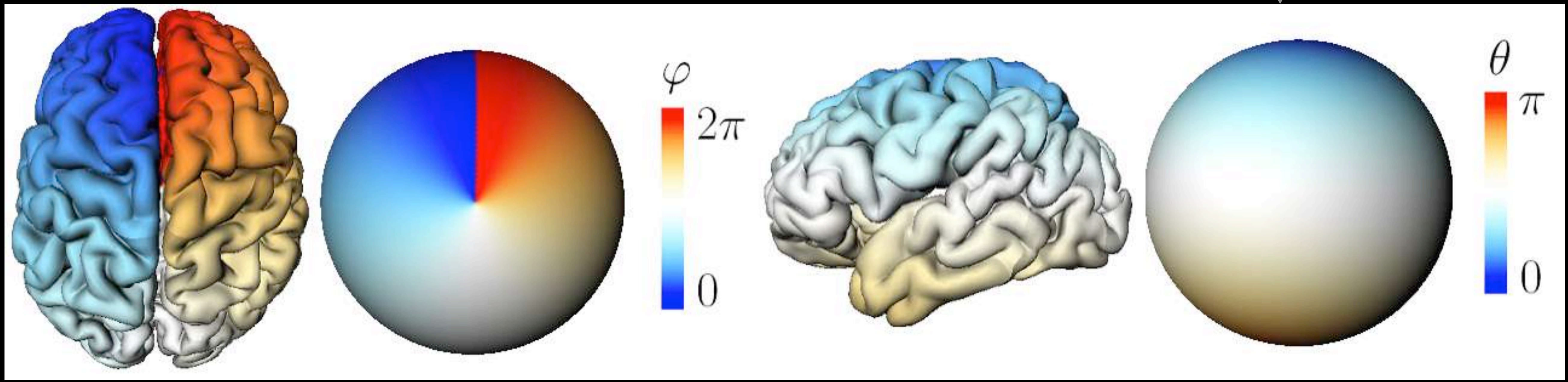


Surface segmentation



Surface flattening

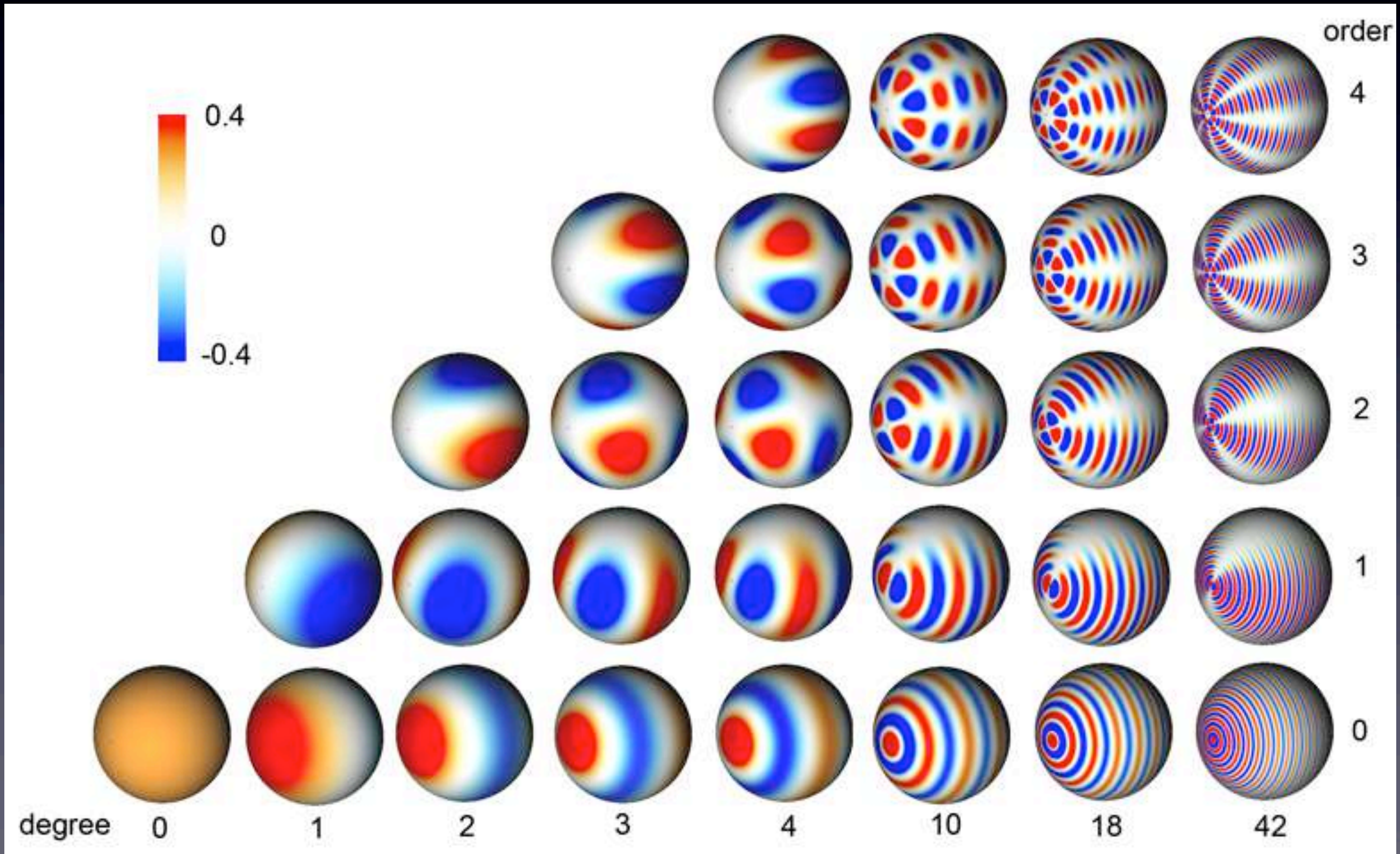




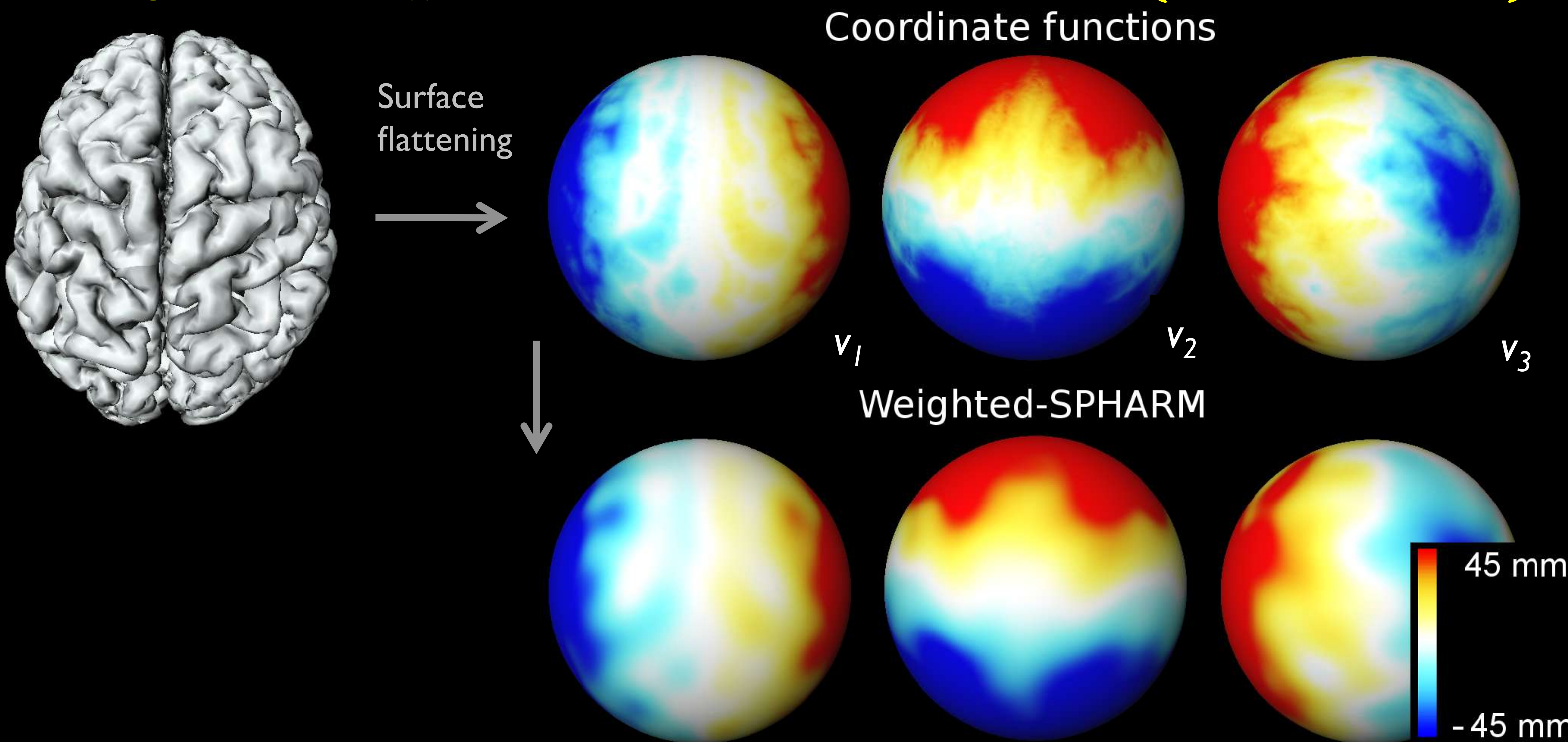
Spherical angle based coordinate system

Spherical harmonic of degree *l* and order *m*

$$Y_{lm} = \begin{cases} c_{lm} P_l^{|m|}(\cos\theta)\sin(|m|\varphi), & -l \le m \le -1, \\ \frac{c_{lm}}{\sqrt{2}} P_l^0(\cos\theta), & m = 0, \\ c_{lm} P_l^{|m|}(\cos\theta)\cos(|m|\varphi), & 1 \le m \le l, \end{cases}$$



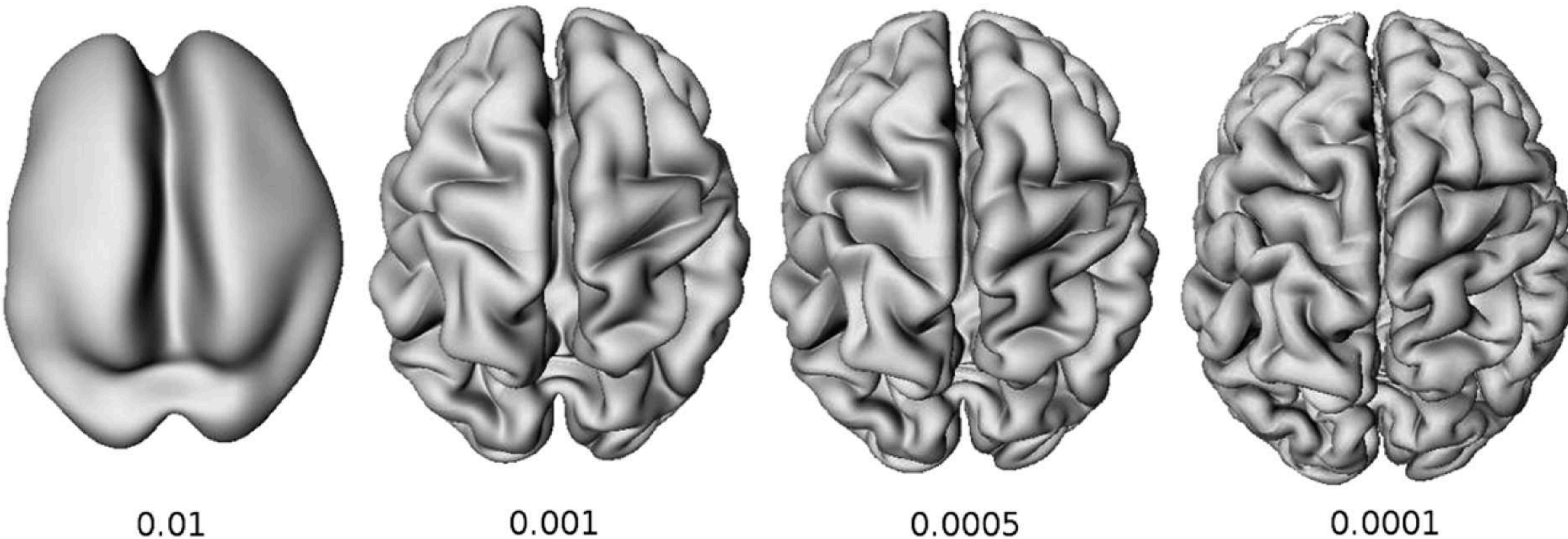
Weighted-Spherical harmonics (SPHARM)



$$K_\sigma(\theta, \phi, \theta', \phi') = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} Y_{lm}(\theta, \phi) Y_{lm}(\theta', \phi')$$

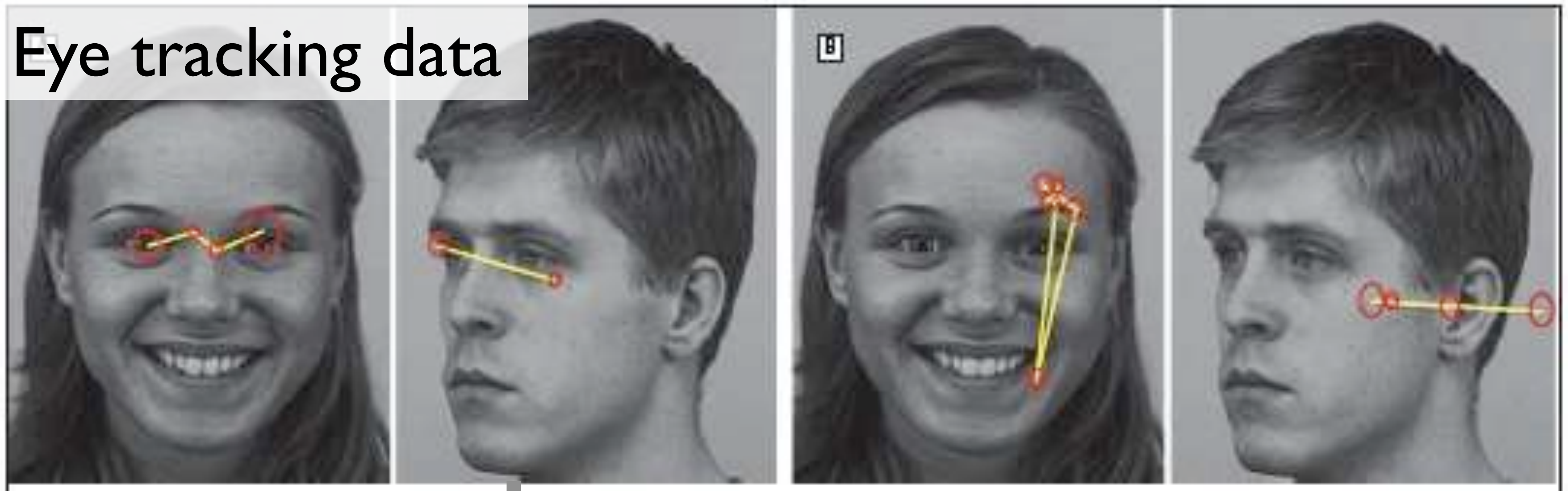
$$K_\sigma * v_i(\theta, \phi) = \int_0^{2\pi} \int_0^{\pi} K_\sigma(\theta, \phi, \theta', \phi') v_i(\theta', \phi') \ \sin\theta' d\theta' d\phi'$$

Heat kernel smoothing of surface coordinates



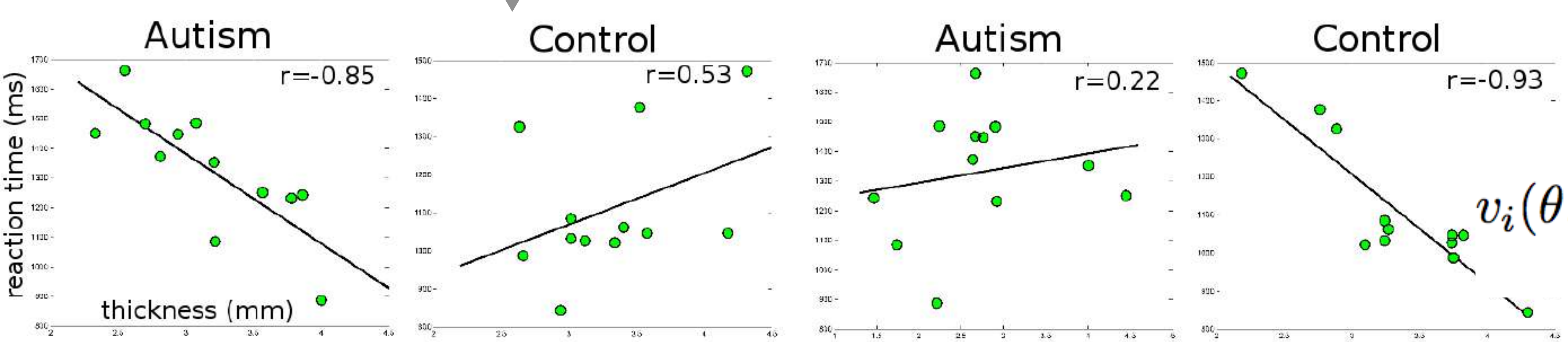
[Chung et al., 2007 IEEE Transactions on Medical Imaging 26:566-581]

Correlating function to structure



Partial correlation mapping

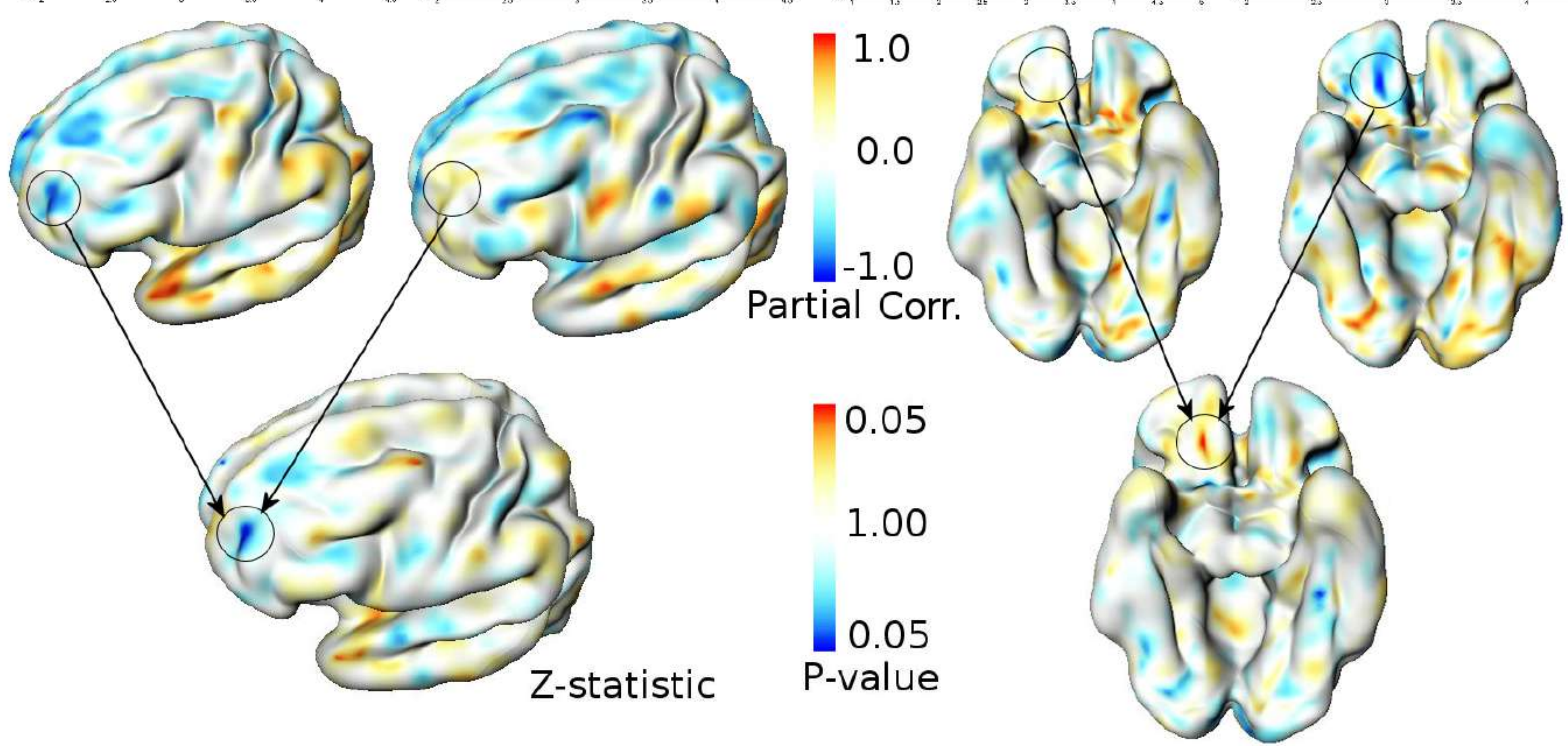
Weighted Fourier representation



$$v_i(\theta,\varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} f_{lm}^{i} Y_{lm}(\theta,\varphi)$$

88.1799	56.6336	5.7367
-12.4775	-11.2552	-2.0791
2.4336	-15.4428	-0.4021
4.3956	2.2733	-0.9354
-0.0106	-0.0674	0.6999
2.1773	-2.4194	-0.1176
0.5808	0.8390	1.2942
0.0615	-0.1893	0.1188
-0.2629	0.7524	0.1089
0.7909	-0.7276	-0.1901
0.5458	0.6236	0.6939

....



Heat kernel smoothing using LB-eigenfunctions

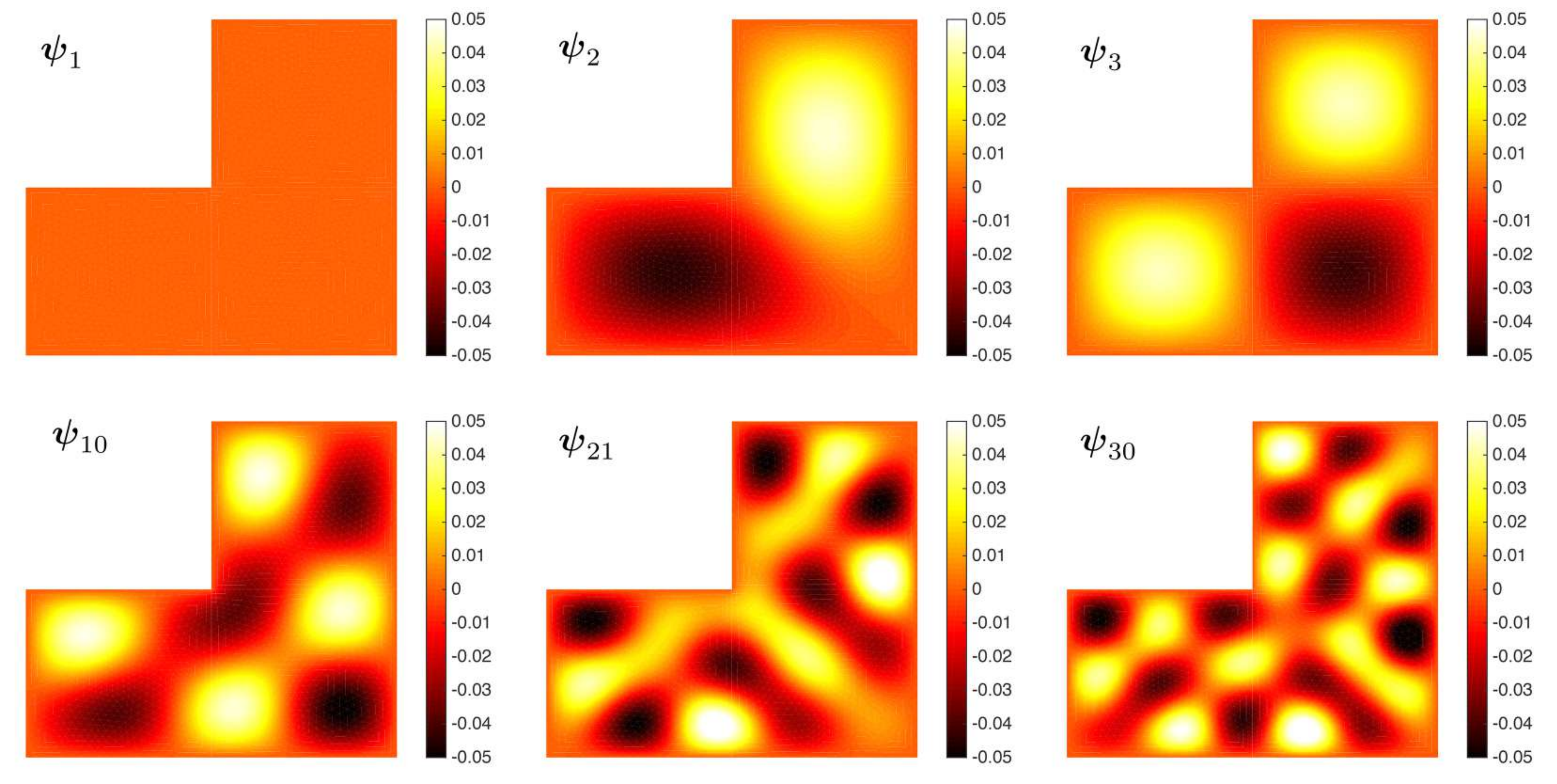
Basis in an arbitrary domain

Steady-state oscillations in a wave equation

↓

Helmholtz equation $$\Delta_X F = \lambda F$$

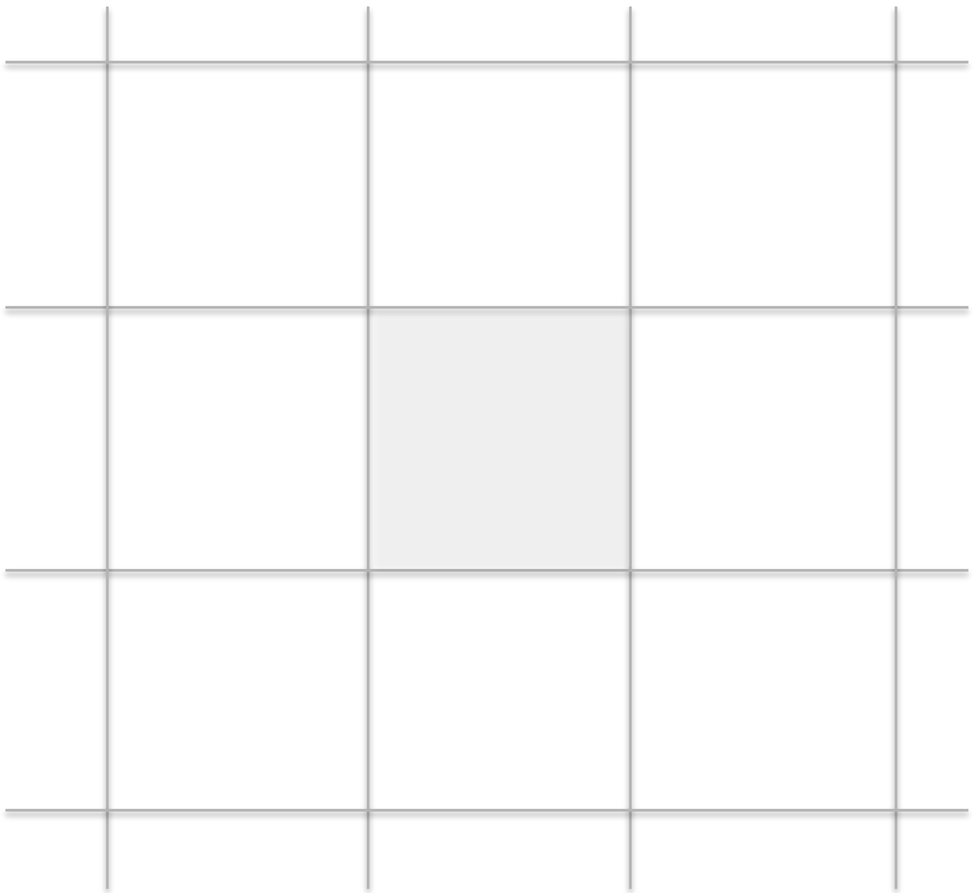
↓

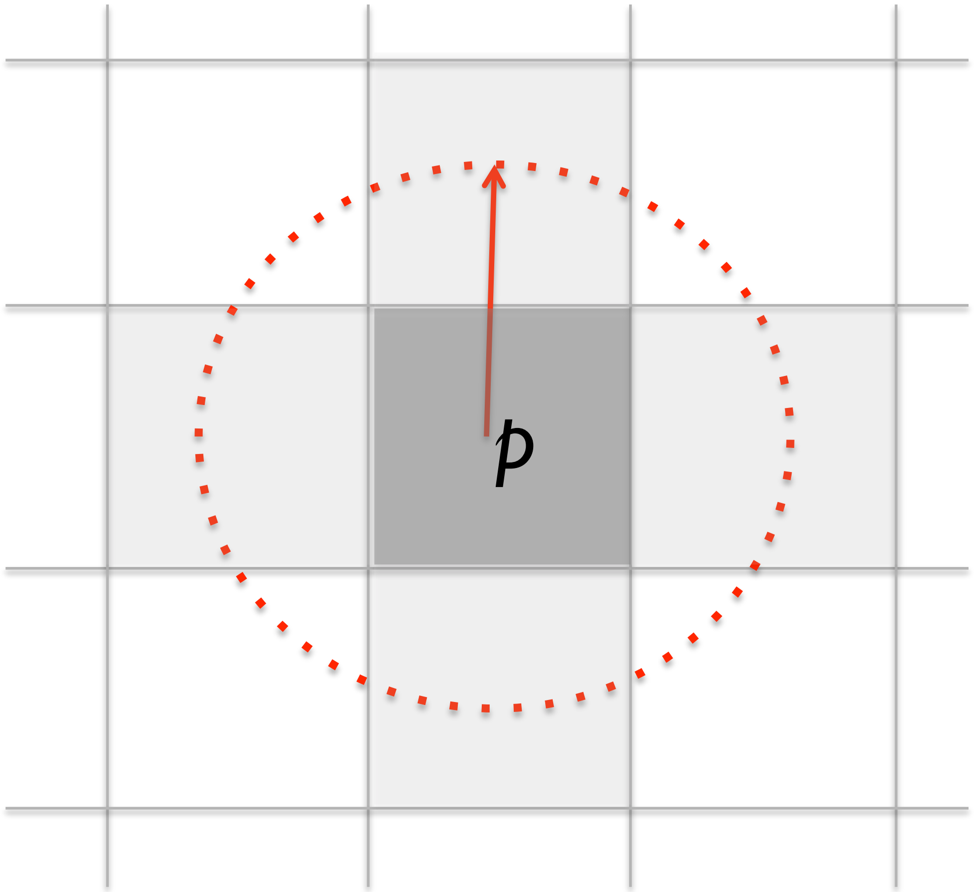


Orthonormal Basis

6 nearest neighbors in 3D

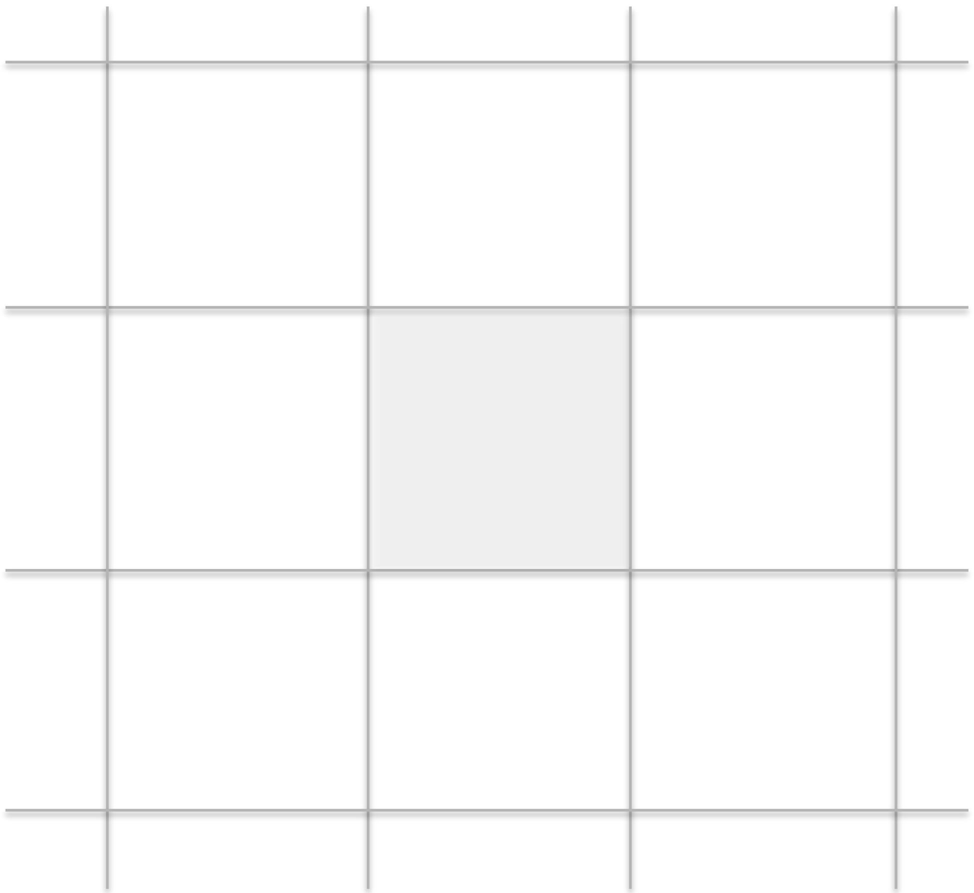
Top layer





Middle layer

Bottom layer

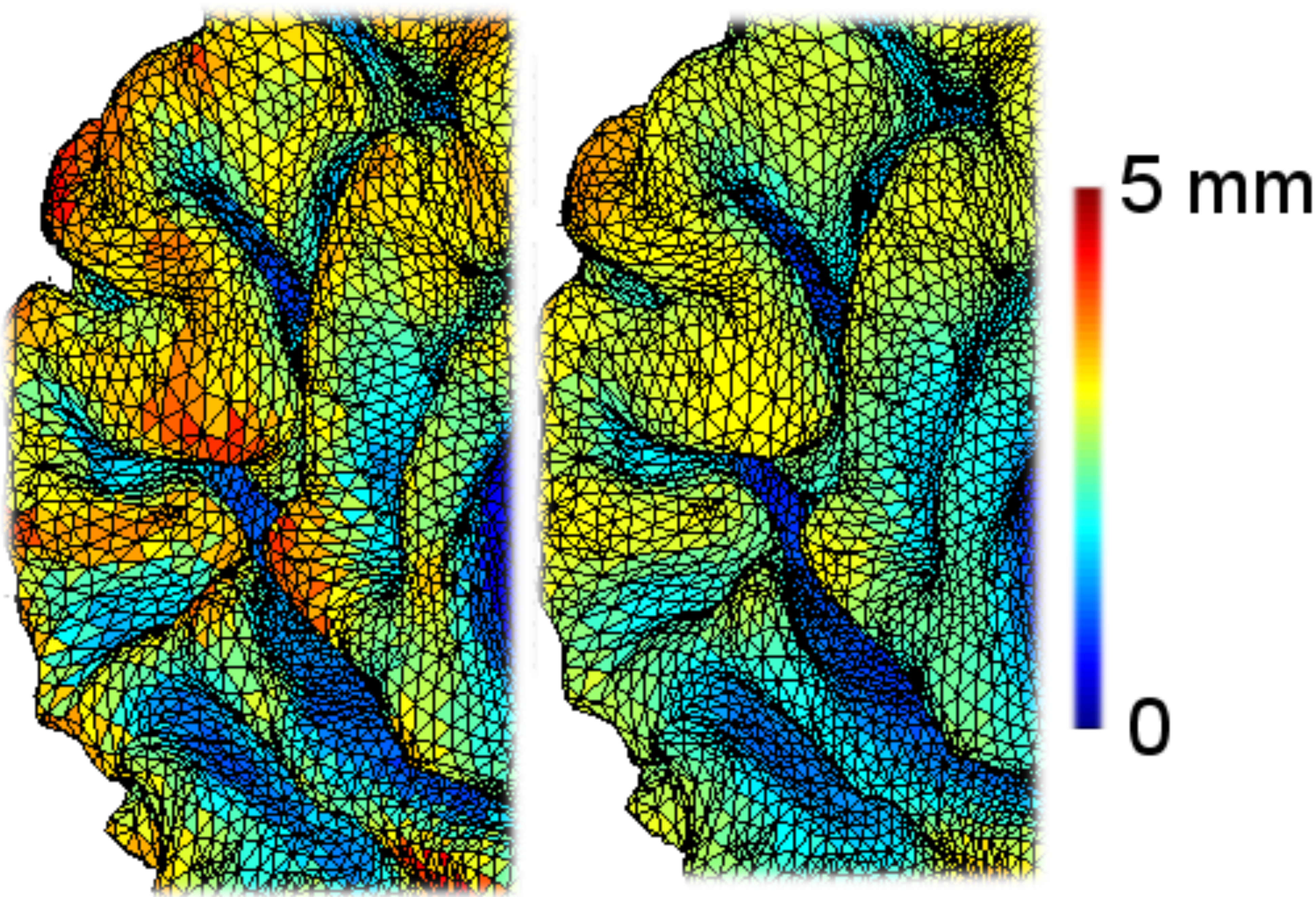


Connect any neighboring voxel with distance less than 1

$$\Delta f(p) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

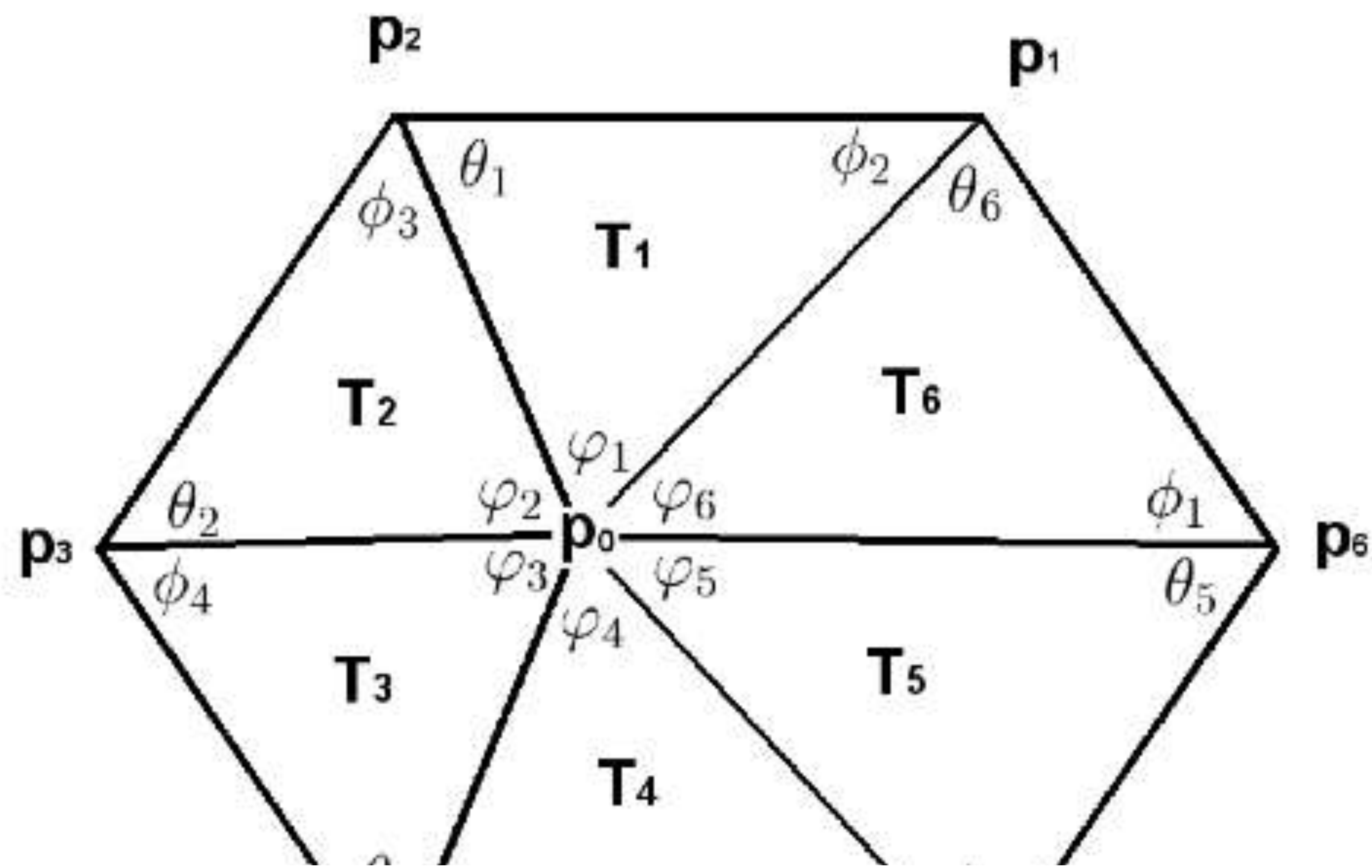
$$\Delta f(p) = \sum_{\delta p} f(p + \delta p) - 6f(p)$$

Surface data on triangle meshes



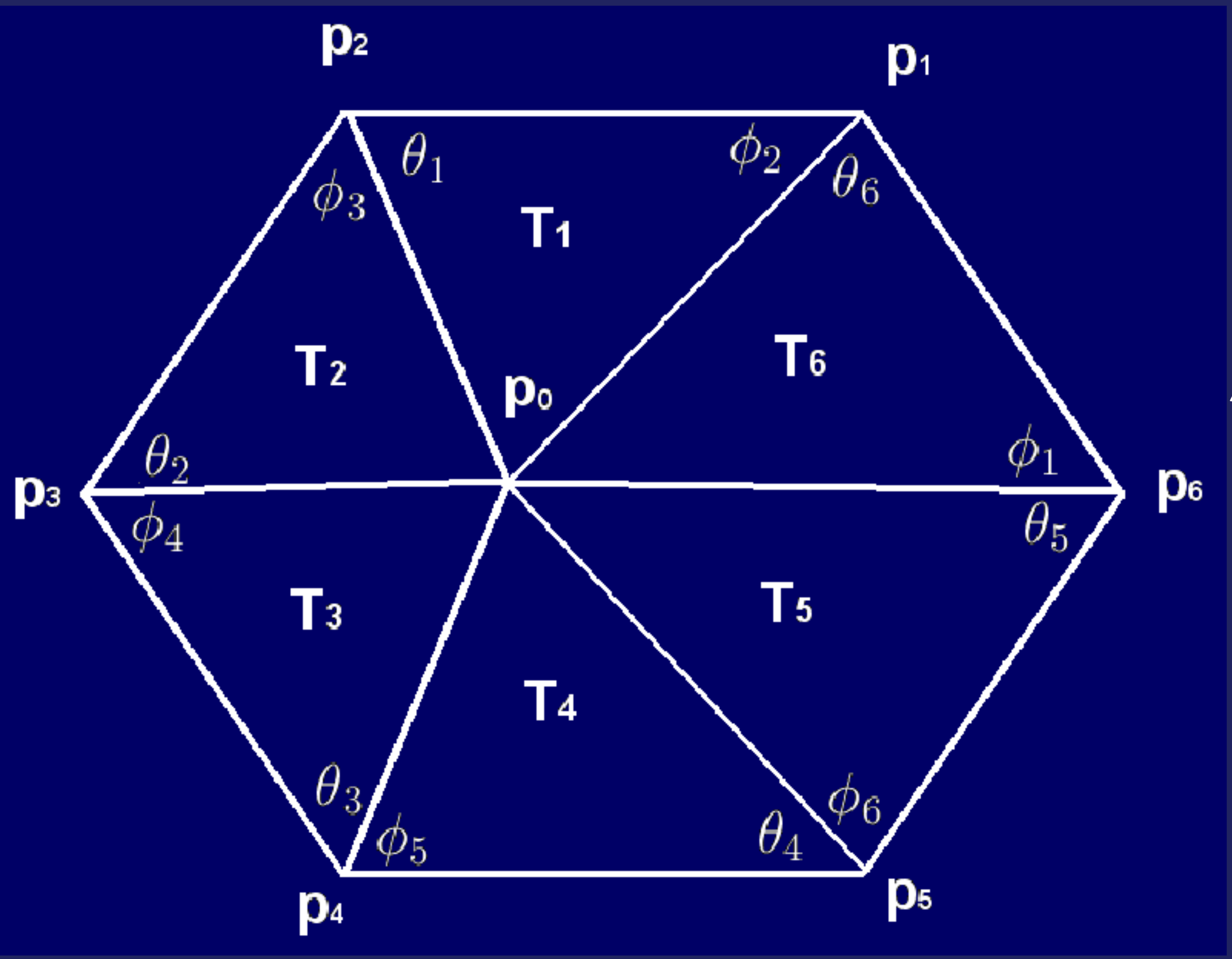
measurement after smoothing

First order neighbor in a triangle mesh



$$\Delta = \frac{1}{\det g^{1/2}} \sum_{i,j=1}^{2} \frac{\partial}{\partial u^i}\Big(\det g^{1/2} g^{ij} \frac{\partial}{\partial u^j} \Big)$$

Cotan Discretization

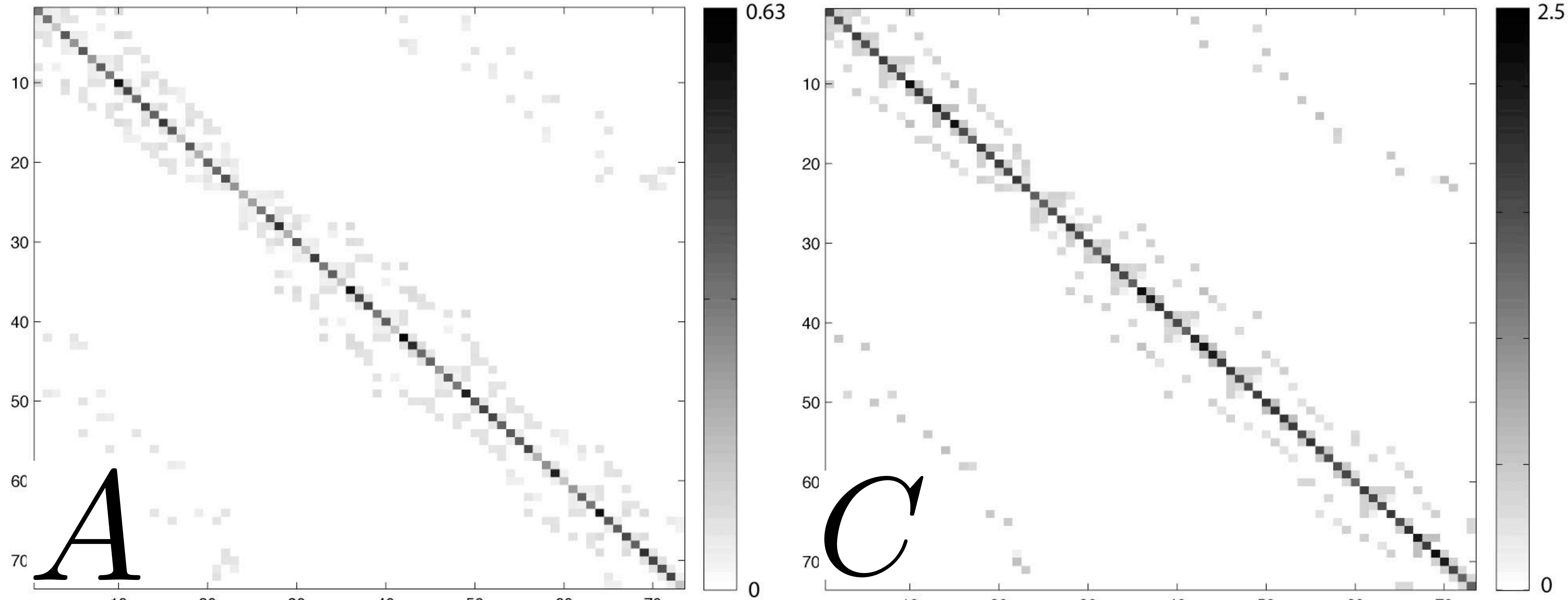


$$\Delta f(p) = \sum_{i=1}^{m} w_i [f(p_i) - f(p)]$$

$$w_j = \frac{\cot \theta_j + \cot \phi_j}{\sum_{j=1}^{m} |T_i|}$$

Chung et al. 2001 NeuroImage 13S:96

Qiu et al., 2006, TMI



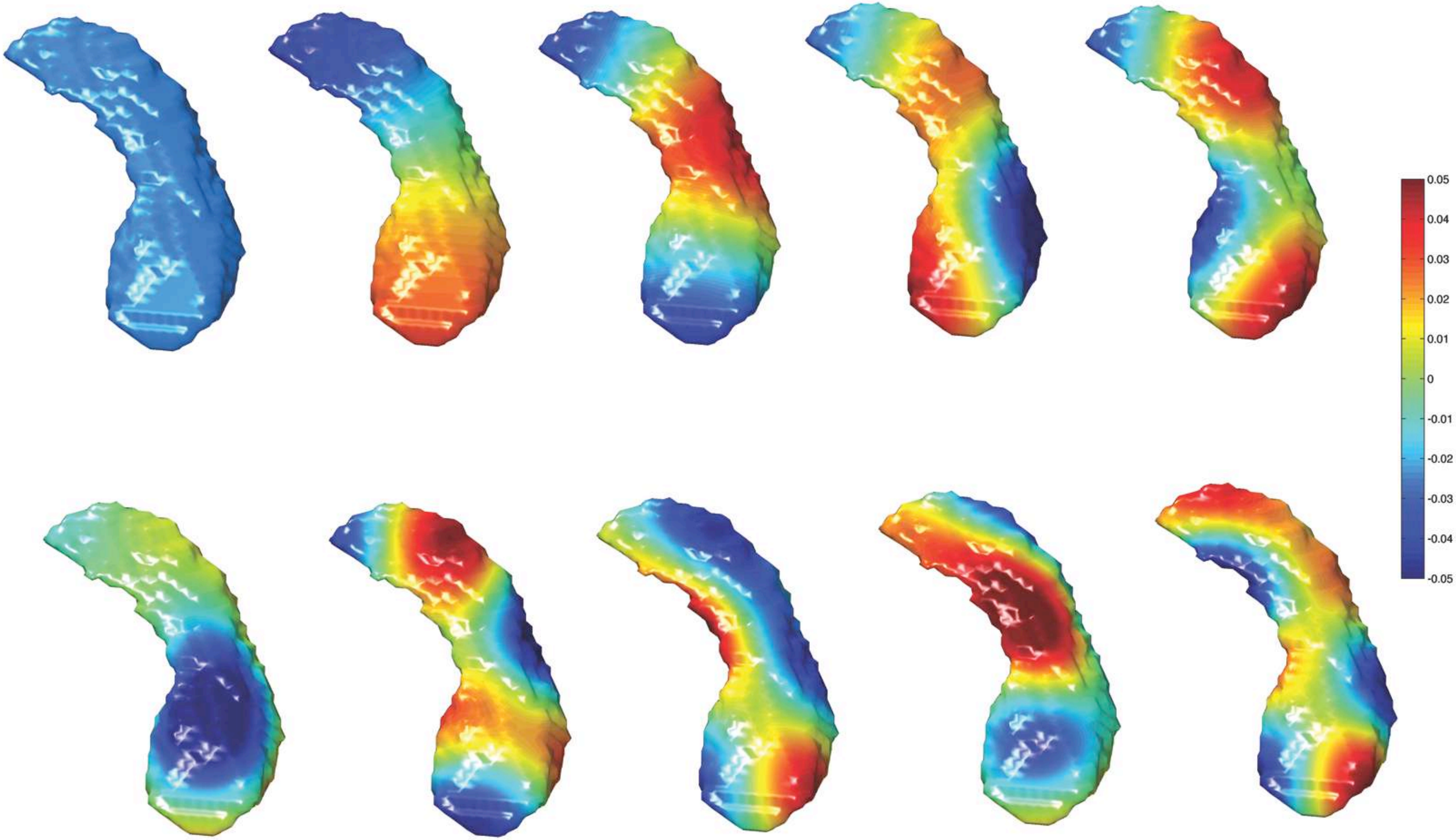
$$\Delta f = \lambda f \longrightarrow C\psi = \lambda A \psi$$

MATLAB code:

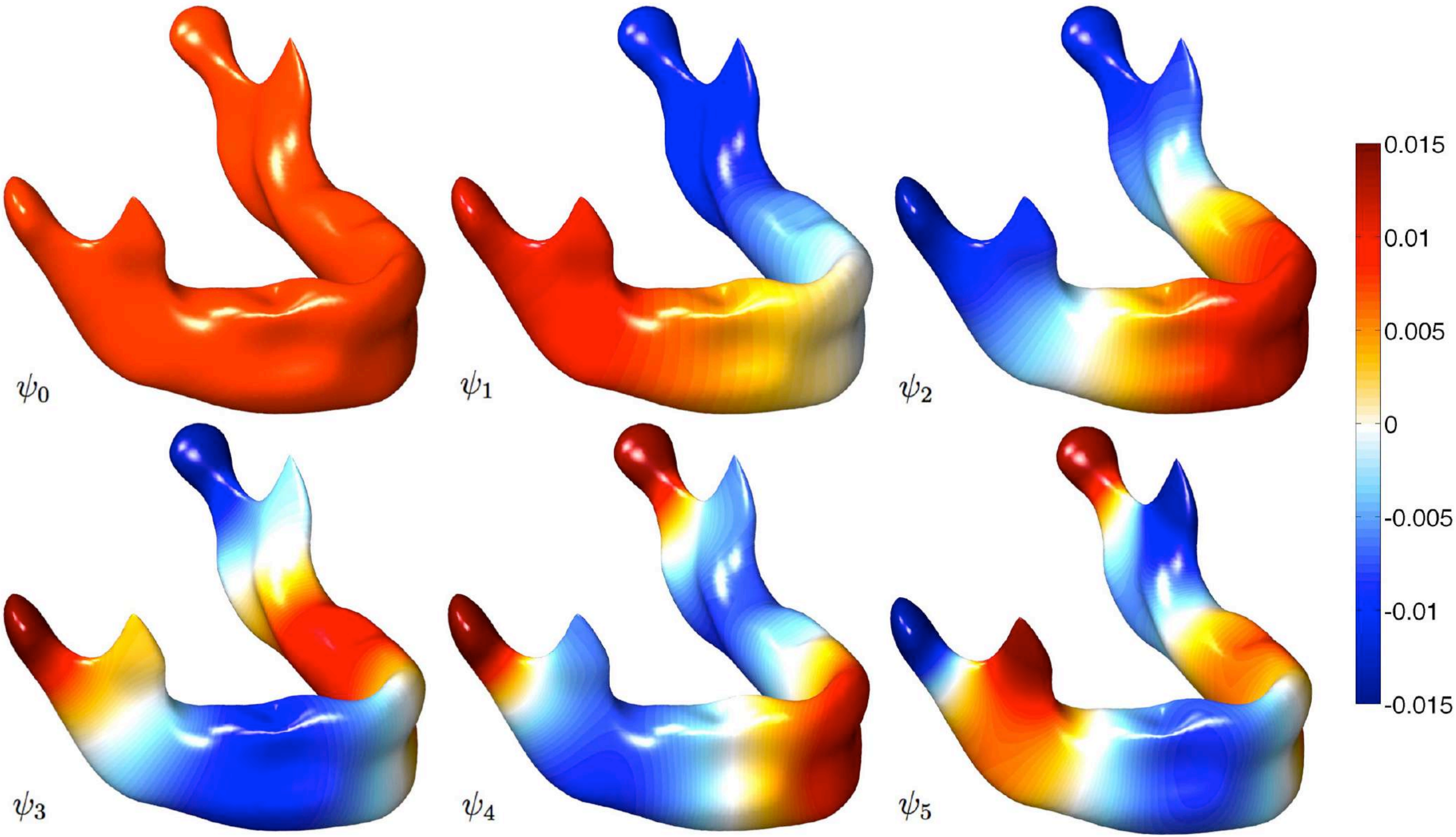
http://brainimaging.waisman.wisc.edu/~chung/lb

Tested for meshes with up to half million vertices

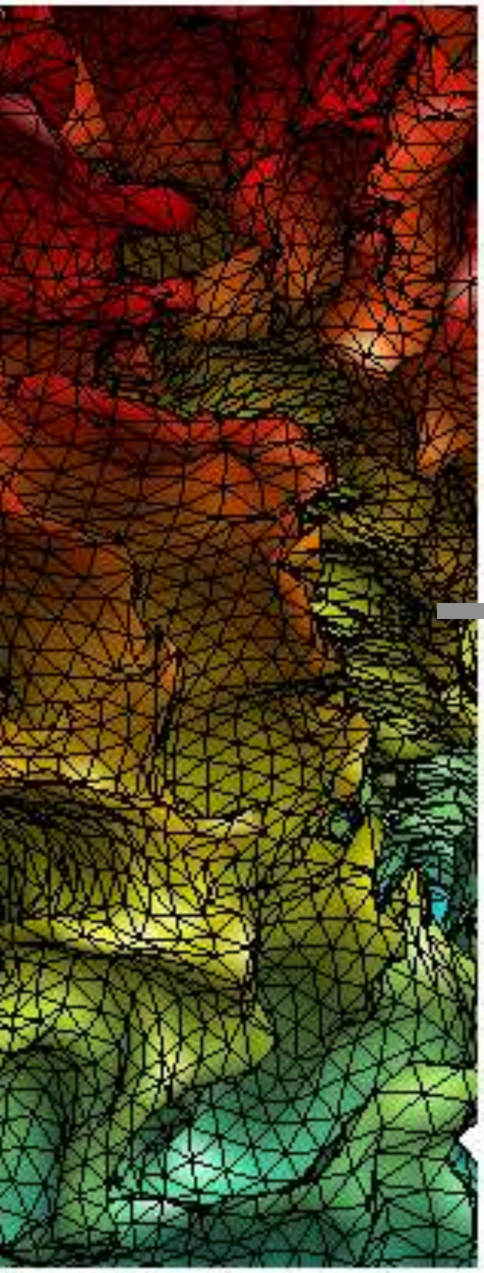
First 10 LB-eigenfunctions on left hippocampus

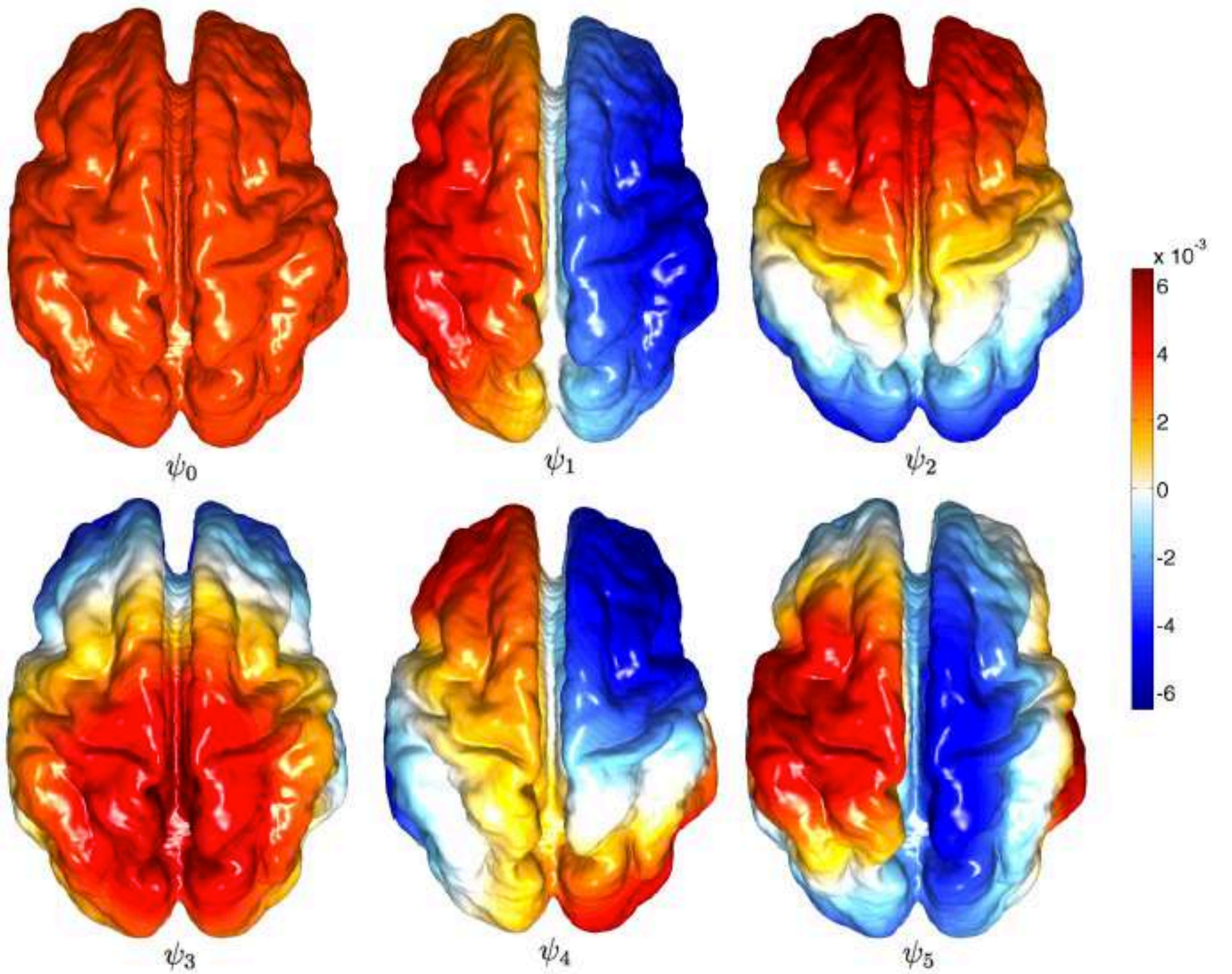


LB-eigenfunctions on mandible $\Delta f = \lambda f$



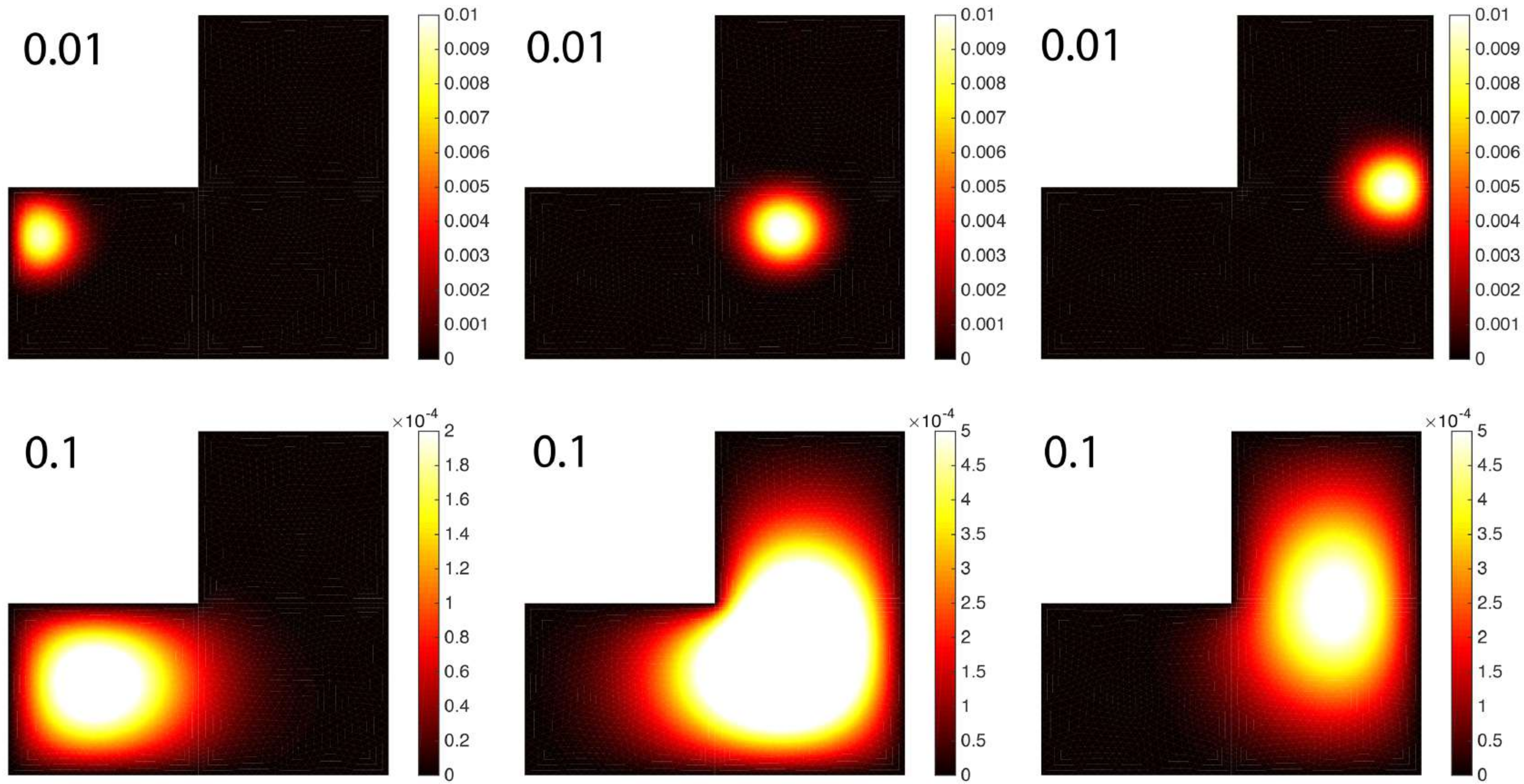
LB-eigenfunctions on brain surface $\Delta f = \lambda f$





Heat kernel

$$K_\sigma(p,q) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} \psi_j(p)\psi_j(q)$$



Fundamental solution of isotropic diffusion on manifolds

Diffusion via heat kernel smoothing

Diffusion equation $$\frac{\partial f}{\partial t} = \Delta f, \; f(x, t = 0) = X(x)$$

$$\sigma = \sqrt{2t}$$

Heat kernel smoothing $$f = K_\sigma * X$$

$$K_\sigma(p, q) = \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} \psi_j(p) \psi_j(q)$$

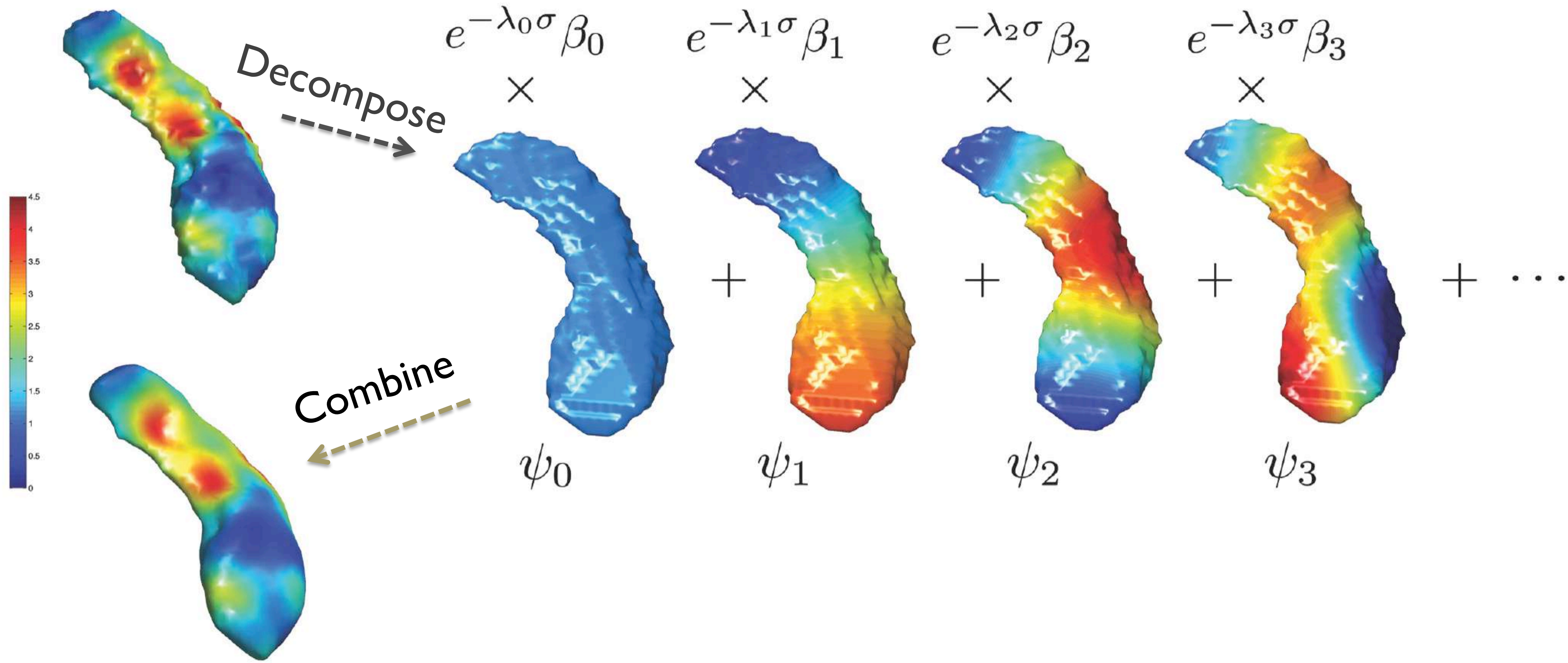
Chung et al., 2005 Information Processing in Medical Imaging (IPMI)

Heat kernel smoothing on manifolds

$$K_\sigma * X(p) = \int_{\mathcal{M}} K_\sigma(p,q) f(q)\, d\mu(q)$$
$$= \sum_{j=0}^{\infty} e^{-\lambda_j \sigma} \beta_j \psi_j(p)$$

Fourier coefficients

$$\beta_j = \int_{\mathcal{M}} X(p) \psi_j(p)\, d\mu(p)$$



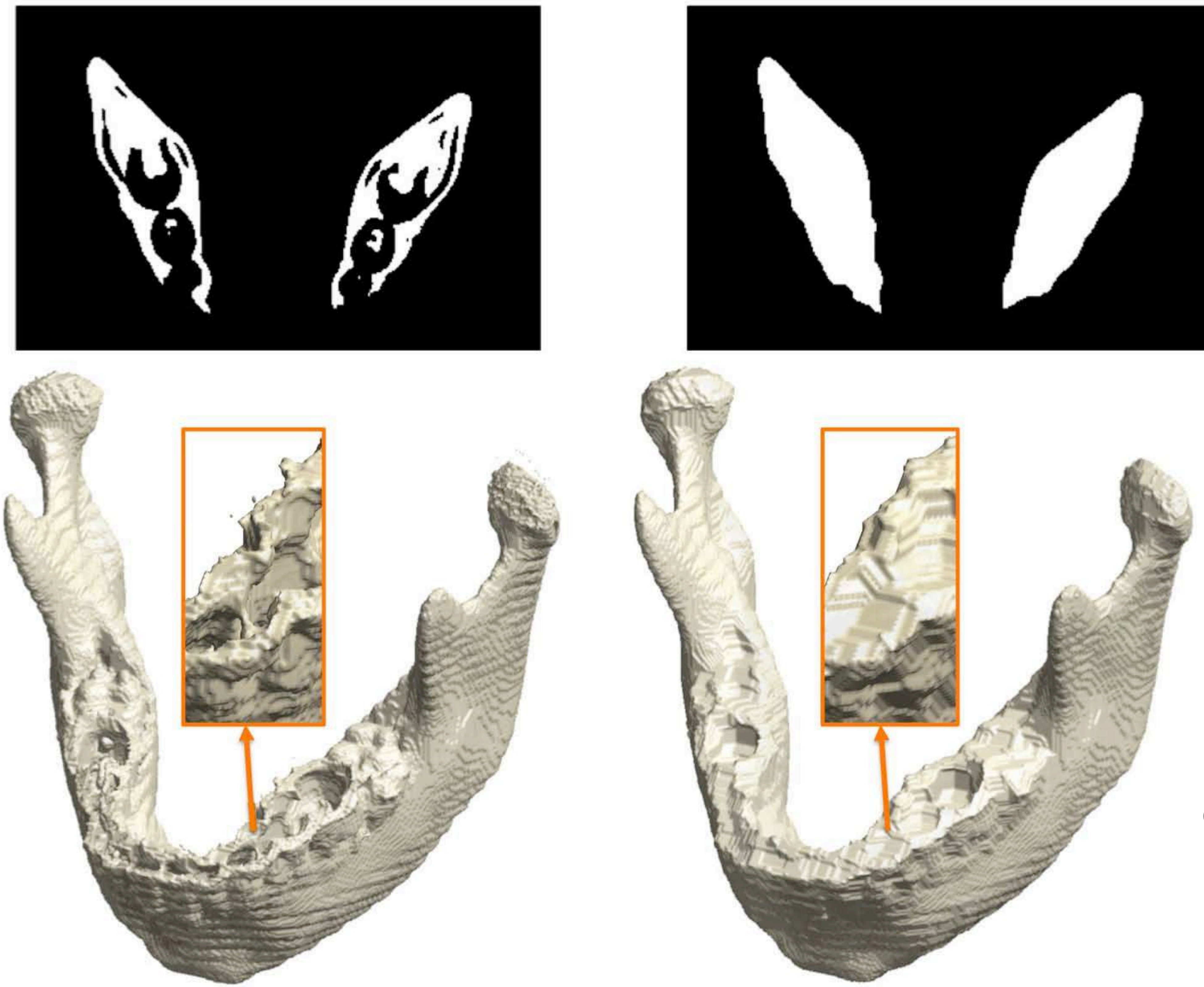
Chung, M.K. 2015 Medical Image Analysis. 22:63-76

Mandible Growth Modeling from CT

Computed Tomography (CT)

Hard tissues: bones, teeth

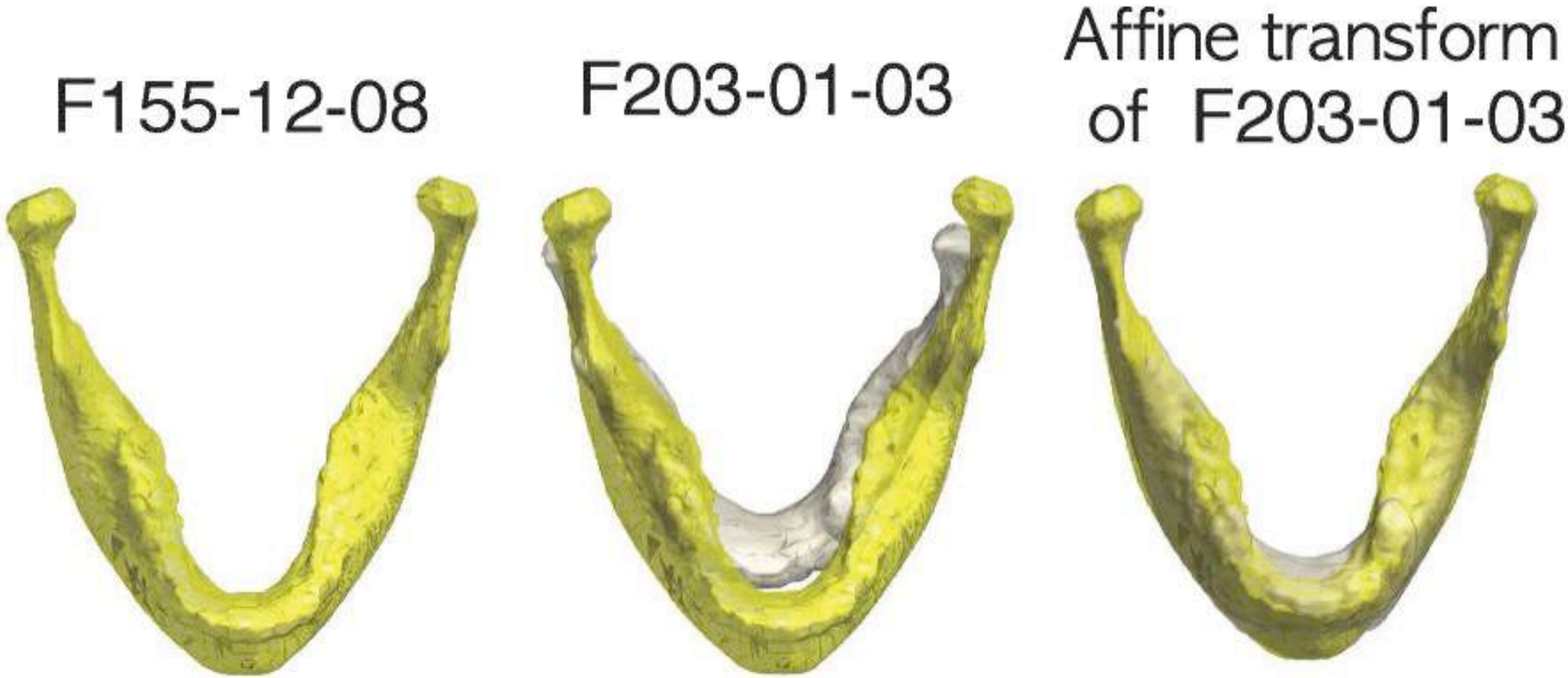
Topology correction in CT



Hole & handles corrected using Euler characteristic

Chung et al. 2015 Medical Image Analysis. 22:63-76

Initial affine registration



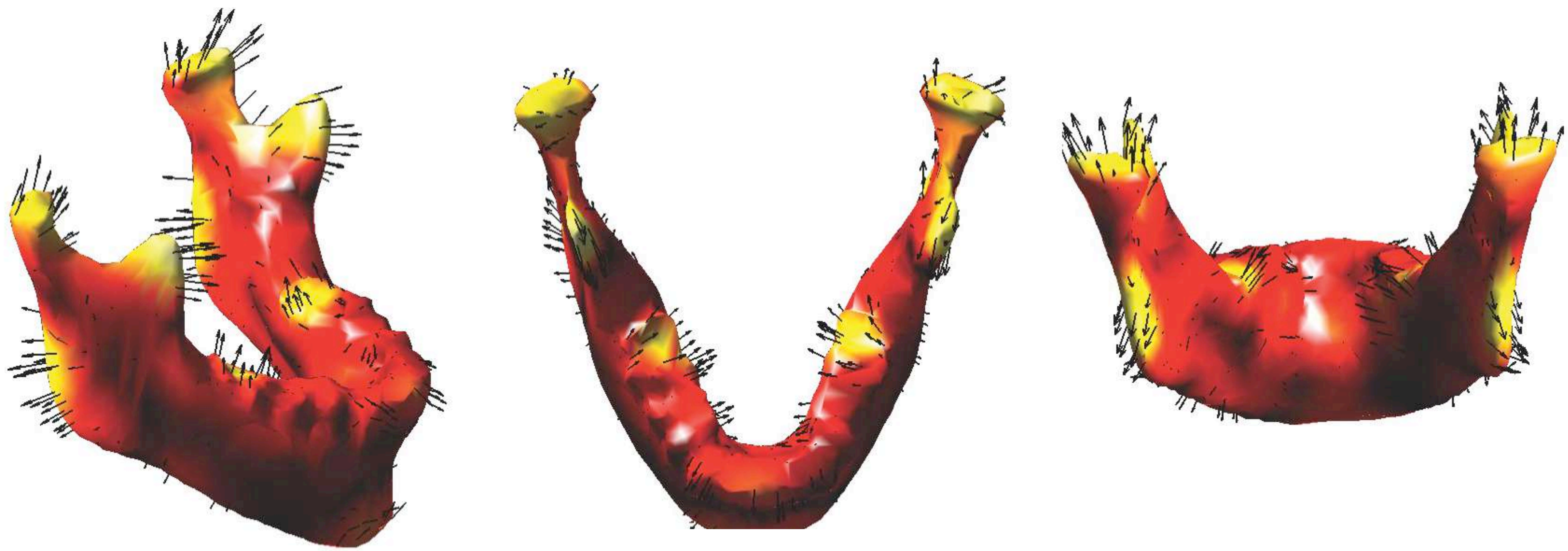
Nonlinear diffeomorphic registration

F155-12-08

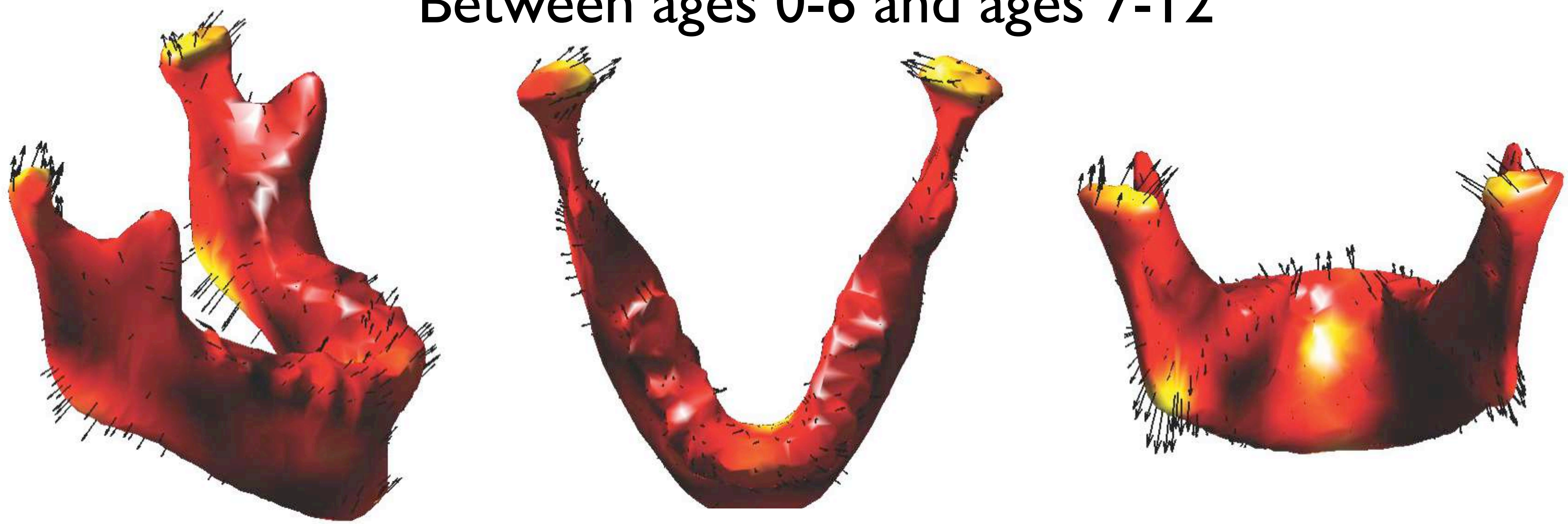
Affine registered surfaces

Final diffeomorphic registration

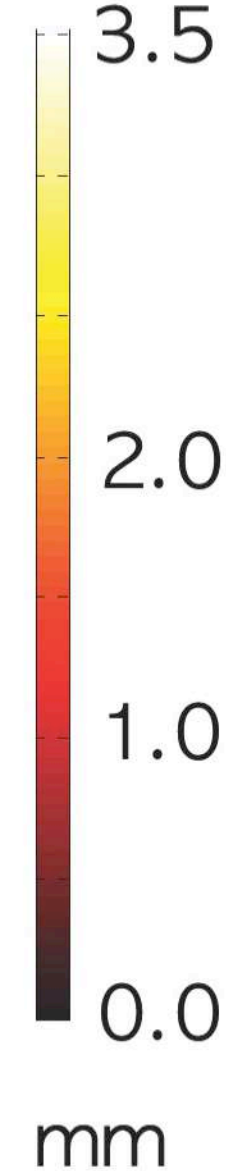
Average mandible growth pattern in children



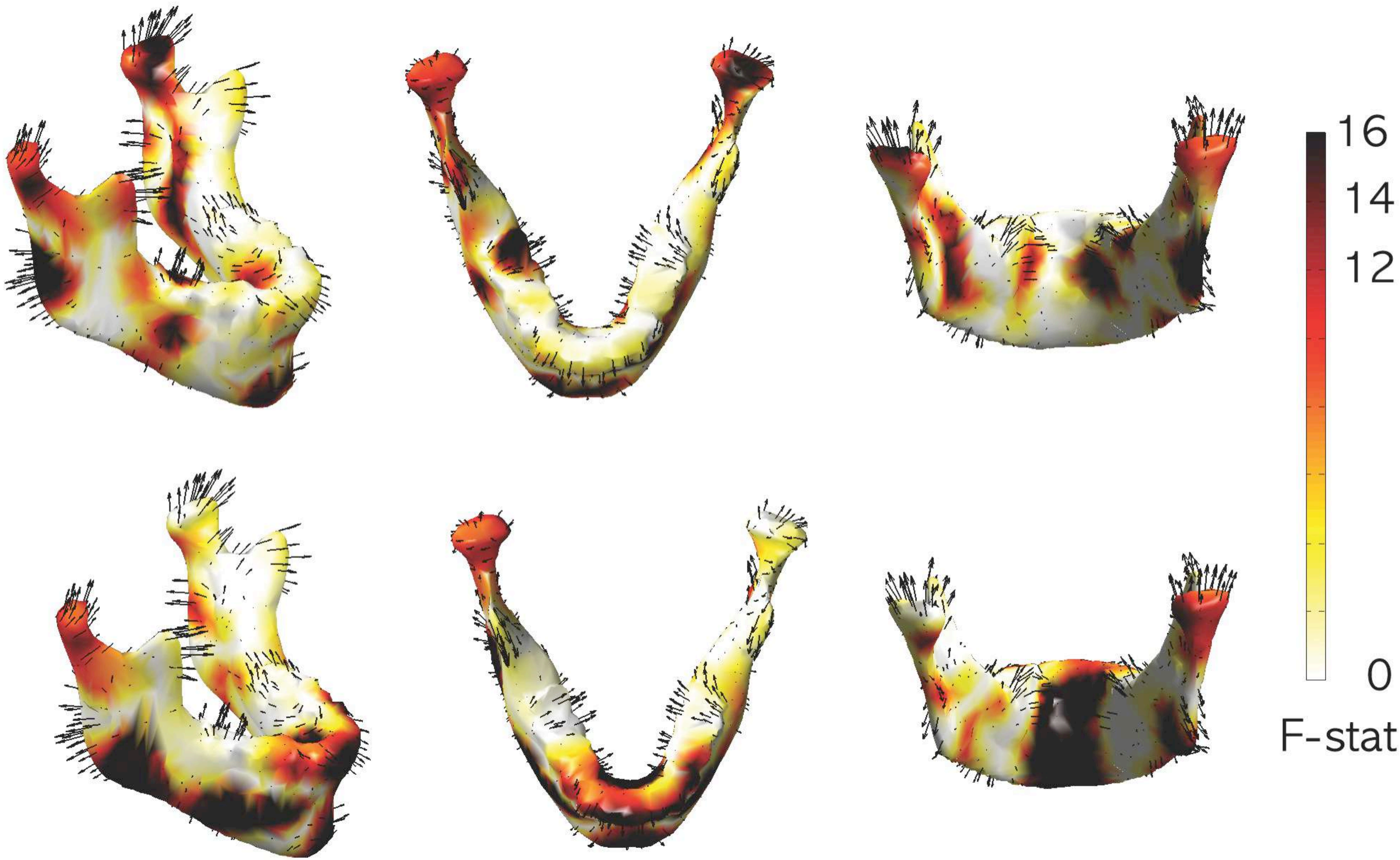
Between ages 0-6 and ages 7-12



Between age 7-12 and age 13-19

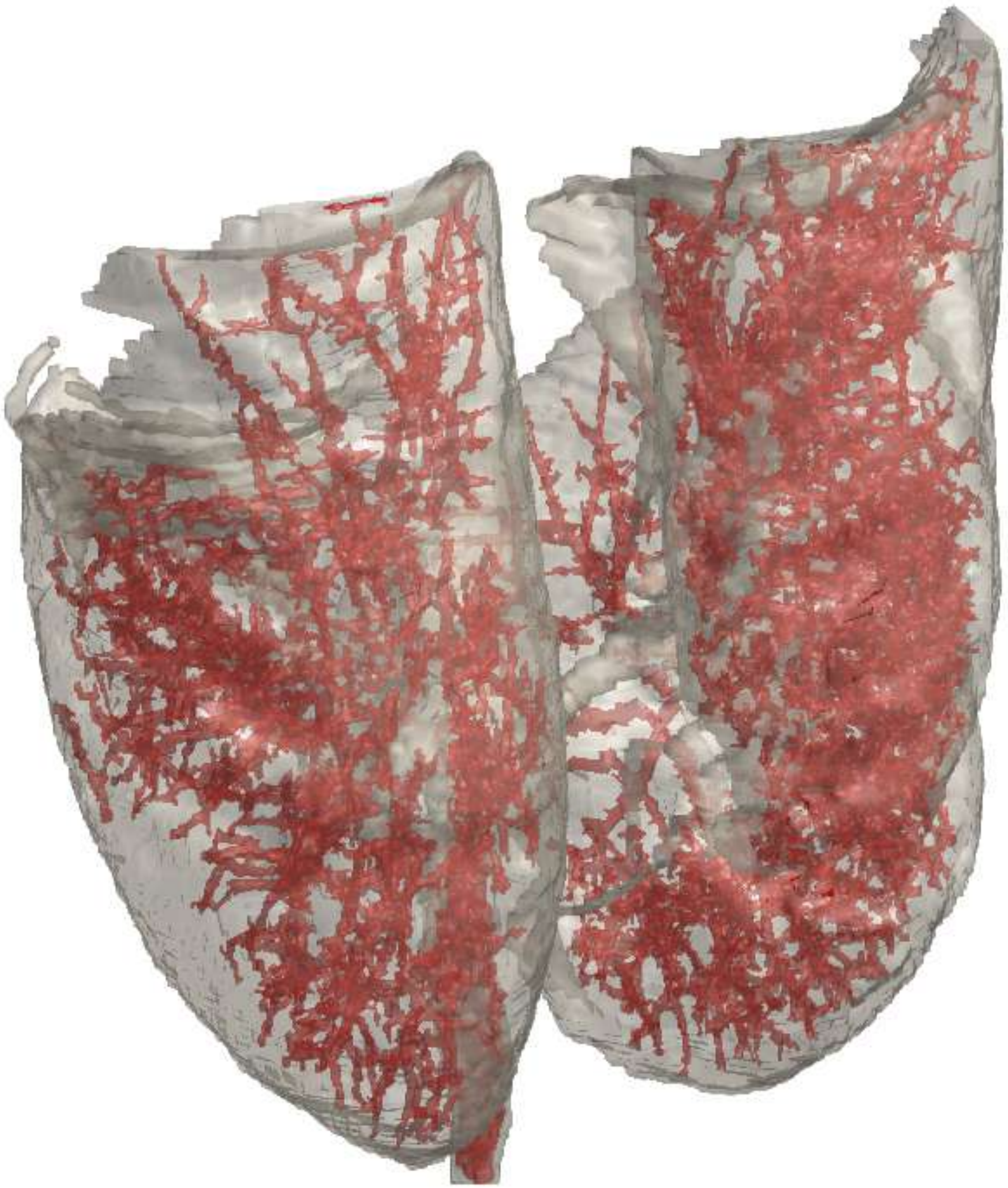


Statistically significant regions (F-stat) of mandible growth in age range between 0 and 20 years

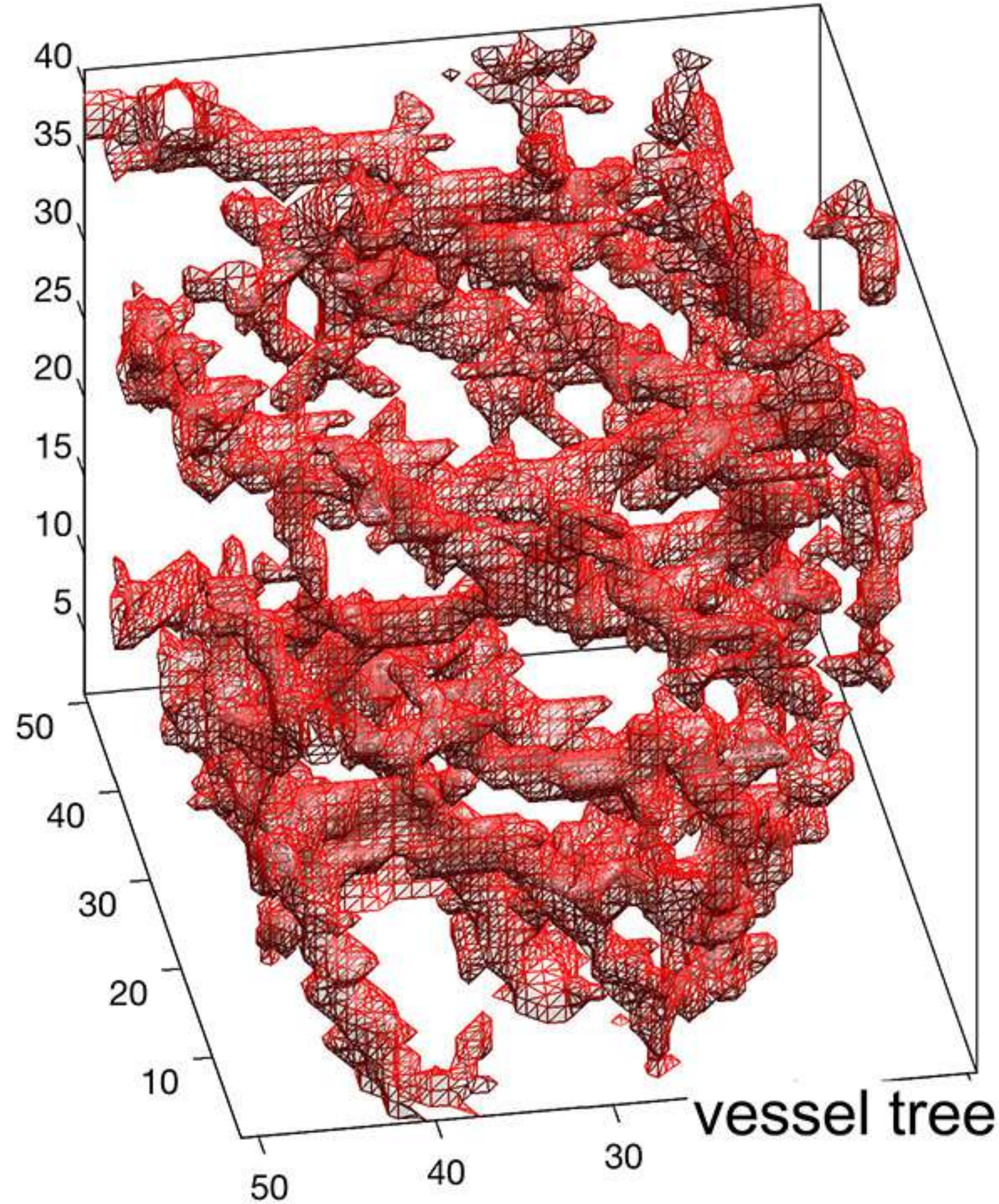


Skeleton Representation of Lung Blood Vessel

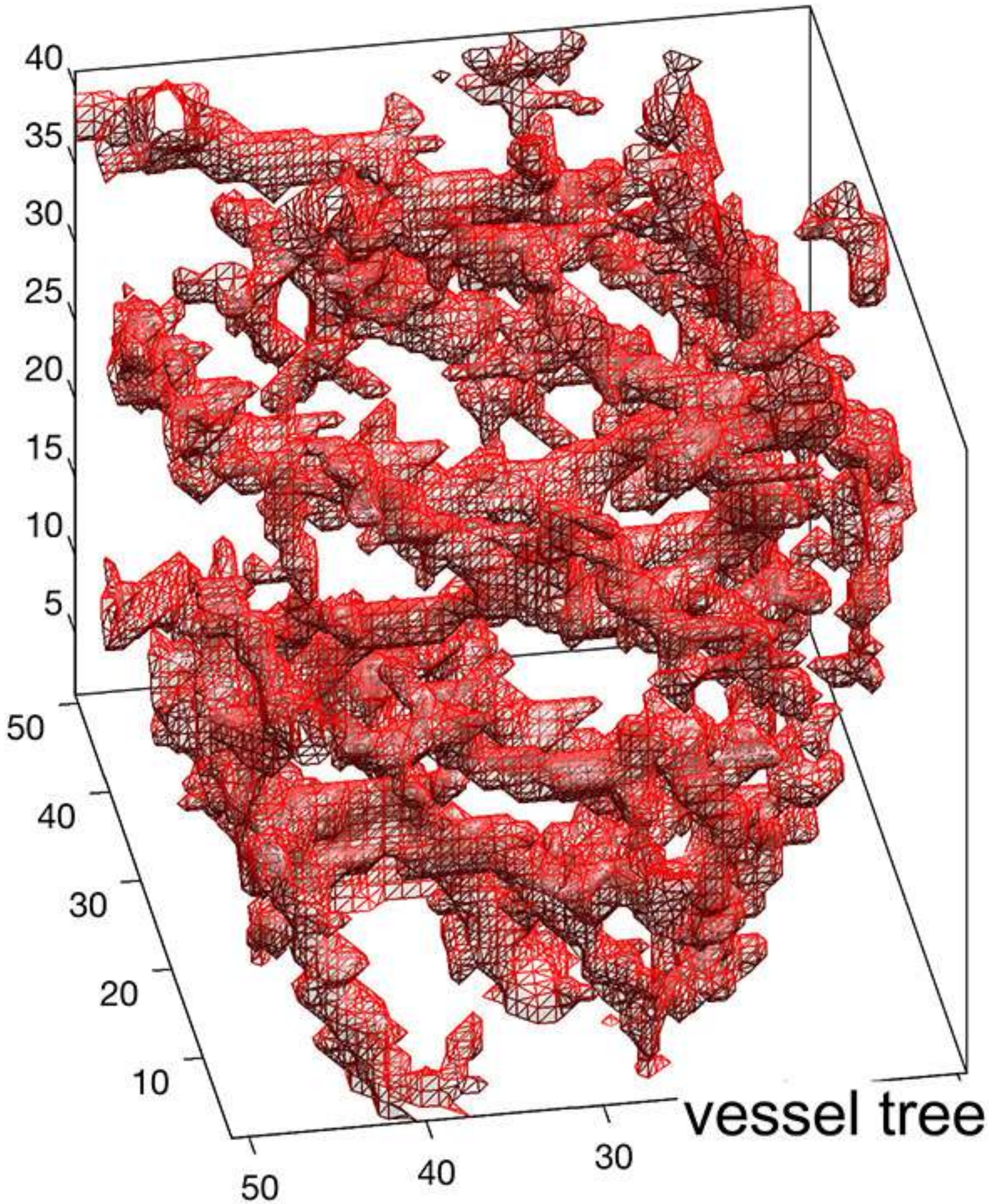
3D computed tomography

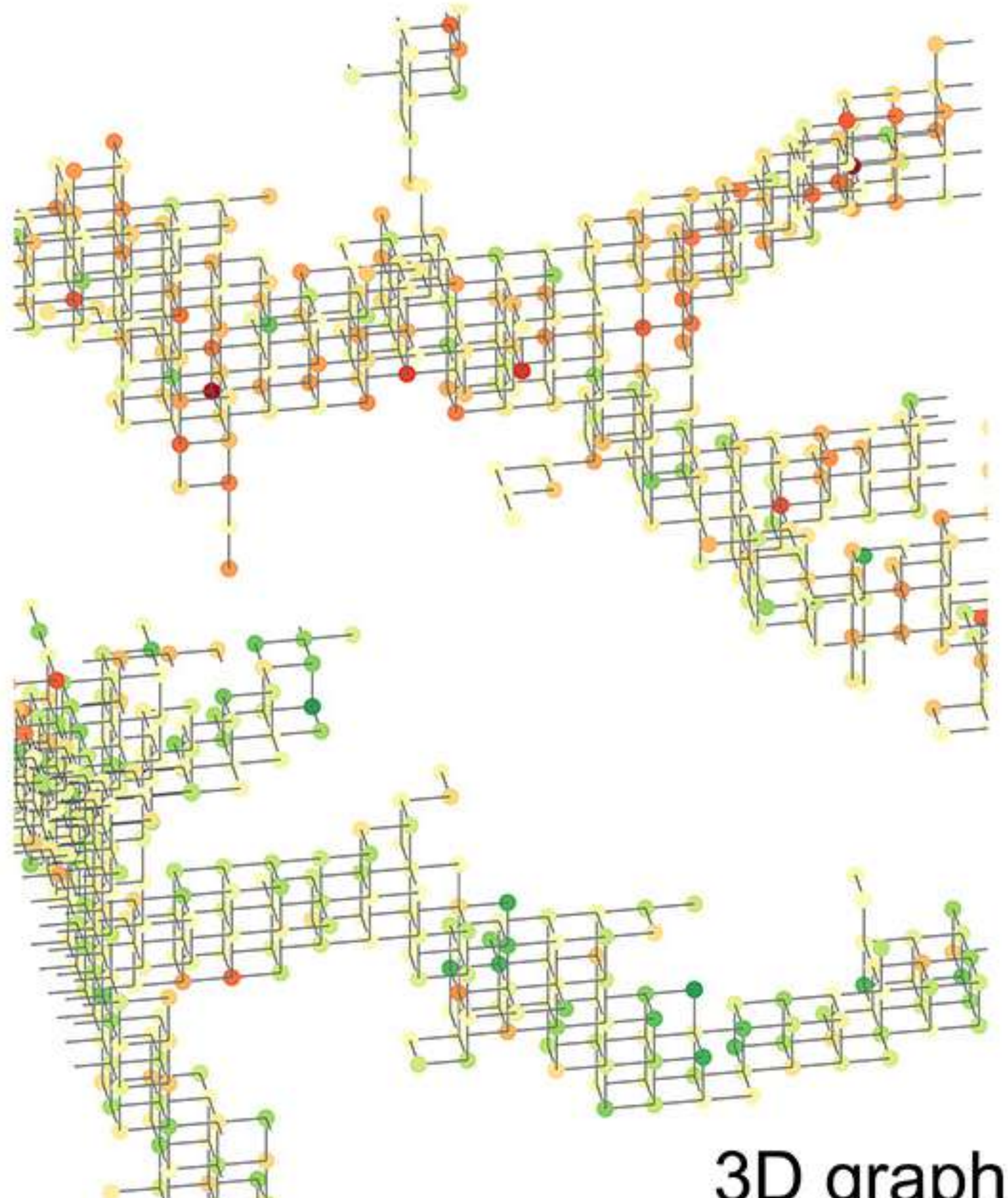


3D binary segmentation



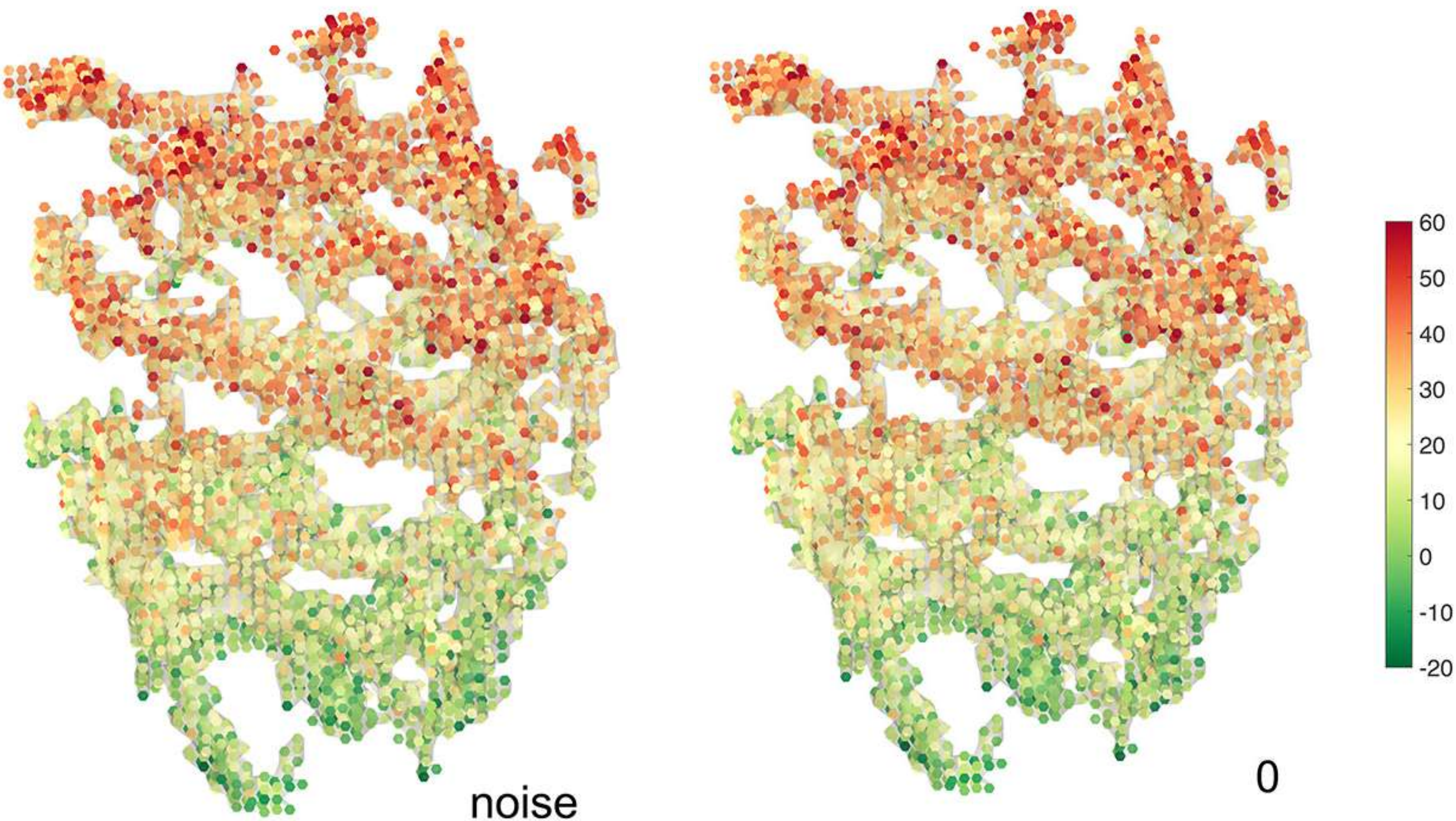
3D binary segmentation 3D graph using 6-neighbors



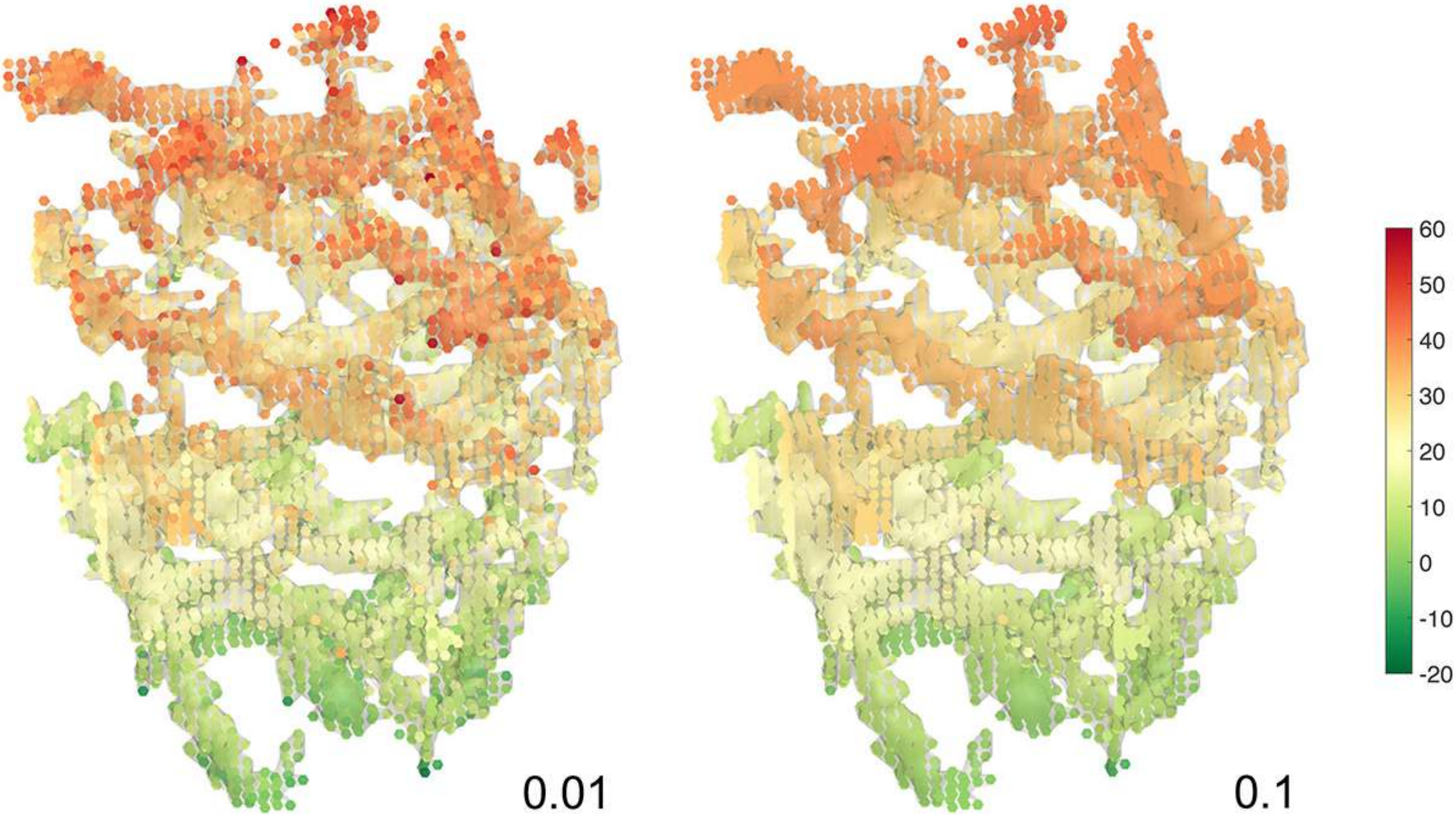


Z-coordinate
+ Gaussian noise

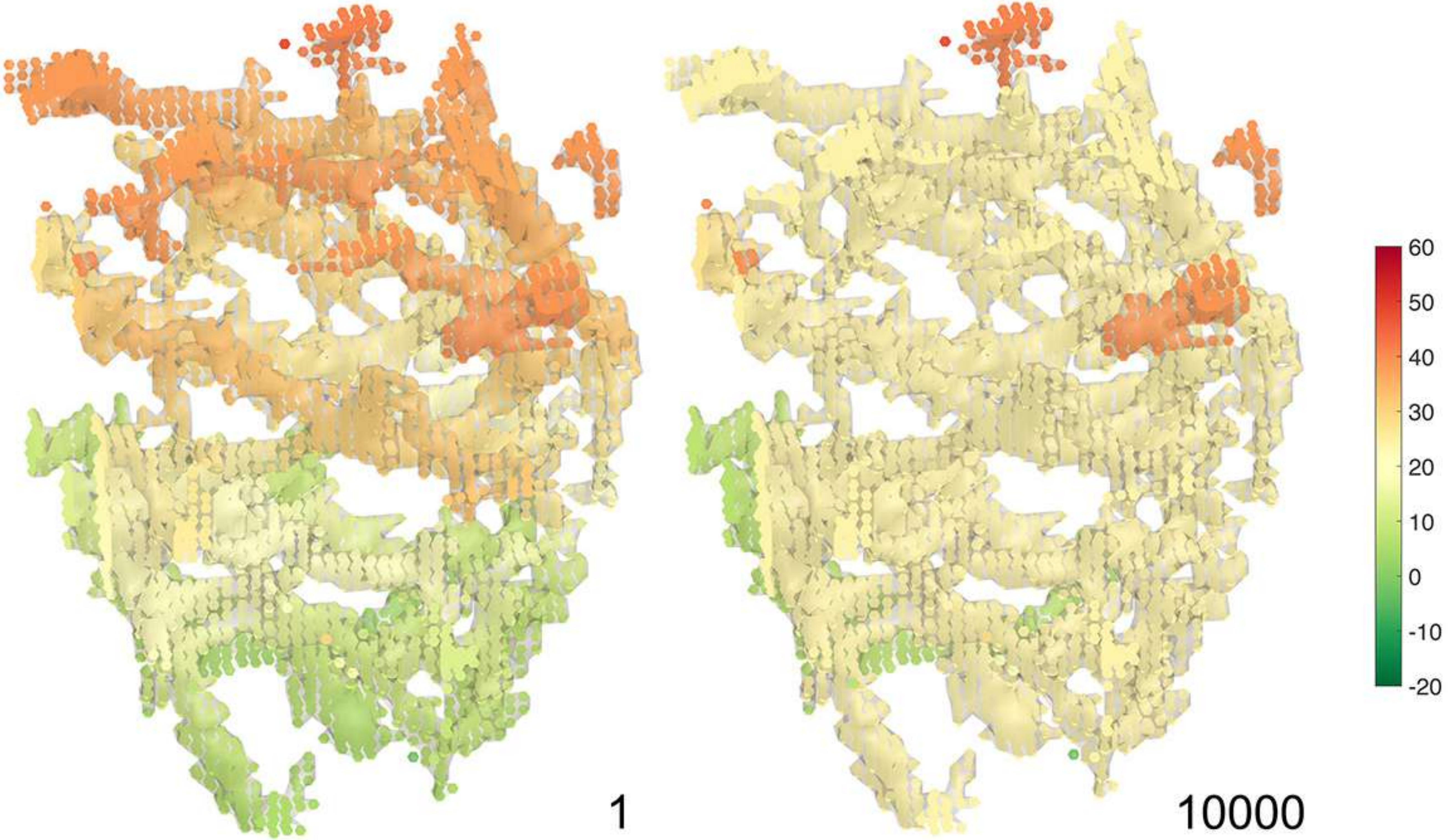
Fourier series expansion
with 6000 basis



Heat kernel smoothing with 6000 basis

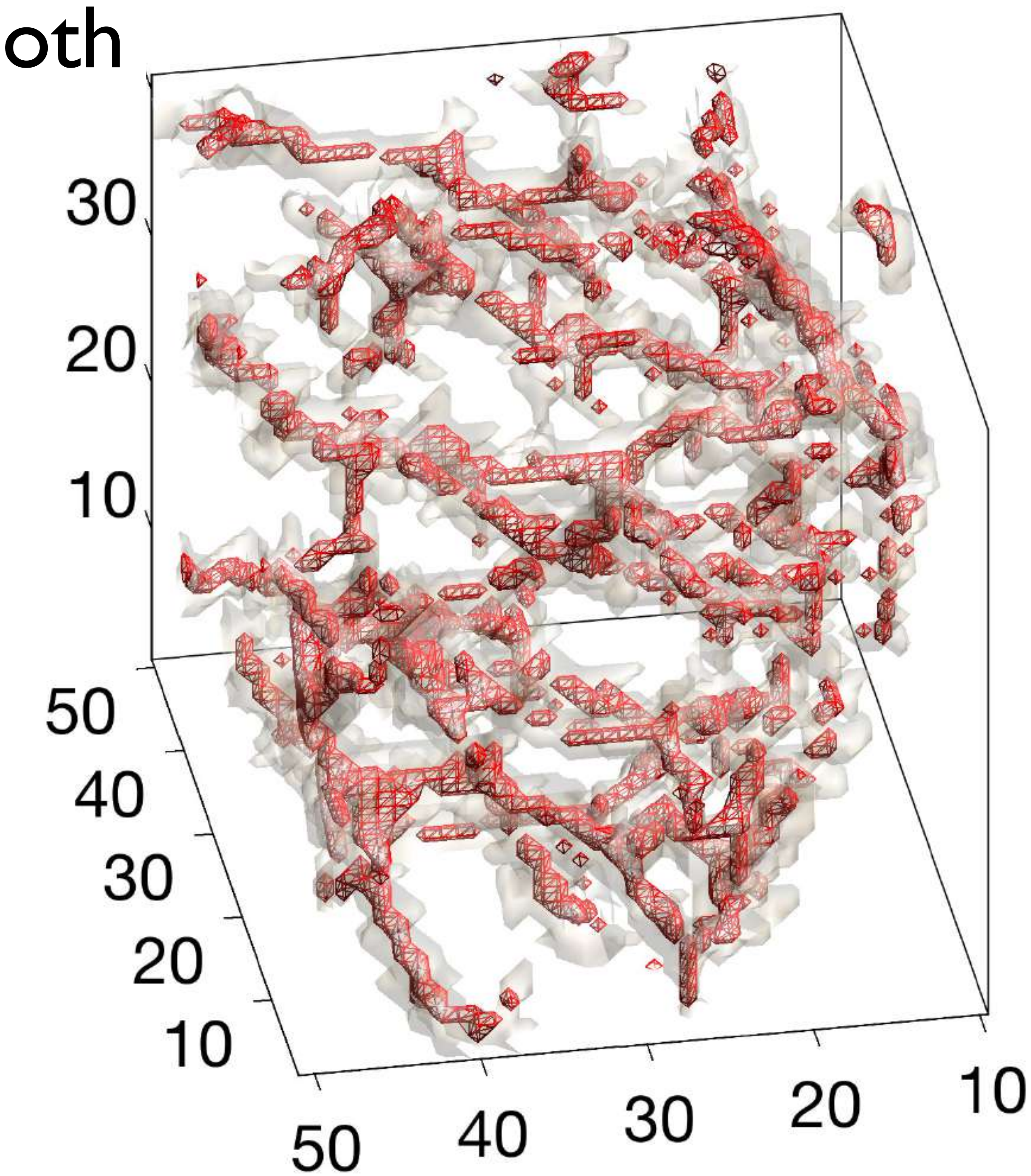


Heat kernel smoothing

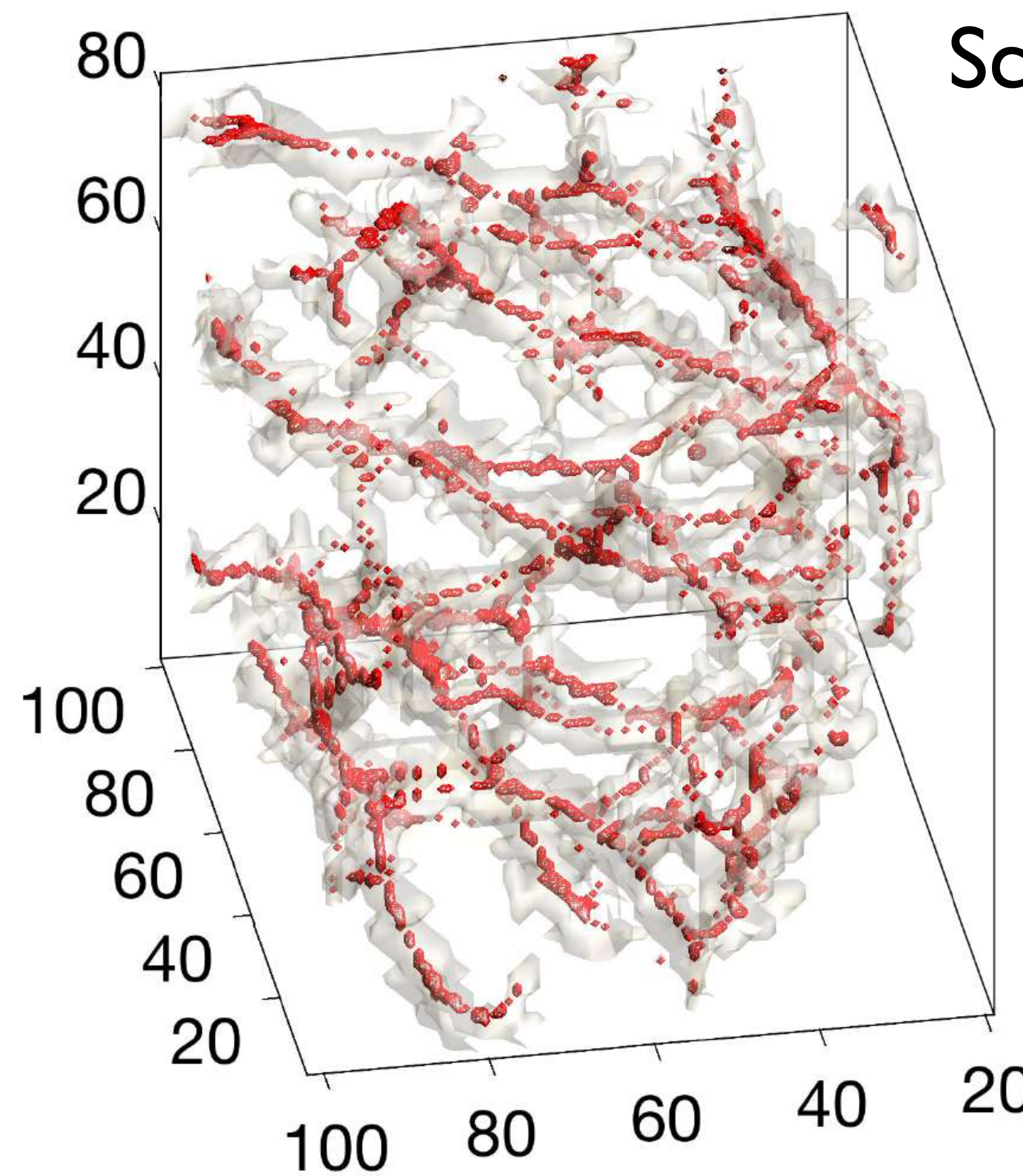


Skeleton representation of blood vessel

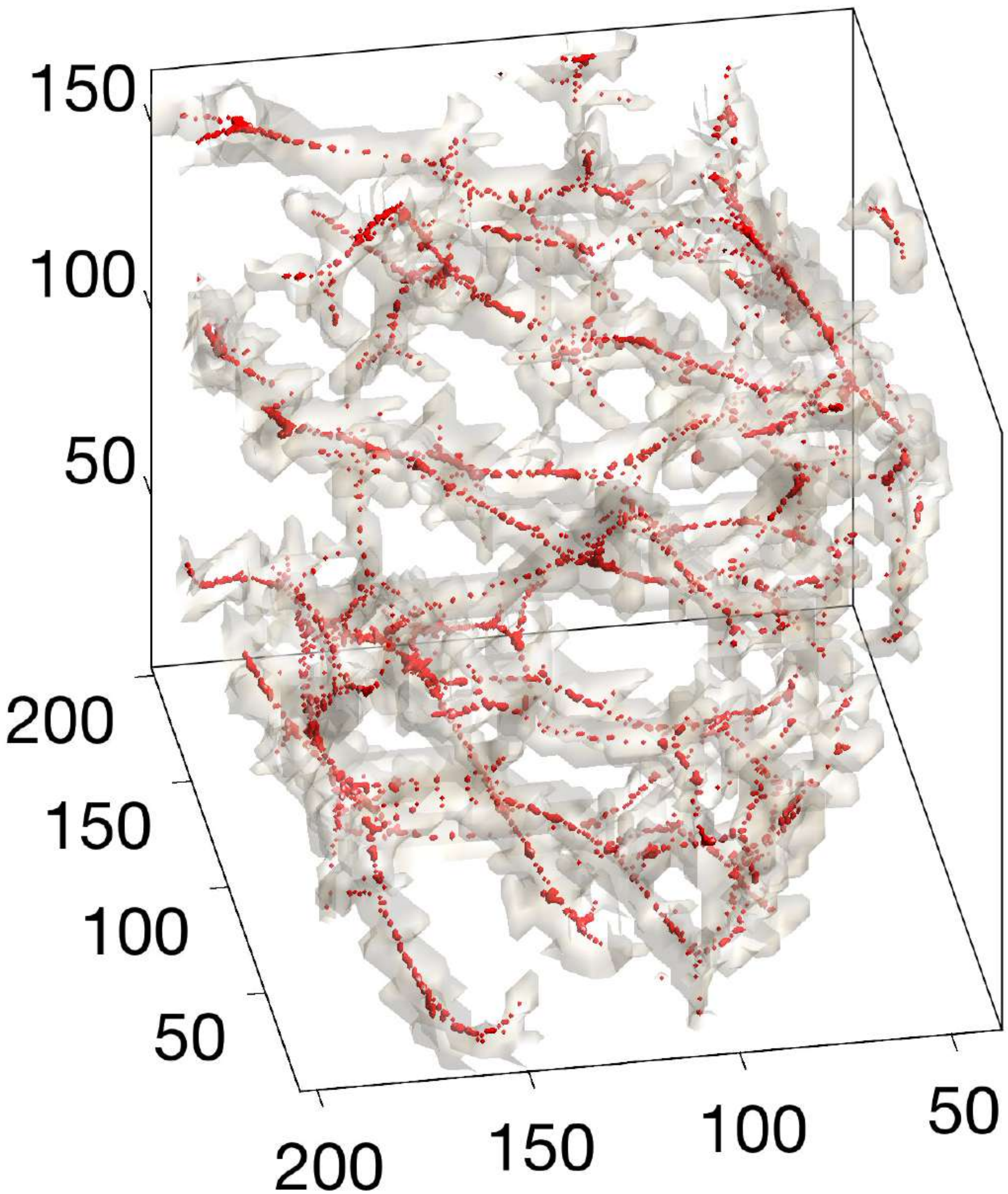
Smooth



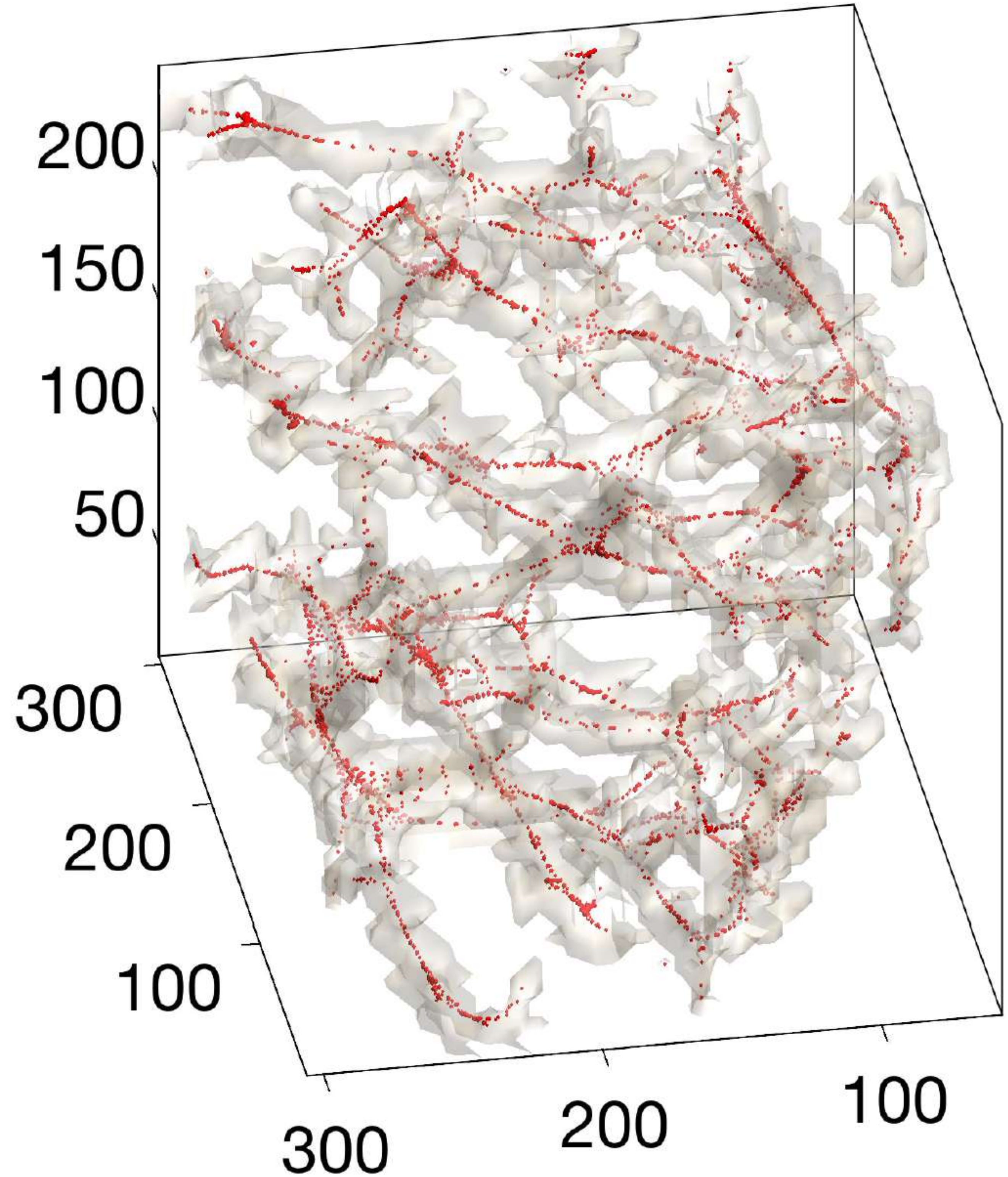
Scale up



Smooth + scale up



Smooth + scale up



Thank you

Any inquiry and collaboration request to

mkchung@wisc.edu