

Medical Image Analysis

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Lecture 04.

Tensor-based Morphometry (TBM)

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Tensor-based Morphometry (TBM)

- It uses higher order spatial derivatives of deformation fields to construct morphological tensor maps.
- From these tensor maps, 3D statistical parametric maps (SPM) are created to quantify the variations in the higher order change of deformation.

Jacobian determinant (JD)

- The Jacobian determinant J of the deformation field is mainly used to detect volumetric changes.

- Notation:

Voxel position $p = (p_1, p_2, p_3)'$

Deformation $d = (d_1, d_2, d_3)'$

Jacobian determinant

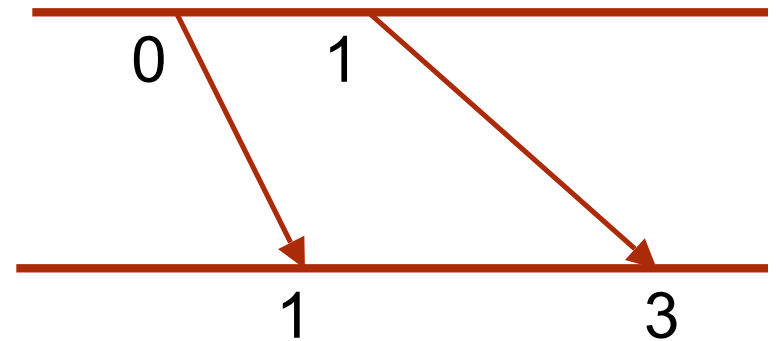
$$J(p) = \det \frac{\partial d(p)}{\partial p'} = \det \left(\frac{\partial d_j}{\partial p_i} \right)$$

Interpretation of Jacobian determinant

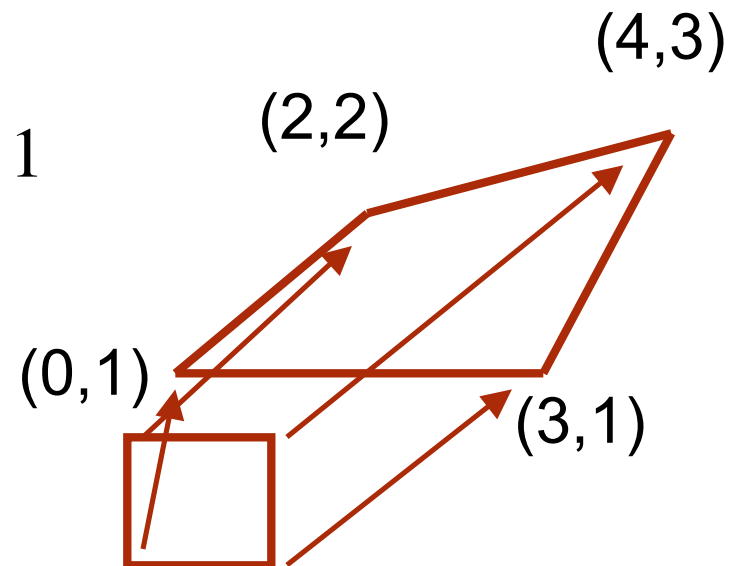
- JD measures the volume of the deformed unit-cube after registration.
- In images, a voxel can be considered a unit-cube
- JD measures how a voxel volume changes after registration.

JD Computation

- 1D example: $x' = 2x + 1$
 $J(x) = 2$



- 2D example: $x' = 2x + y + 1$
 $y' = x + 2y$
 $J(x, y) = 3$



Is it possible to perform TBM using only an affine registration?

- *For affine registration $p' = Ap + B$, the Jacobian determinant is $\det(A)$.*
- Every voxel will have the same scalar value.
- Affine registration based TBM only detect global size difference.

Normality of JD

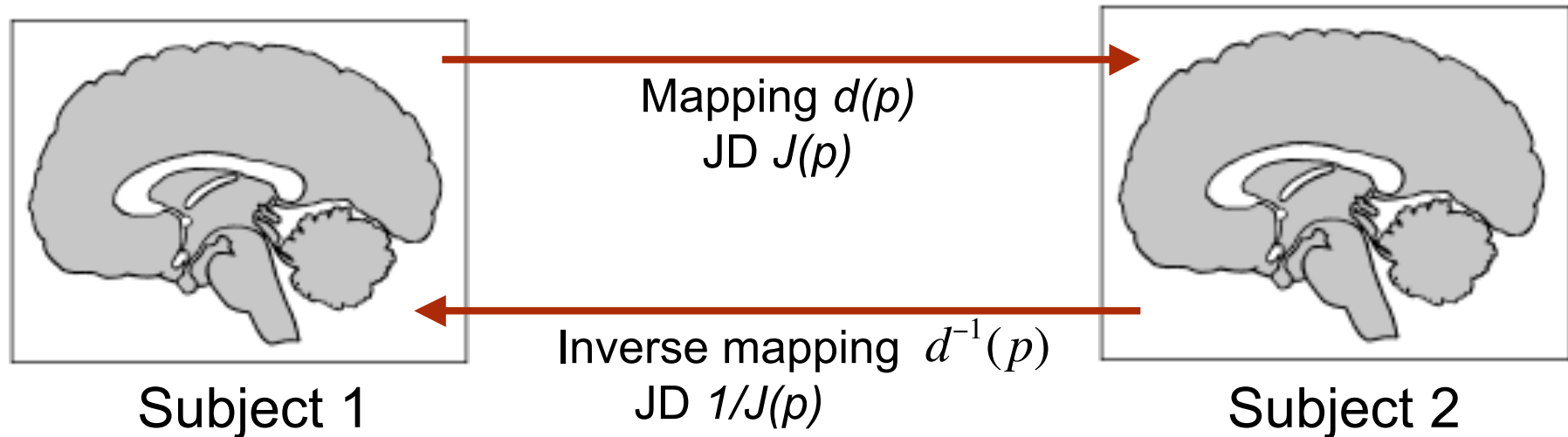
$$d(p) = p + U(p)$$

$$\frac{\partial d(p)}{\partial p'} = I + \frac{\partial U(p)}{\partial p'}$$

$$J(p) \cong 1 + \text{tr}\left(\frac{\partial U}{\partial p'}\right) = 1 + \frac{\partial U_1}{\partial p_1} + \frac{\partial U_2}{\partial p_2} + \frac{\partial U_3}{\partial p_3}$$

Note that we modeled displacement U to be a Gaussian random field. Any linear operation (derivative) on a Gaussian random field is again Gaussian. So J is approximately a Gaussian random field.

Properties of JD



- $J(p) > 0$ for one-to-one mapping
- $J(p) > 1$ volume increase; $J(p) < 1$ volume decrease
- Due to symmetry, the statistical distribution of $J(p)$ and $1/J(p)$ should be identical.

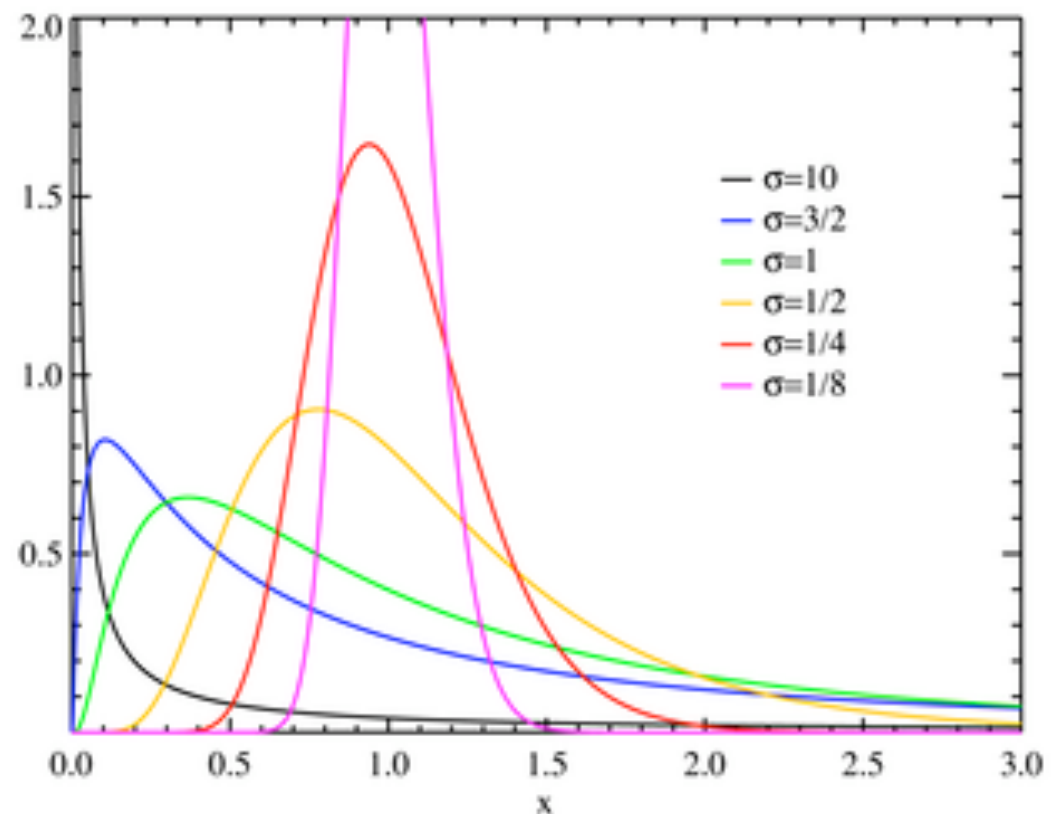
Lognormality of JD

- Domain $-\infty < \log J(p) < \infty$
- If $J(p)=1$, $\log J(p) = 0$
- Symmetry: $\log[J^{-1}(p)] = -\log J(p)$
- These 3 properties show that JD can be modeled as *lognormal distribution*.

Lognormal distribution

- Random variable X is log-normally distributed if $\log X$ is normally distributed.
- For $E \log X = 0$, the shape of density:

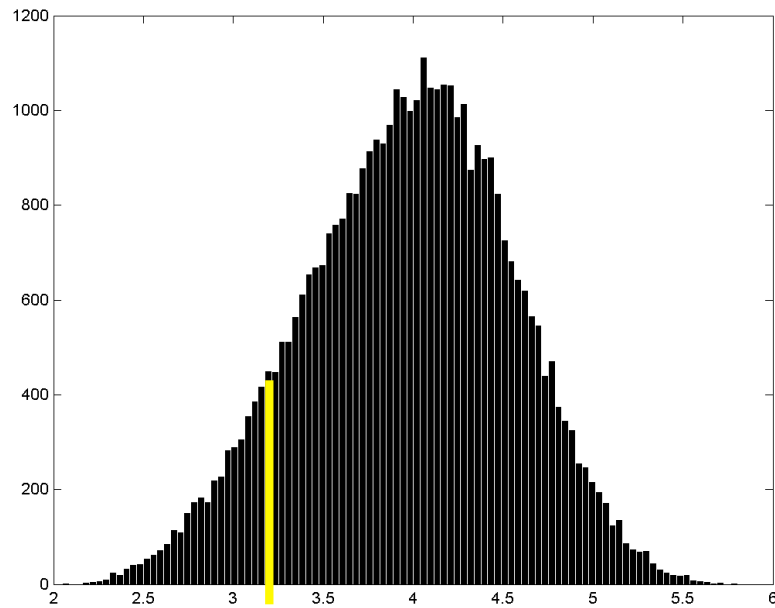
Some lognormal distribution looks normal so how do we check if data follows normal or lognormal?



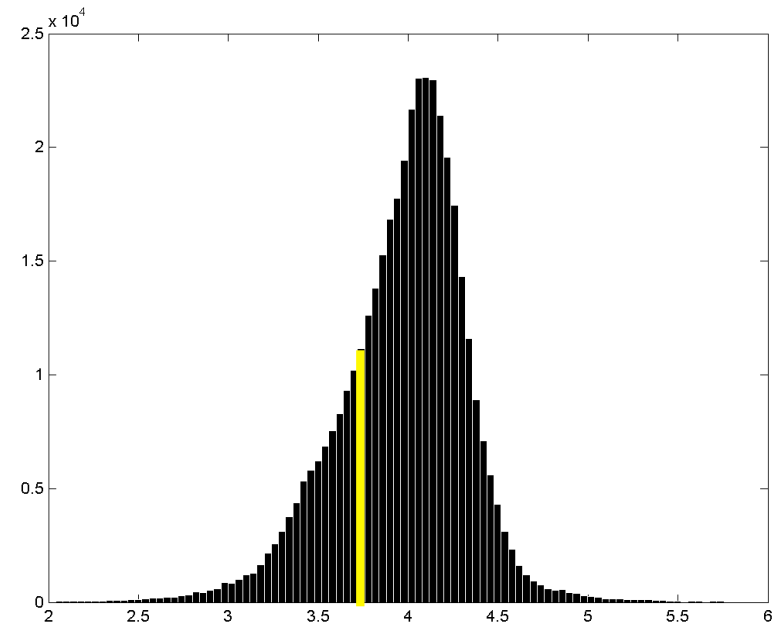
Testing normality of data

- How do we check if JD is normal or lognormal emphatically?
- Quantile-quantile (QQ) plot can be used. For given probability p , the p -th quantile of random variable X is the point q that satisfies $P(X < q) = p$.
- See supplementary lecture material.

QQ-plot compares quantiles



x



y

y

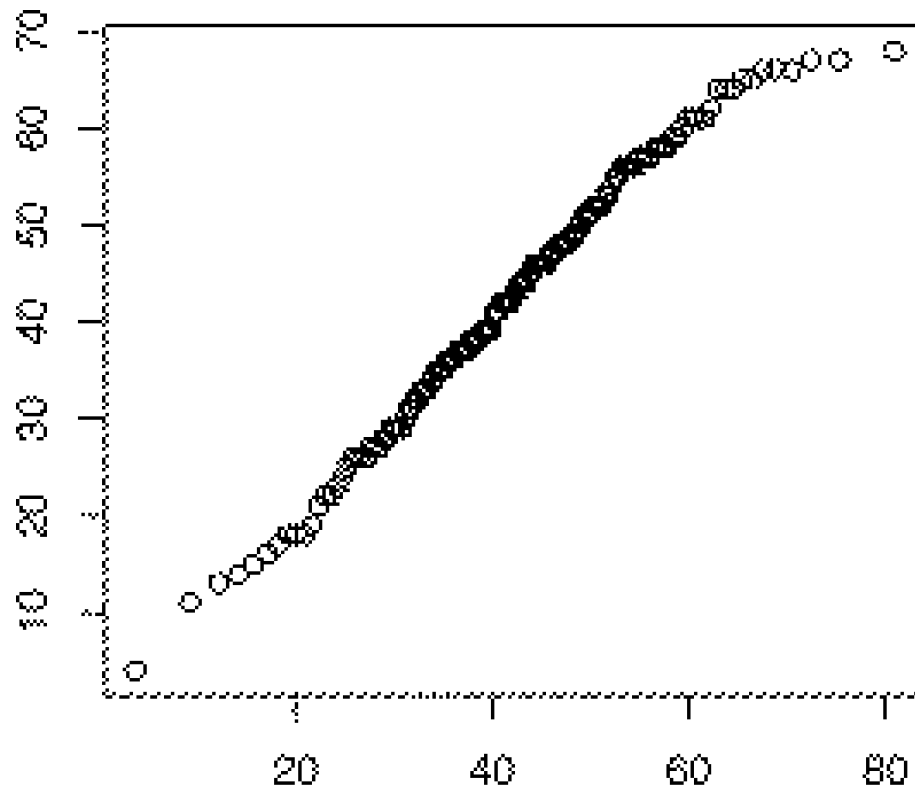


x

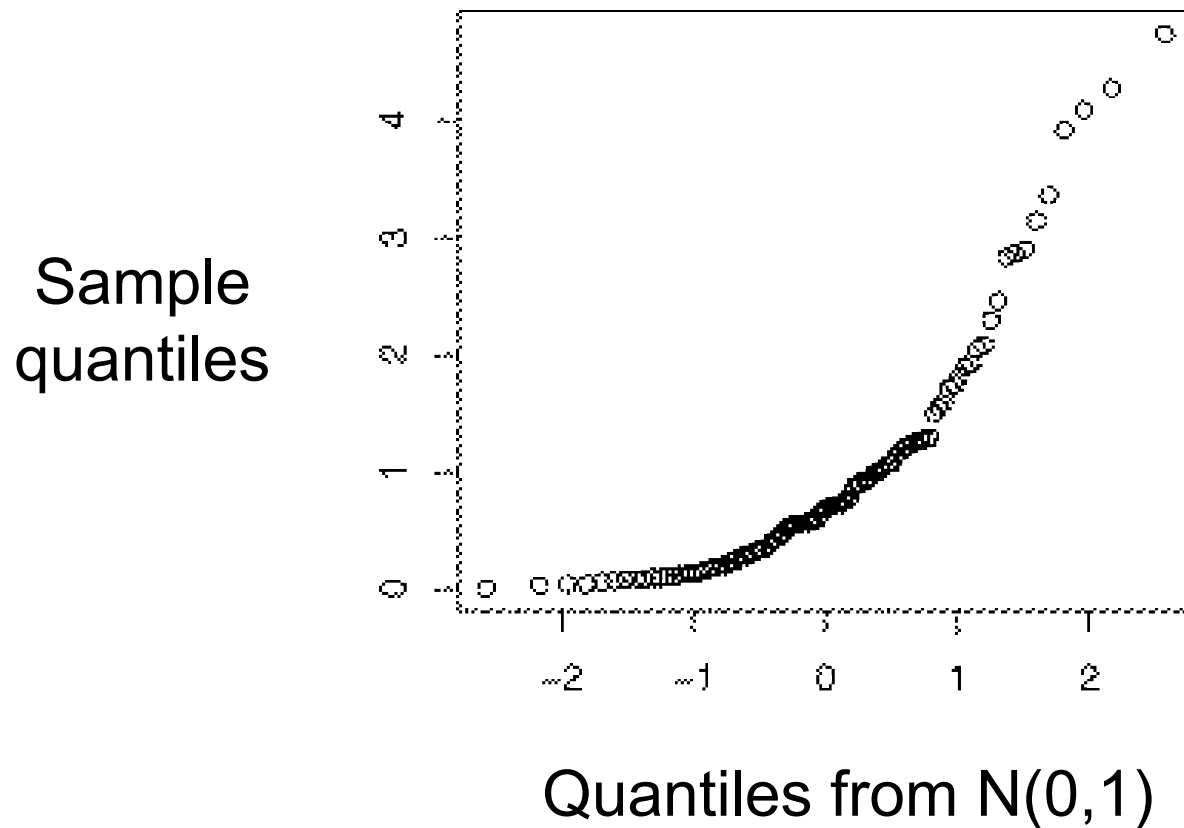


QQ-plots

Plots are generated by R-package <http://www.r-project.org>

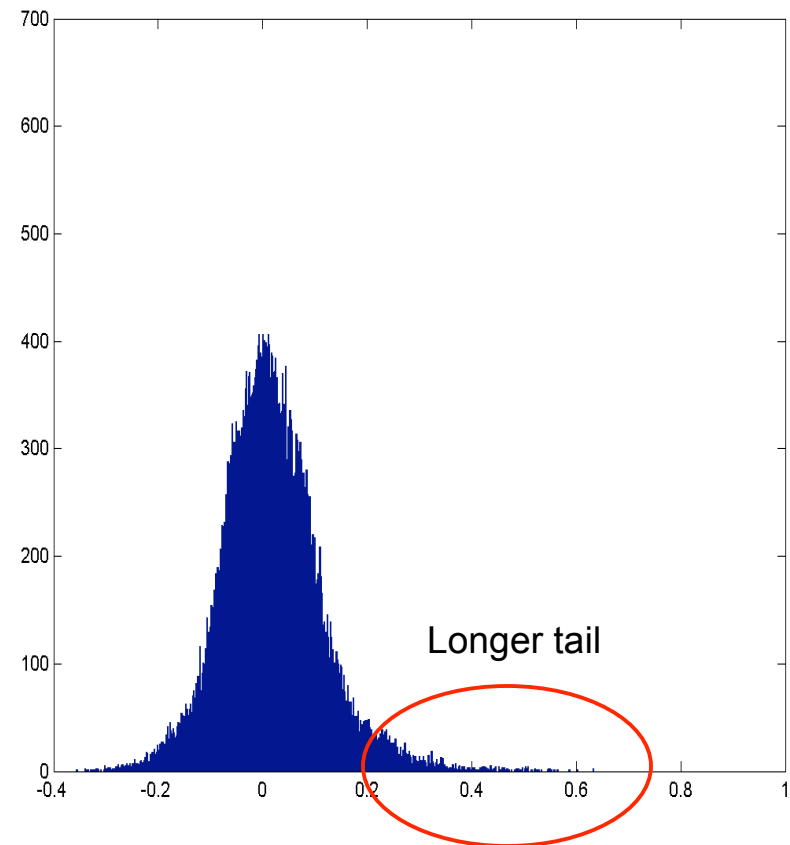
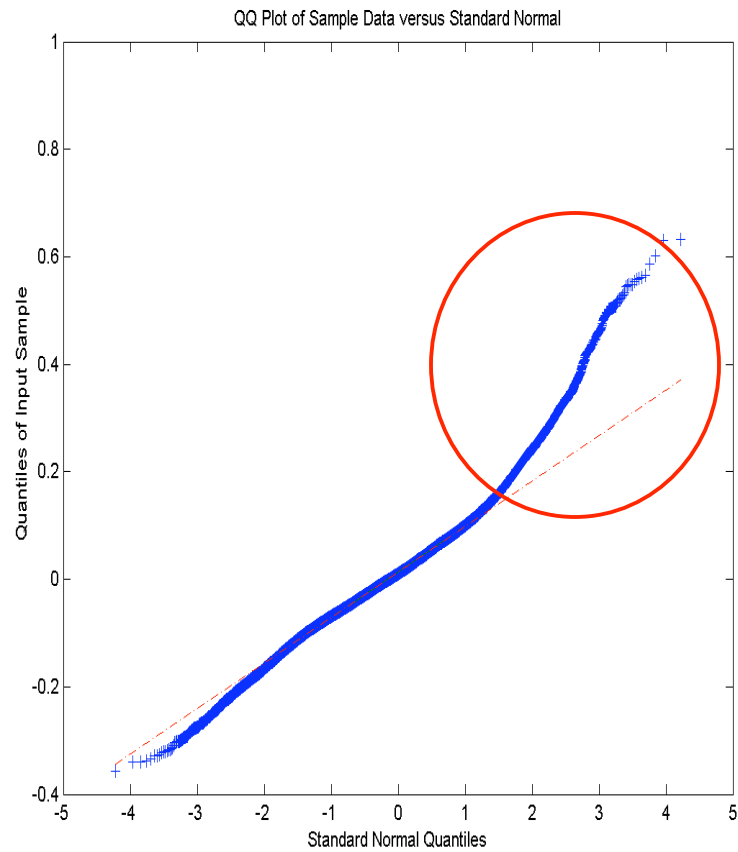


Normal probability plot

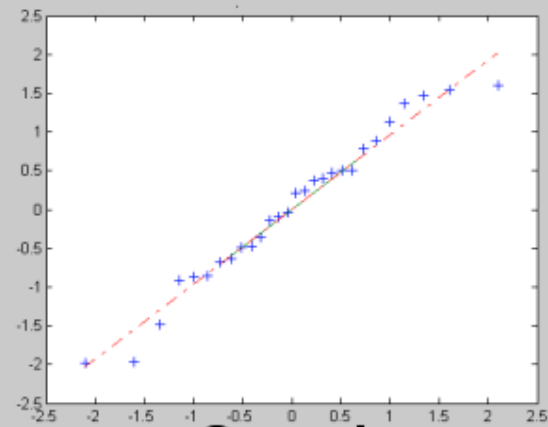


The sample has longer tails.

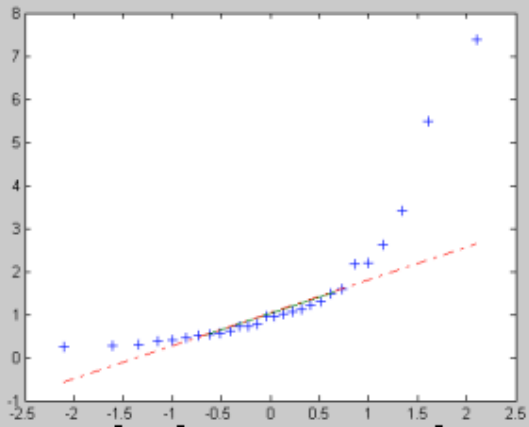
Normal probability plot showing asymmetric distribution



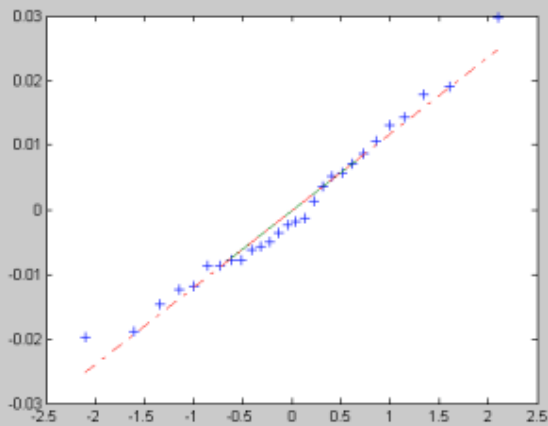
Checking normality across subjects



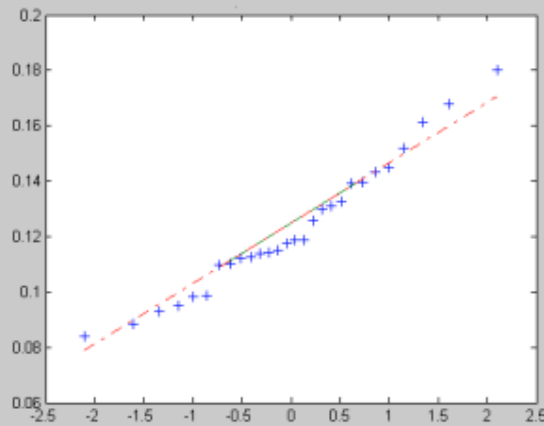
a. Gaussian



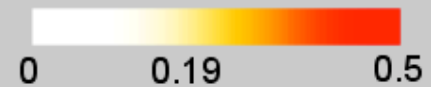
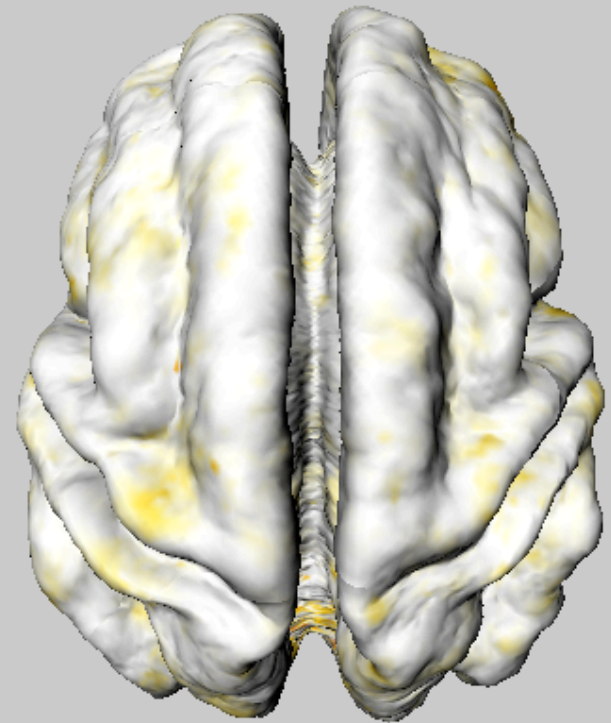
b. Lognormal



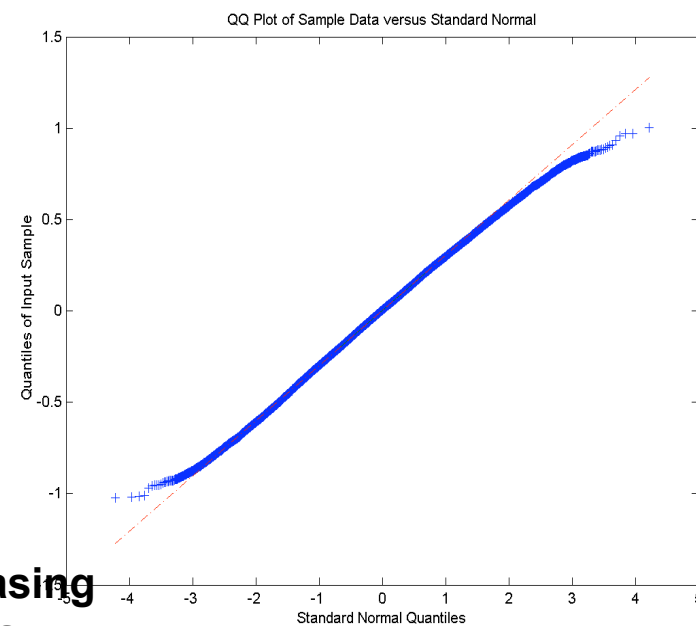
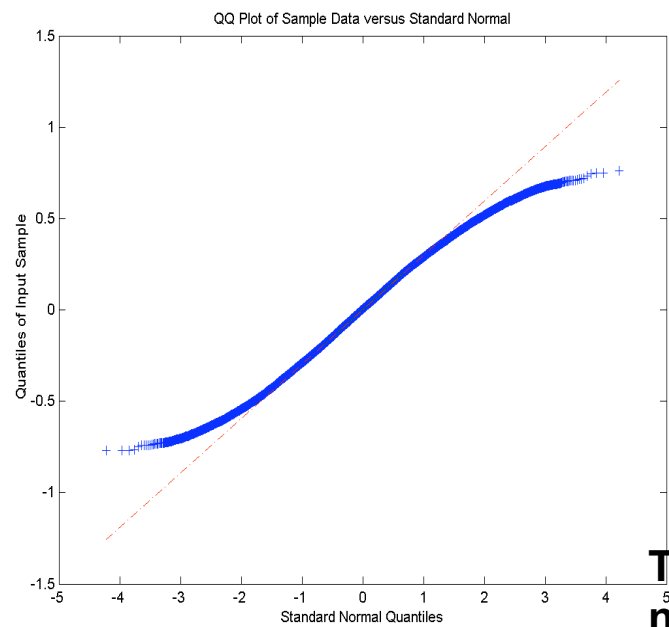
c. Vertex 40546



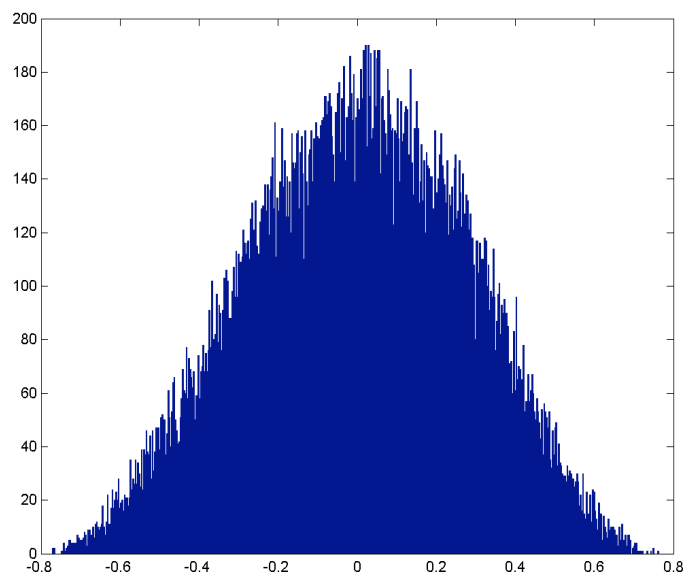
d. Vertex 14300



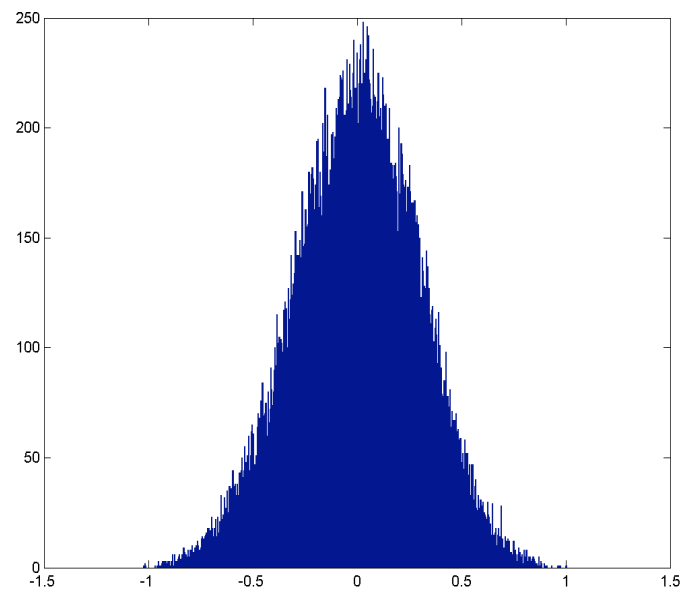
e. Lilliefors Statistic



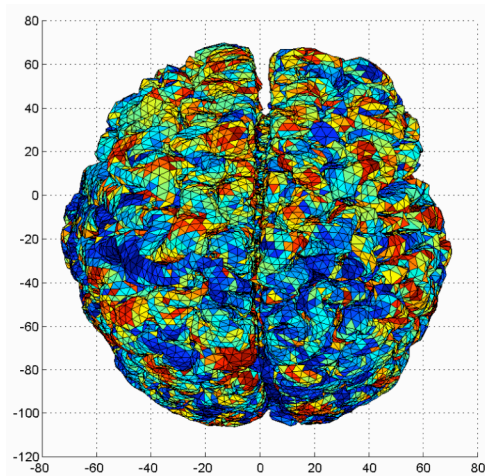
**Tricks for increasing
normality of data**



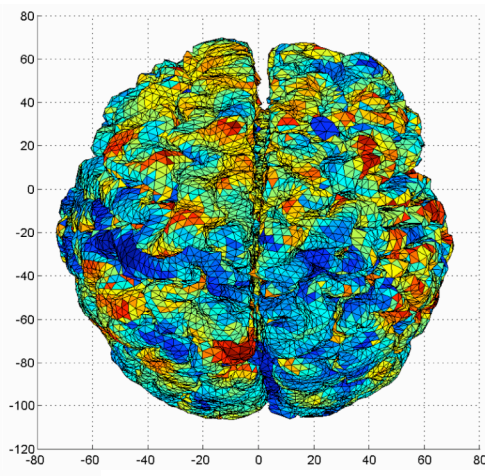
Fisher's
Z transform
on correlation



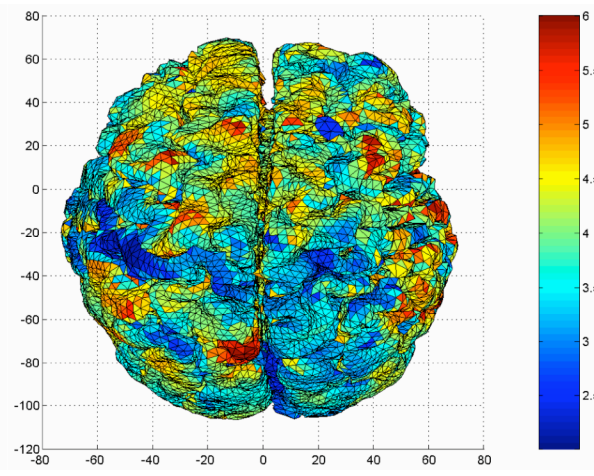
Increasing normality of surface-based smoothing



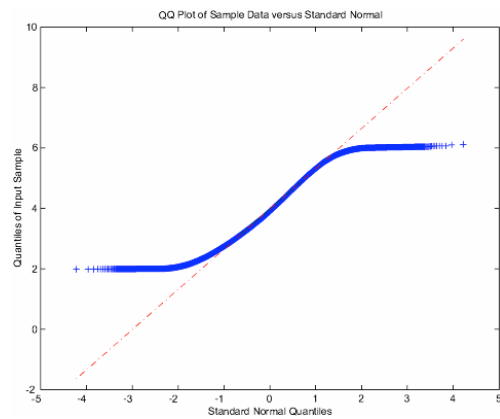
Thickness



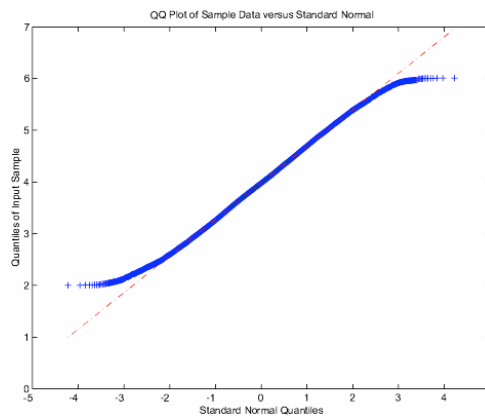
50 iterations



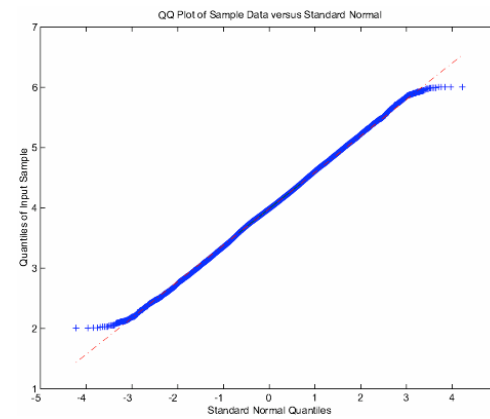
100 iterations



QQ-plot

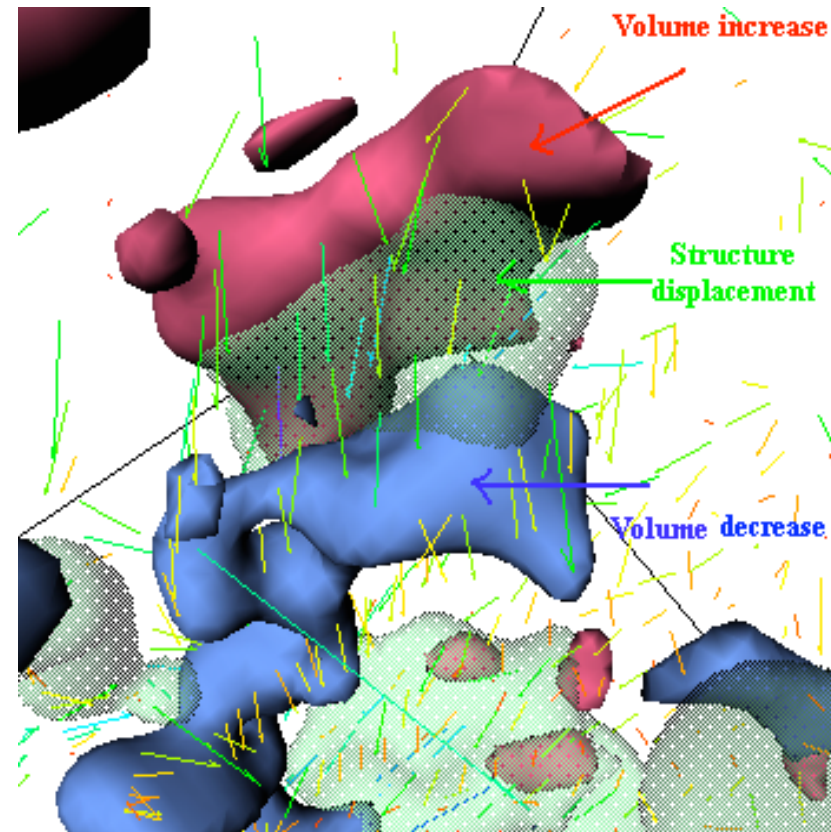
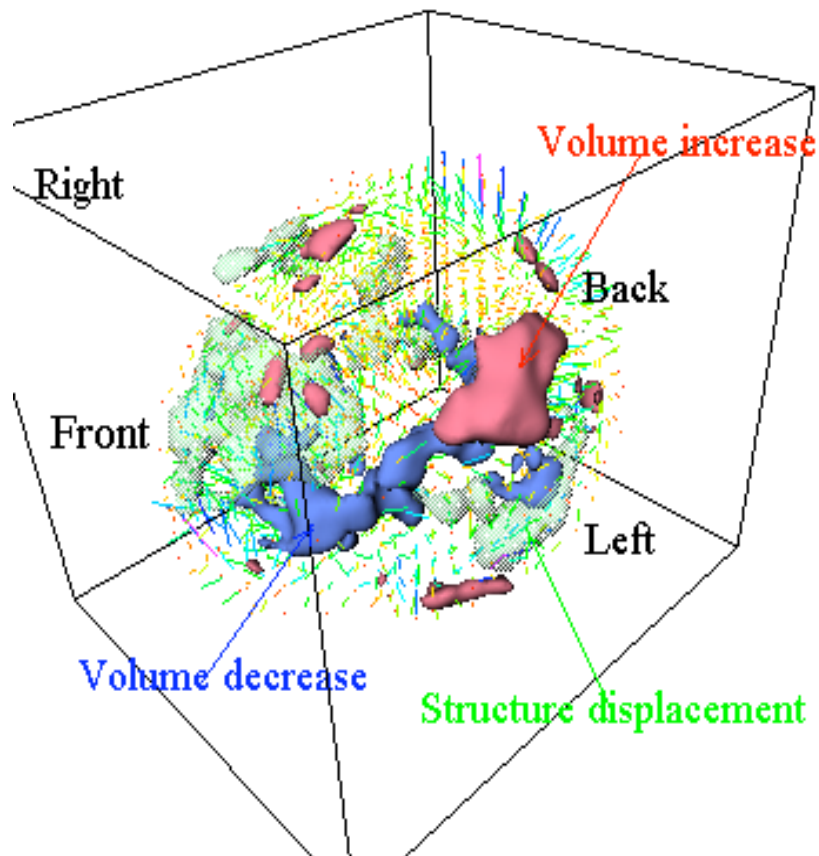


QQ-plot

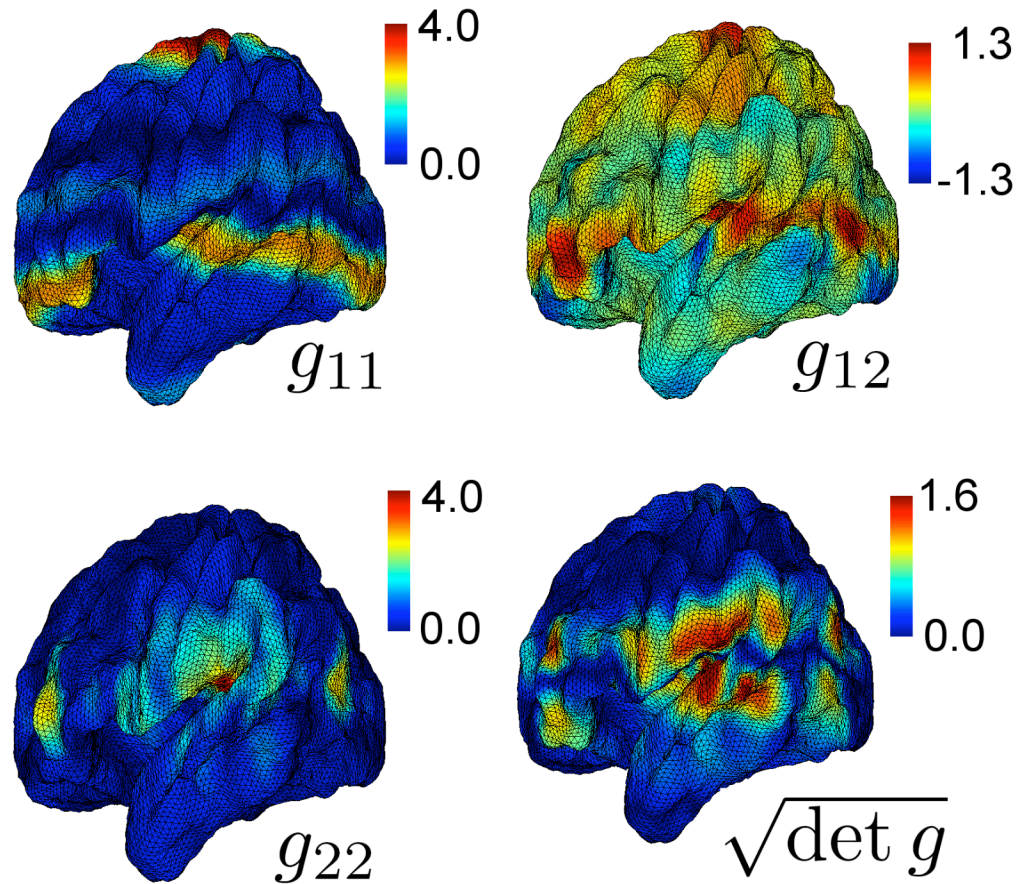


QQ-plot

Statistically significant regions of local volume change
JD > 1 volume increase, JD < 1 volume decrease over time



Generalization of Jacobian determinant in arbitrary manifold
= determinant of Riemannian metric tensors
= local volume (surface area) expansion



Lecture 5 topics

Surface-based Morphometry (SBM)
and
Cortical thickness