

Medical Image Analysis

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Lecture 3.

Deformation-based Morphometry (DBM)

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Deformation based Morphometry (DBM)

- It uses deformation fields obtained by nonlinear registration of brain images.
- *Automated image registration* (AIR) package (<http://bishopw.loni.ucla.edu/AIR5/index.html>) uses polynomial basis.
- SPM package uses cosine basis.

Image Registration

- Process of transforming one image to another.
- Transformation types: linear (rigid-body), affine (non rigid-body), nonlinear.
- Type of data obtained: 3D vector fields (displacement, deformation).

Affine Transformation matrix for (rigid, non rigid)

- R : 3 x 3 matrix of rotation, scaling and shear
- $p' = Rp + c$, where

$$p' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, c = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$$

Linear transform

- T is a linear transform if $T(ap+bq) = aT(p)+bT(q)$ for all numbers a, b .
- Checking if the affine transform is linear. Let $T(p)=Rp+c$. Note $T(ap+bq)=R(ap+bq)+c=aT(p)+bT(q)+c(1-a-b)$.
- This shows the affine transform is nonlinear.

Rigid-body:

- Rotation and Translation only



Original Image



Shape doesn't change

Source: A. Chowdhury,
University of Georgia

Non rigid-body

Scaling, Translation, Rotation, Reflection,
Shear



Original Image



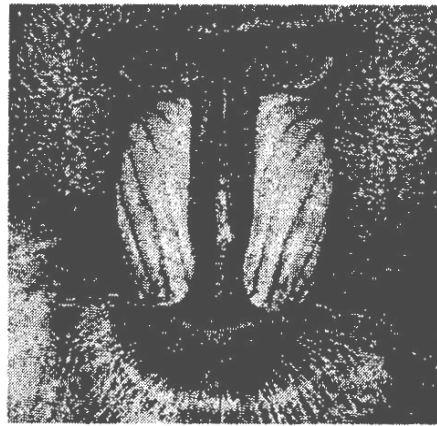
Non rigid-body

Nonlinear transform

- Anatomical variability is encoded in the nonlinear transform.



Original image



Target Image



Result of warping

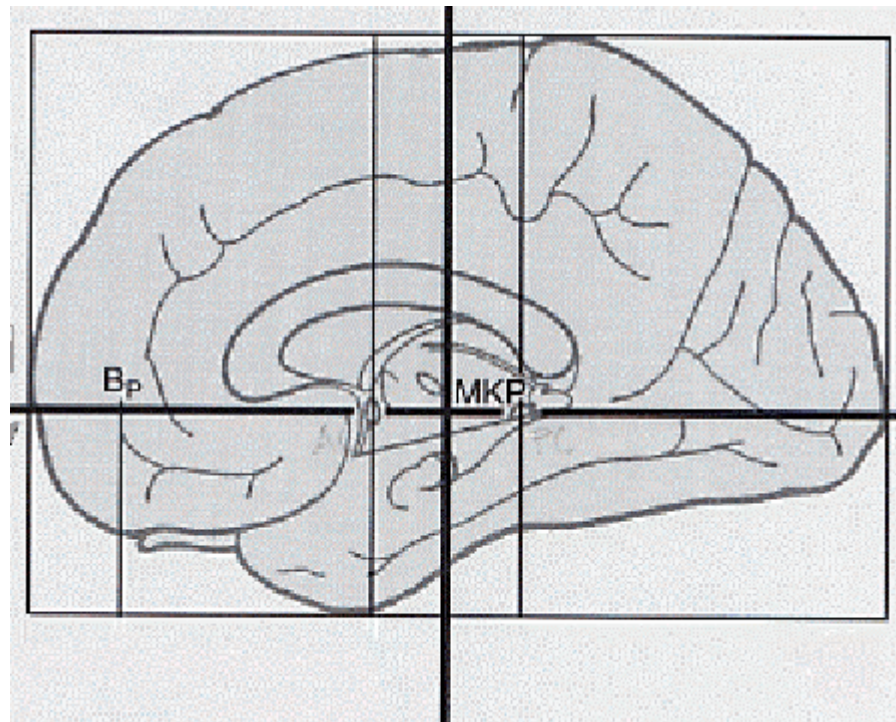
Piecewise Affine

- The Talairach normalization approach
- It uses a different matrix transformation for each of the 12 pieces of the Talairach Grid
- This is a really old technique based on an old subject.
- It provides an easy reference for comparing different study results.
- Other than for comparing results, it should not be used.

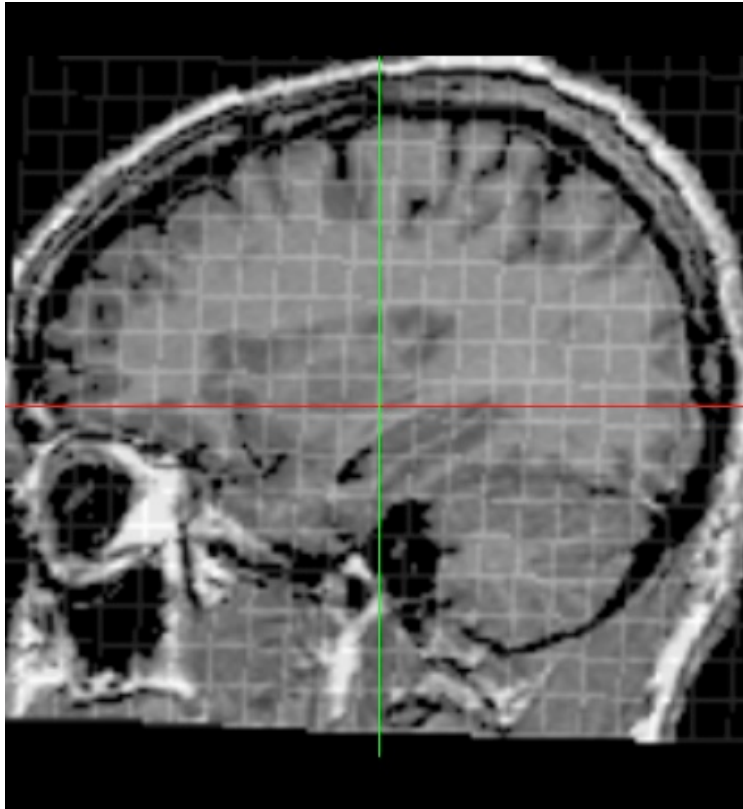
Talairach Definition

- Interhemispheric plane (3+ landmarks) $\Rightarrow$ 2 rotations and 1 translation
- Anterior and posterior commissure (AC, PC) $\Rightarrow$ 3rd rotation, 2 translations
- Scale to anterior, posterior, left, right, inferior, superior landmarks (7 parameters)
- Each cerebral hemispheres divided into six associated blocks (interhemispheric plane, AC-PC axial plane, 2 coronal planes through AC and PC).

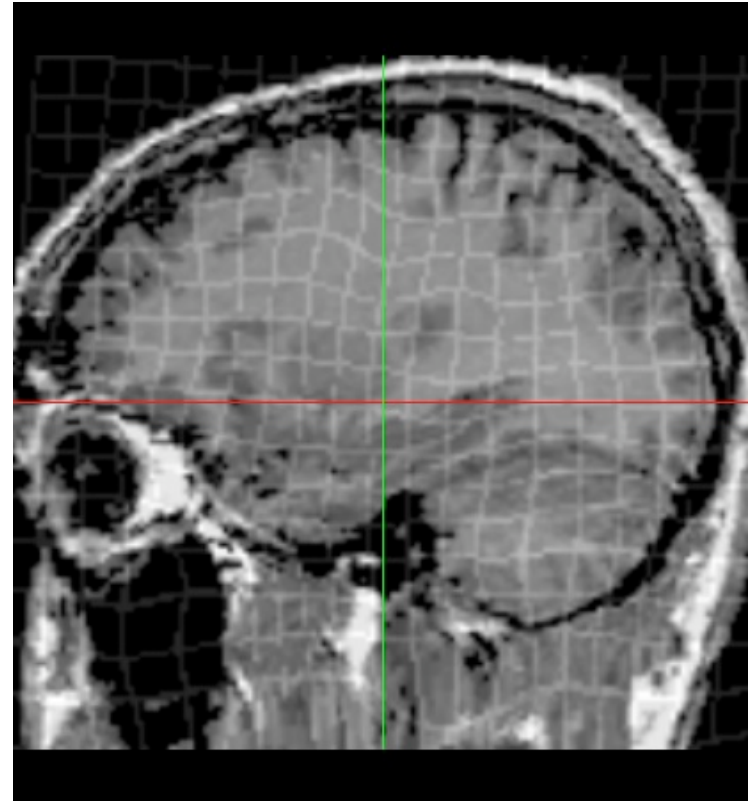
Talairach Template



Affine vs. Nonlinear



Affine – 12 parameters



Non-Rigid ~ 2000 parameters

Mathematical details of nonlinear registration will be studied later.

Source: Lawrence H. Staib
Yale University

Deformation field

- The deformation field d is a transformation from a subject image to a target image:

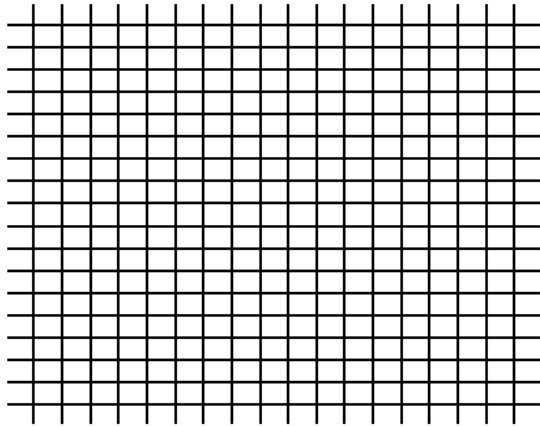
$$d : (x, y, z) \rightarrow (x', y', z')$$

$$(x', y', z') = d(x, y, z)$$

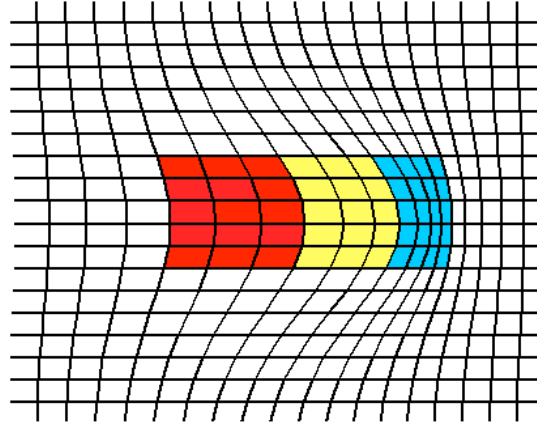
- Example: AIR 3rd degree warping

$$\begin{aligned} x' = & -1.24 - 0.47x - 1.02 \times 10^{-2}y + 0.99z + 1.25 \times 10^{-5}x^2 - 2.66 \times 10^{-5}xy + 4.04 \times 10^{-5}y^2 + 2.35 \times 10^{-6}xz \\ & + 9.23 \times 10^{-5}yz + 5.39 \times 10^{-5}z^2 + 3.60 \times 10^{-8}x^3 + 1.04 \times 10^{-7}x^2y - 9.3 \times 10^{-10}xy^2 + 1.17 \times 10^{-7}y^3 \\ & - 2.2 \times 10^{-8}x^2z + 1.24 \times 10^{-7}xyz - 6.82 \times 10^{-7}y^2z - 1.24 \times 10^{-7}xz^2 - 1.69 \times 10^{-8}yz^2 - 2.76 \times 10^{-7}z^3. \end{aligned}$$

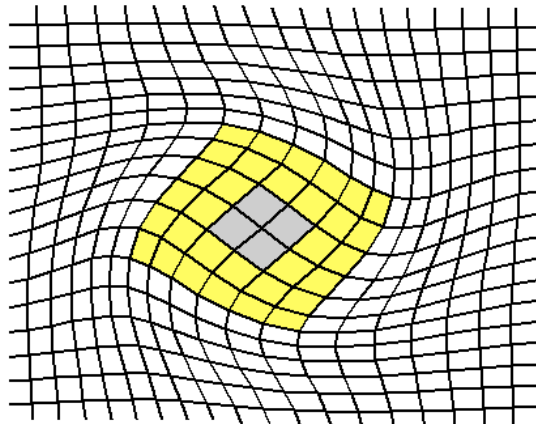
Visualizing Deformation



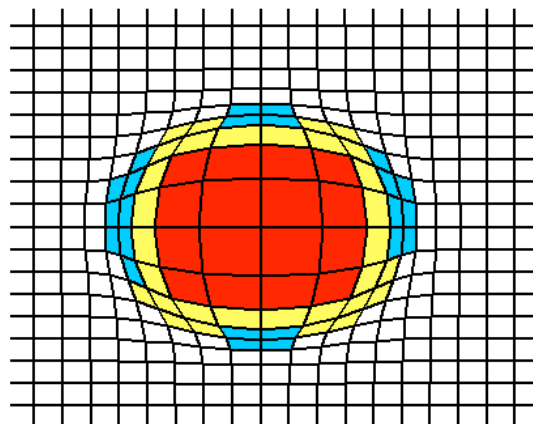
a



b



c



d

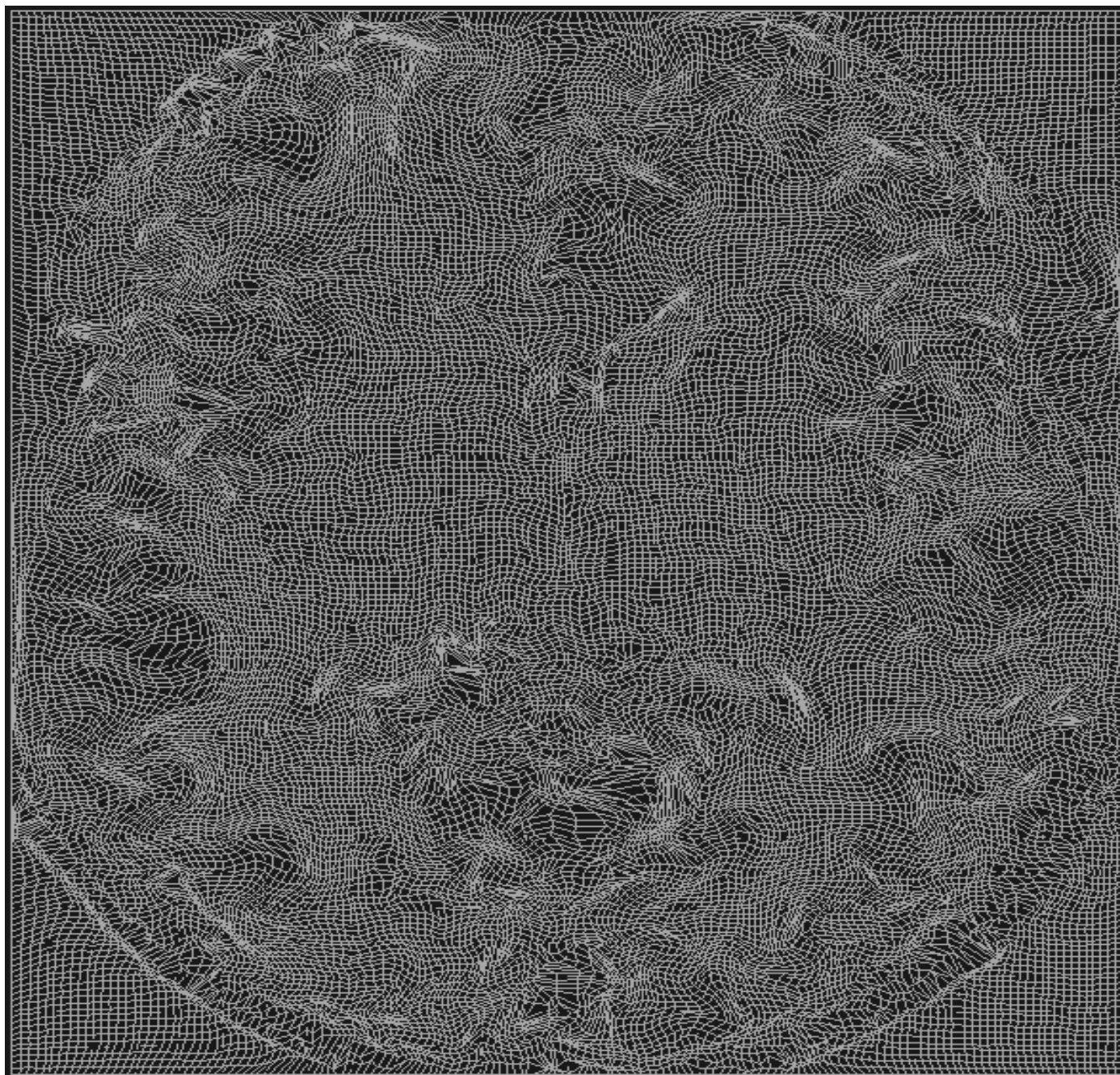
Red: tissue expansion

Blue: tissue shrinking

Yellow: deformation change

**Deformation
field
visualization**

**SPM
result**



Displacement vector fields

- The vector difference between the final position and the original position.

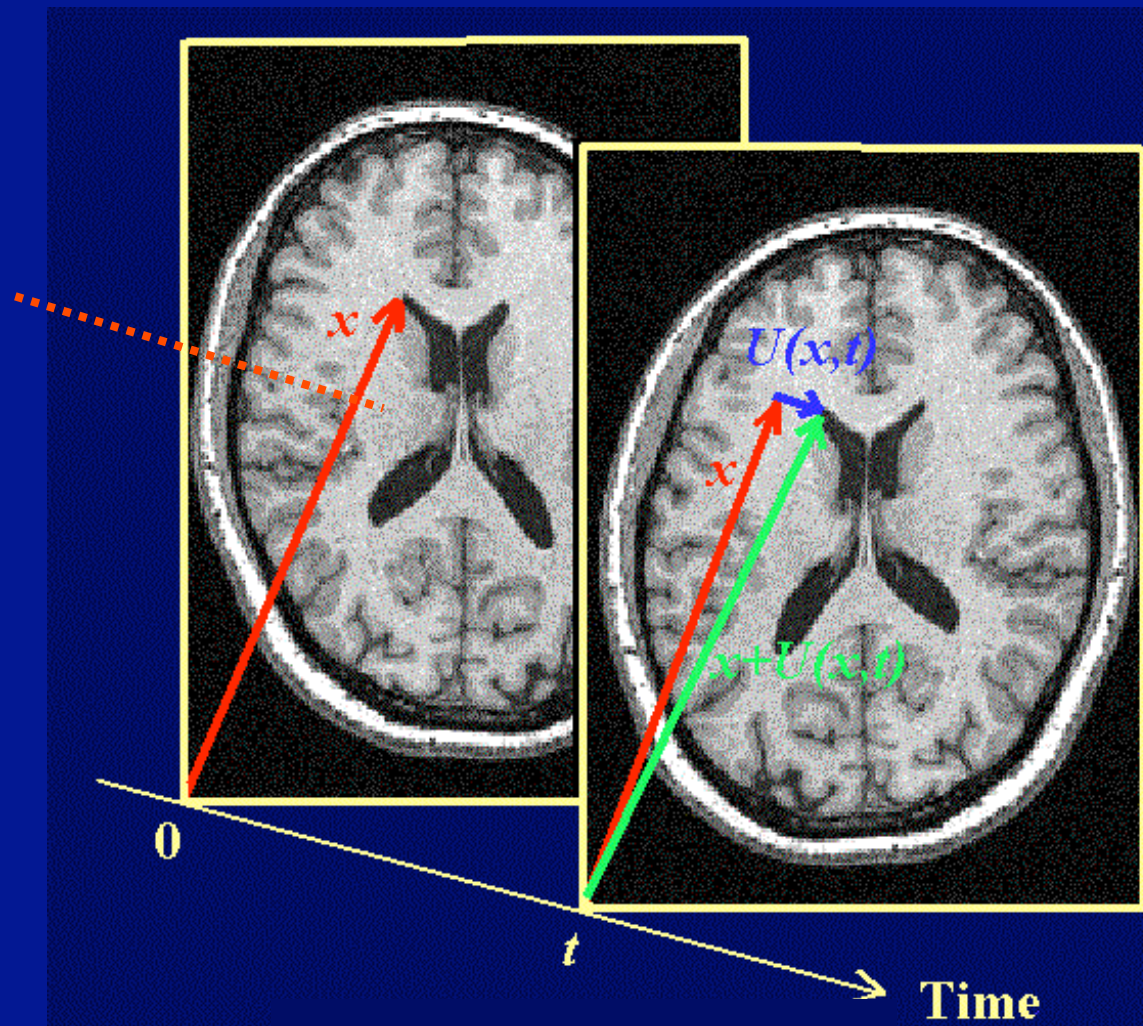
$$(x', y', z') - (x, y, z)$$

$$U(x, y, z) = d(x, y, z) - (x, y, z)$$

- Relation between displacement and deformation

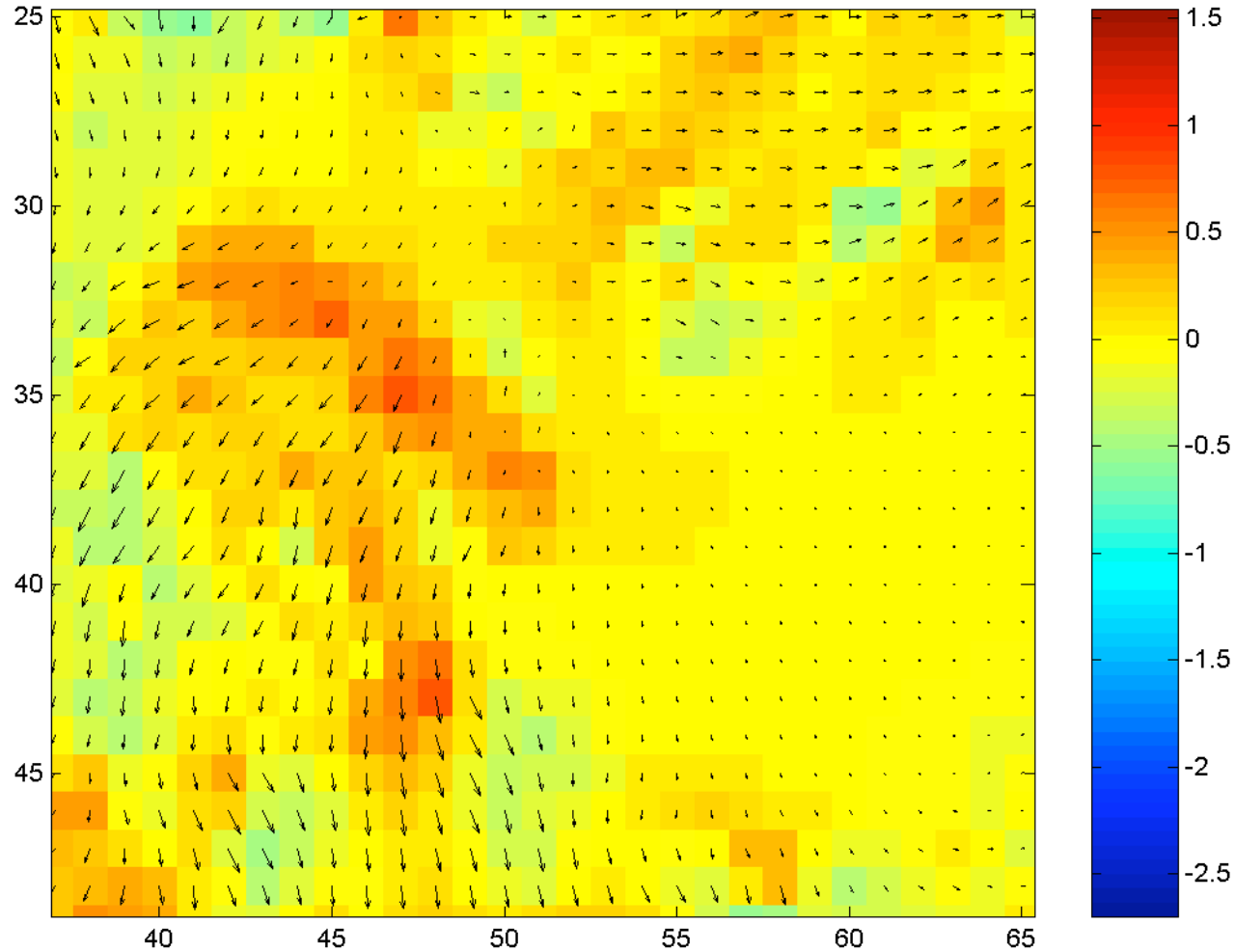
$$d(x, y, z) = (x, y, z) + U(x, y, z)$$

Visualizing Displacement Vector Field



$$x \rightarrow x + U(x, t)$$

Visualizing Displacement Vector Field



Displacement is easier to model statistically than deformation

Real Example: modeling tissue growth

- Data: 28 normal subjects
- Images: 1.5T MRI, 2 scans per subjects
- First scan: 11.5 ± 3.1 year
Second scan: 17.8 ± 3.2 year

Problem: localize the regions of anatomical change over time.

Modeling on the rate of displacement change

Why? Easier than modeling on deformation itself.

$$\frac{\partial U}{\partial t}(x, t) = \mu_0(x) + \Sigma^{1/2}(x)\epsilon(x)$$

Σ : Covariance matrix

ϵ : Gaussian random vector field

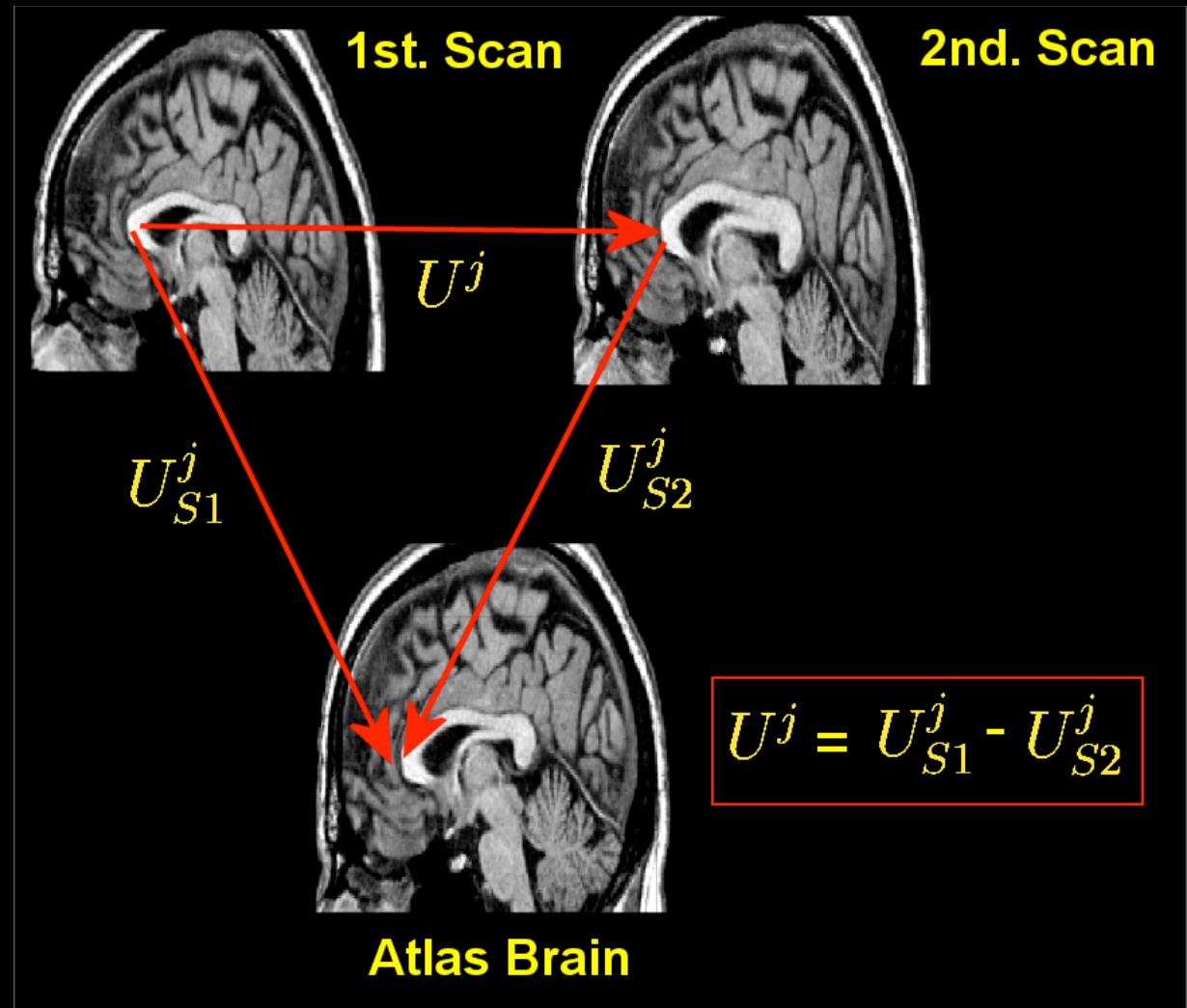
$$H_0 : \mu_0 = 0 \text{ vs. } H_1 : \mu_1 \neq 0$$

Estimating the rate of change

- For subject j , displacement is given by U^j
- Finite difference:

$$\frac{\partial U^j}{\partial t} = \frac{U^j}{\text{age difference}}$$

Registration
procedures in
Chung et al.
(2001)
Is not optimal.



How it should be done correctly? To reduce registration error, 1st scans to be registered to the 2nd scans.

Hotelling's T-square statistic

Most efficient way of computing it in MATLAB

- Rate of displacement V^j
- Sample mean $\bar{V} = \sum_{j=1}^n V^j$
- Sample covariance $\hat{\Sigma} = \frac{1}{n-1} \sum_{j=1}^n (V^j - \bar{V})(V^j - \bar{V})^T$
- Hotelling's T-square $H = \bar{V}^T \hat{\Sigma}^{-1} \bar{V} \approx cF_{3,n-3}$
(related to Mahalanobis distance)

Multiple comparison correction via random field theory

We will study this topic later.

Corrected P-value computation

fMRI-STAT package by Keith Worsley
has the MATLAB m code.

$$P\left(\max_{x \in \Omega} H(x) > y\right) \approx \mathbb{E}\chi(A_y) \quad A_y = \{x : H(x) > y\}$$

$$\mathbb{E}\chi(A_y) = \phi_3(\Omega)\rho_3(y) + \phi_2(\Omega)\rho_2(y) + \phi_1(\Omega)\rho_1(y) + P\left(H(x) > y\right)$$

3D SPM

Right

Back

Front

Tissue growth $p < 0.025$

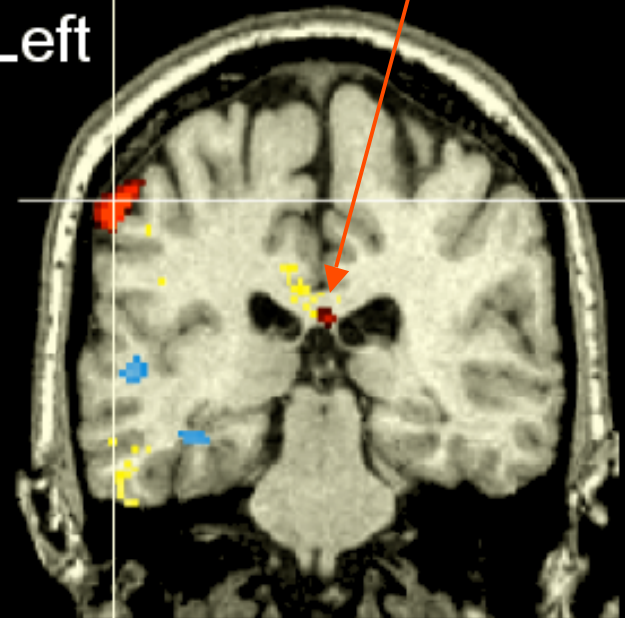
Tissue loss $p < 0.025$

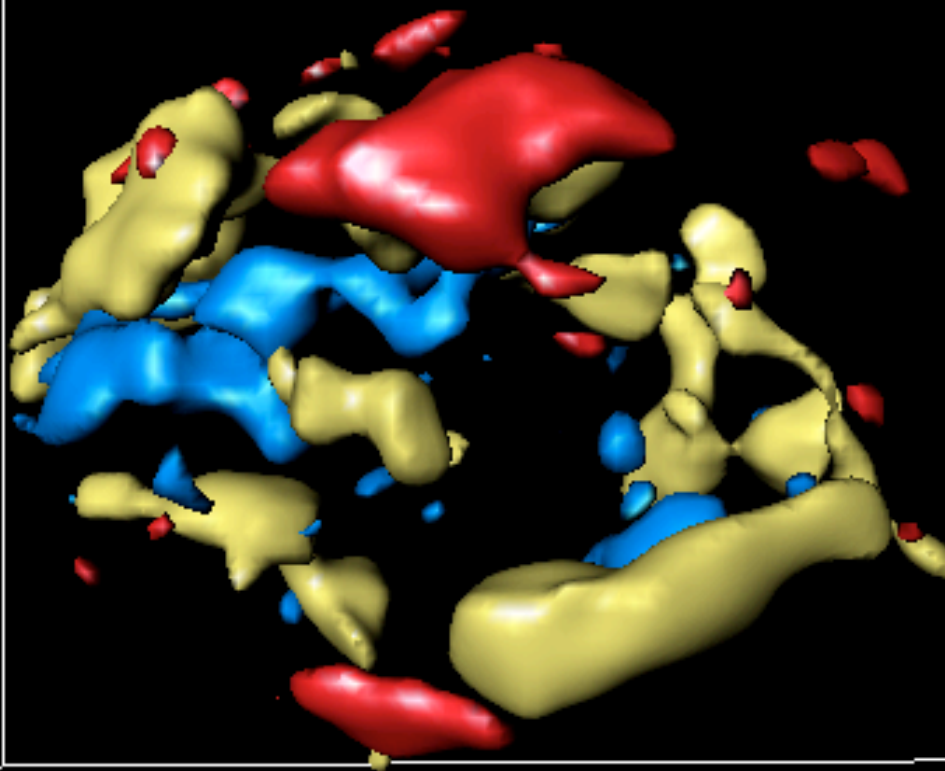
Structure displacement $p < 0.05$

These 3D blobs of signal are meaningless without superimposing them onto MRI.

Corpus Callosum

Left

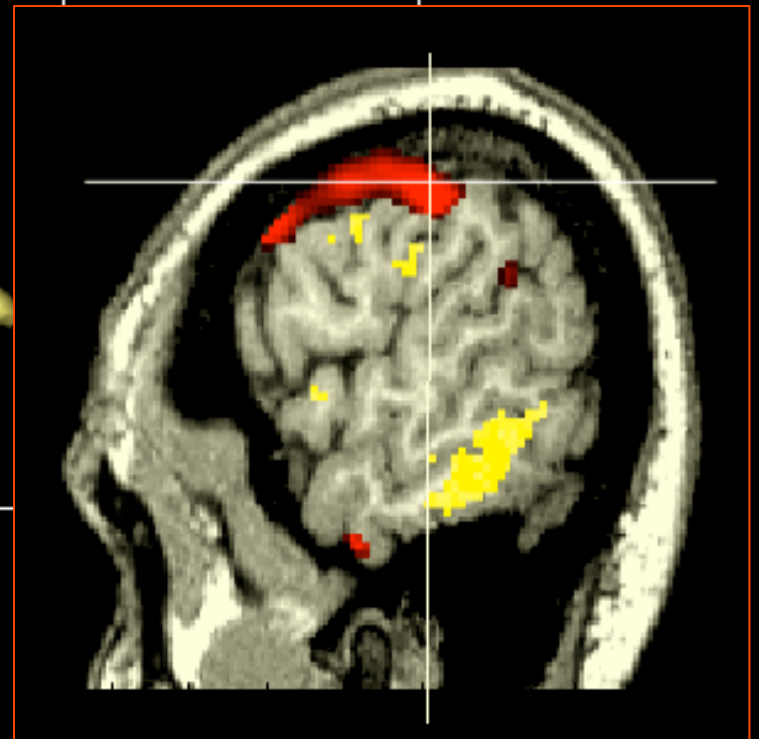




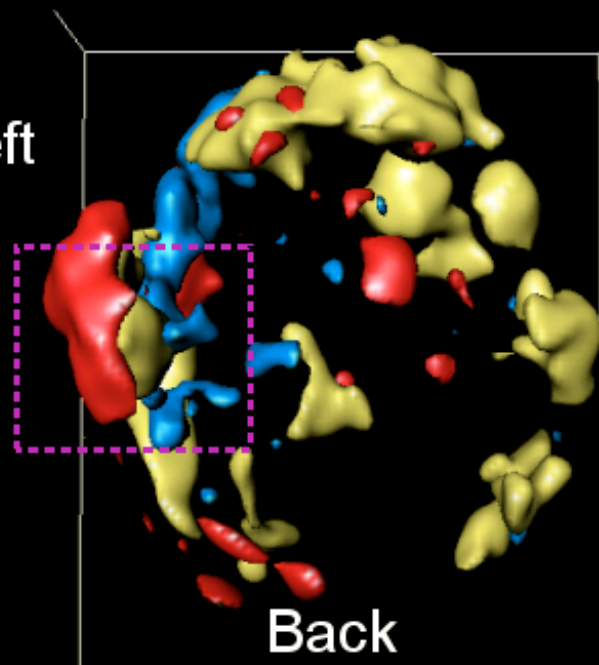
Red: Tissue growth

Blue: Tissue loss

Yellow: Structure displacement

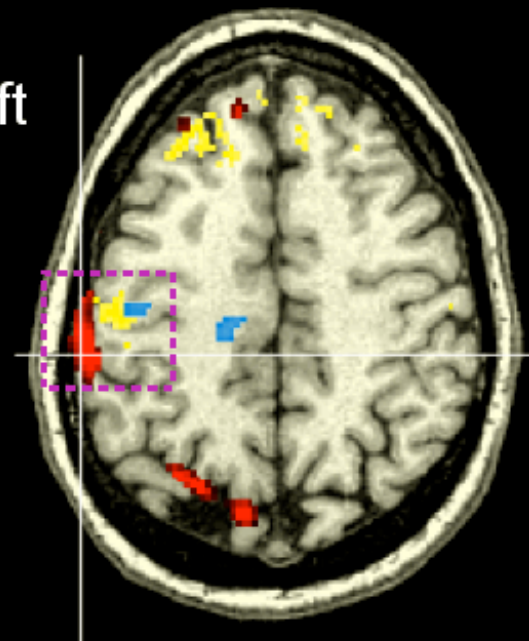


Left

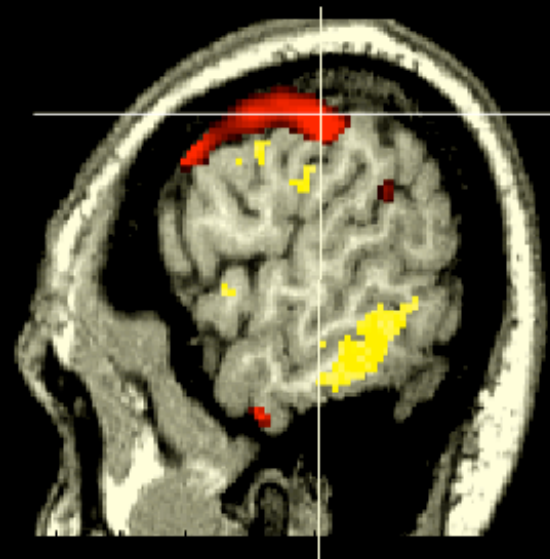


Back

Left



Back



3D SPM

Back

Right

Front

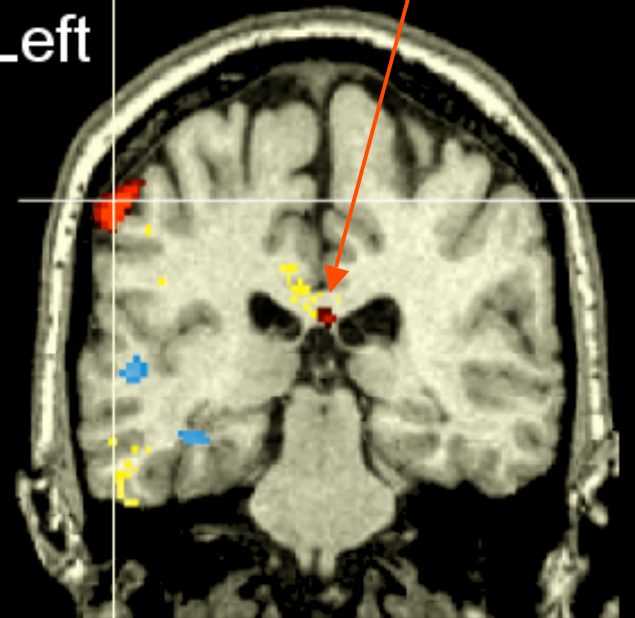
Red: Tissue growth $p < 0.025$

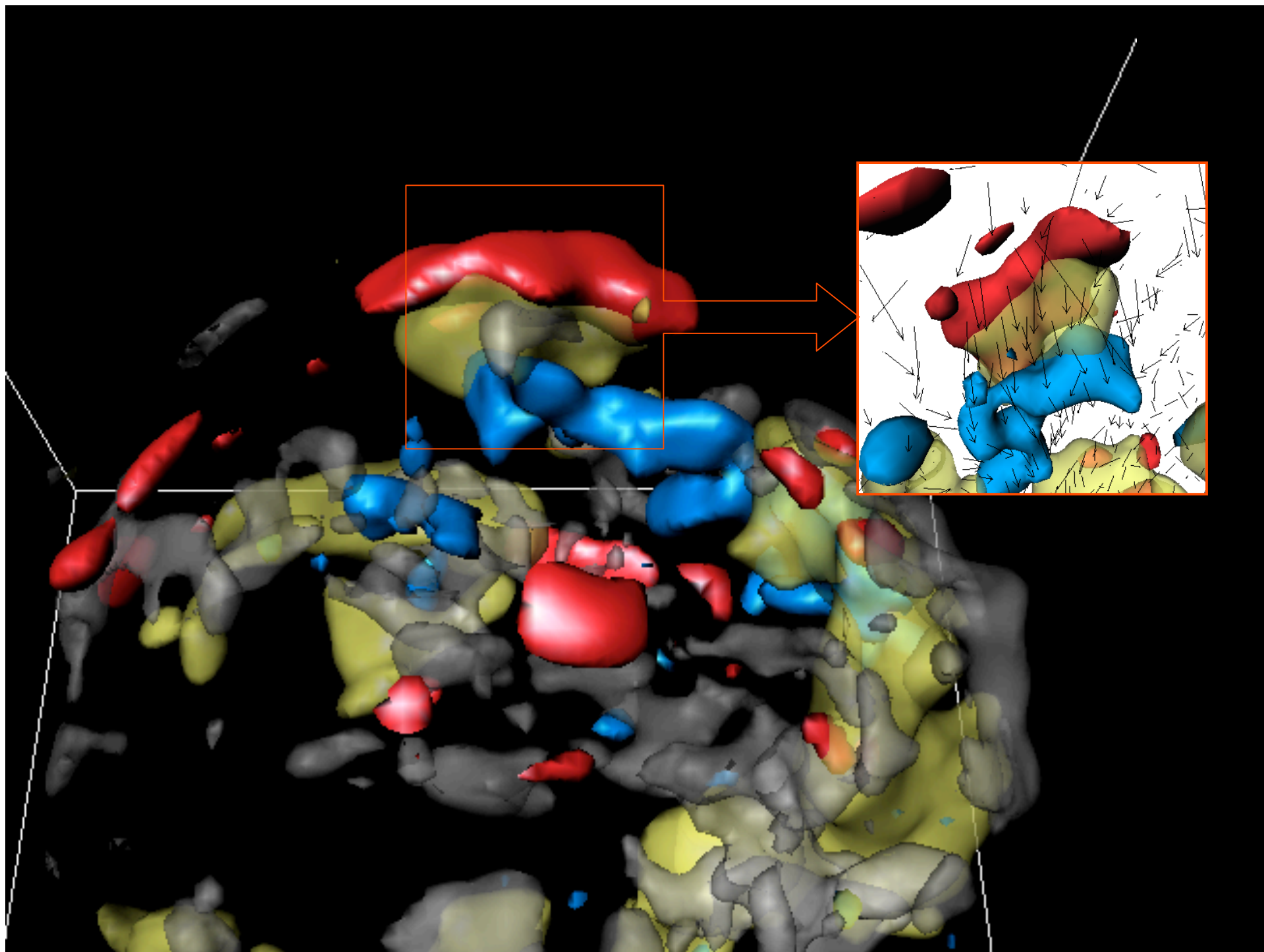
Blue: Tissue loss $p < 0.025$

Yellow: Structure displacement $p < 0.05$

Corpus Callosum

Left





Hotelling's T-square for two sample test

Group index j , subject index i

$$V_{ij} = \mu_j + \Sigma^{1/2} e_{ij}$$

$$H_0 : \mu_1 = \mu_2 \text{ vs. } H_1 : \mu_1 \neq \mu_2$$

- Sample means: $\bar{V}_1, \bar{V}_2$

- Sample covariance: pool the variance across groups

$$\hat{\Sigma} = \frac{1}{n+m-2} \left[\sum_{i=1}^n (V_{i1} - \bar{V}_1)(V_{i1} - \bar{V}_1)^T + \sum_{i=1}^m (V_{i2} - \bar{V}_2)(V_{i2} - \bar{V}_2)^T \right]$$

- Hotelling's T-square statistic for two samples

$$H = \left(\bar{V}_2 - \bar{V}_1 \right)^T \hat{\Sigma}^{-1} \left(\bar{V}_2 - \bar{V}_1 \right) \approx cF_{3,n+m-4}$$

Lecture 4 topics

Tensor based Morphometry (DBM)
and
Jacobian determinant