Medical Image Analysis

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Lecture 05.

Surface-based Morphometry (SBM)

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Surface-based Morphometry (SBM)

- Surface-specific morphometric technique.
- More sensitive to surface-specific changes.
- Example: substantial cortical changes were found in AD and other dementias and it is likely that different clinical populations will exhibit different cortical shape variability.

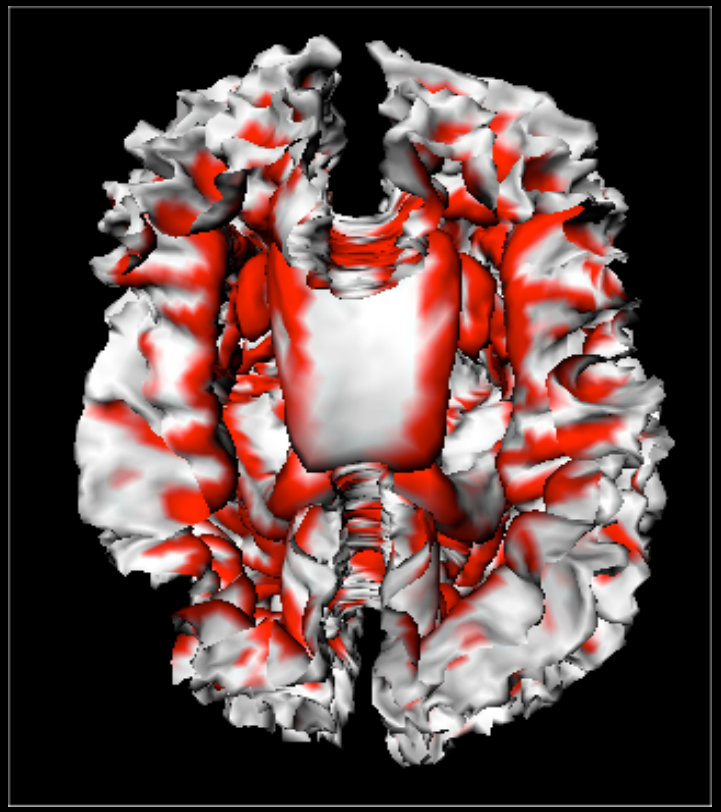
Motivation for SBM

Compared to other 3D volumetric techniques, SBM can quantify cortical variations better.



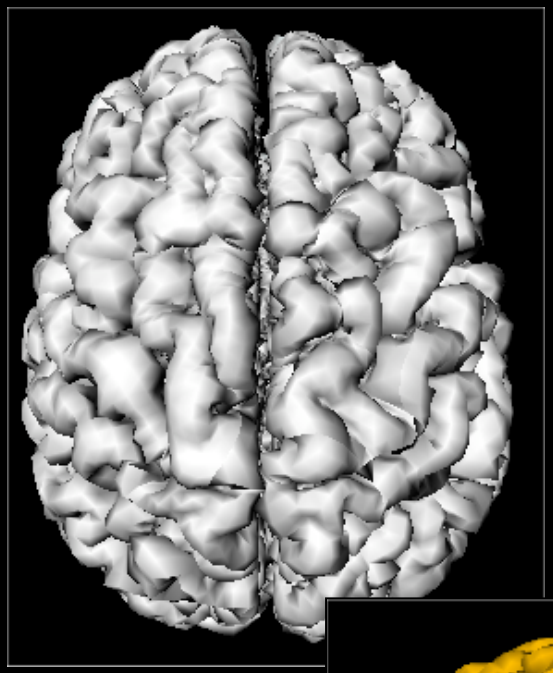
Age 14

Ventricle enlargement



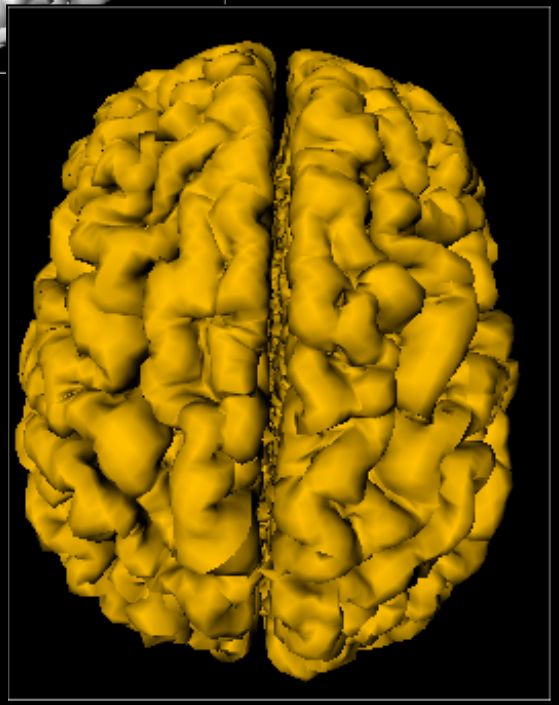
Age 19

Comparison of cortical surface

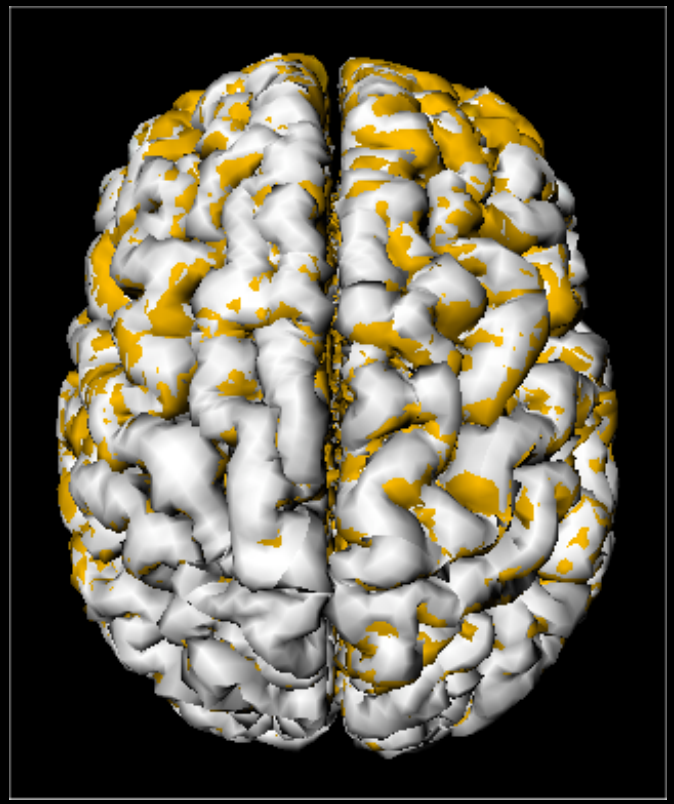


Age 14

Age 19



Superimposition



MNI cortical image processing pipeline

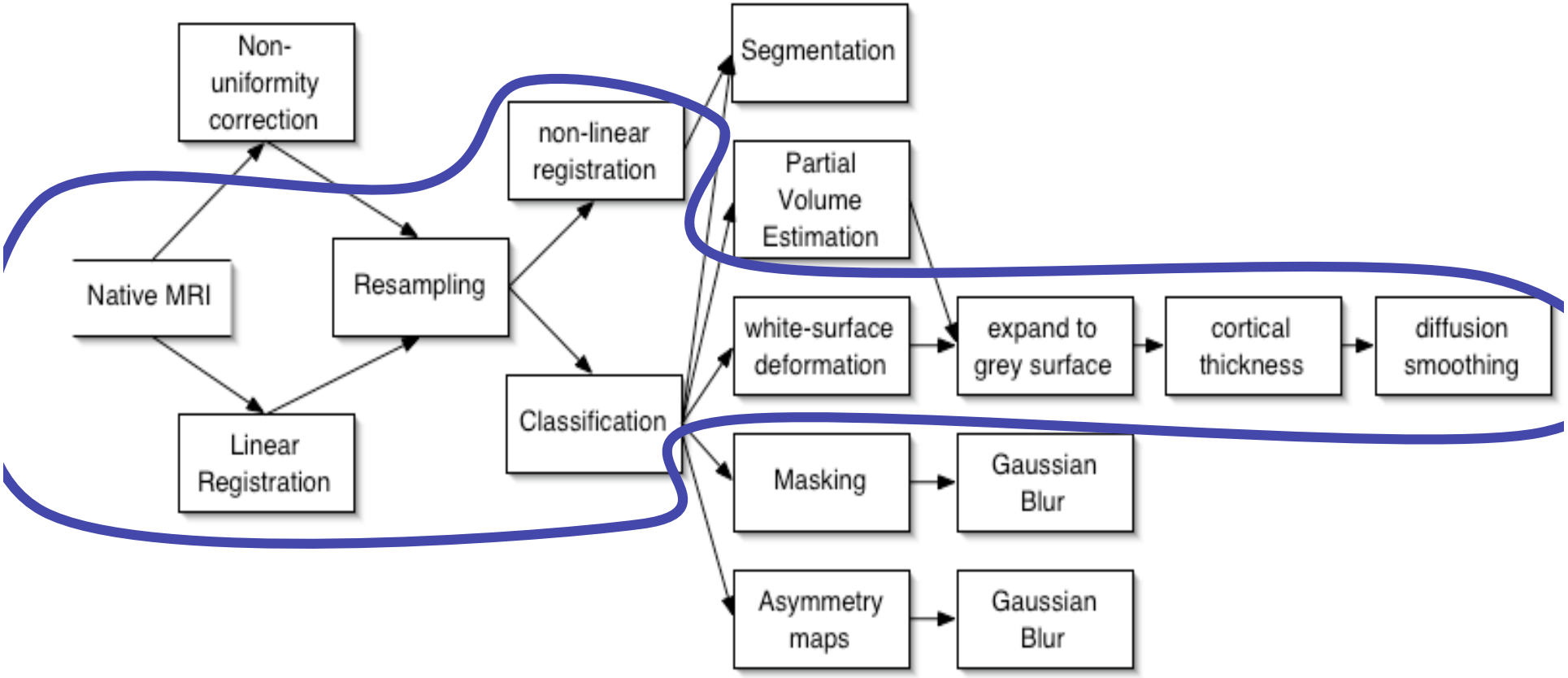
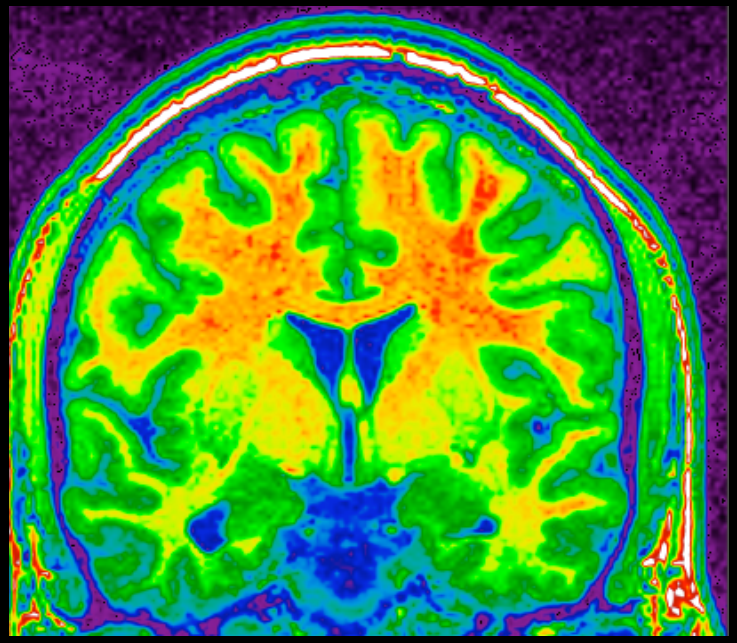
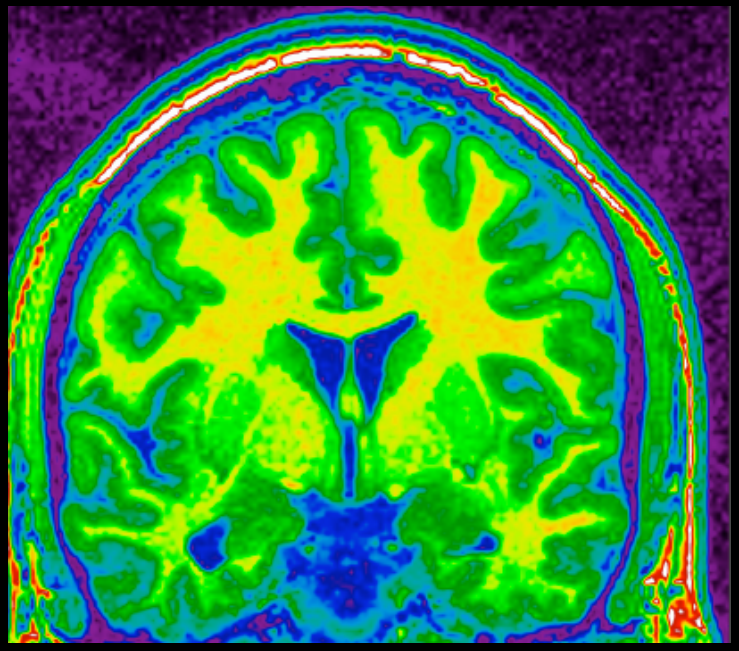


Image intensity Nonuniformity correction N3 algorithm

Original data

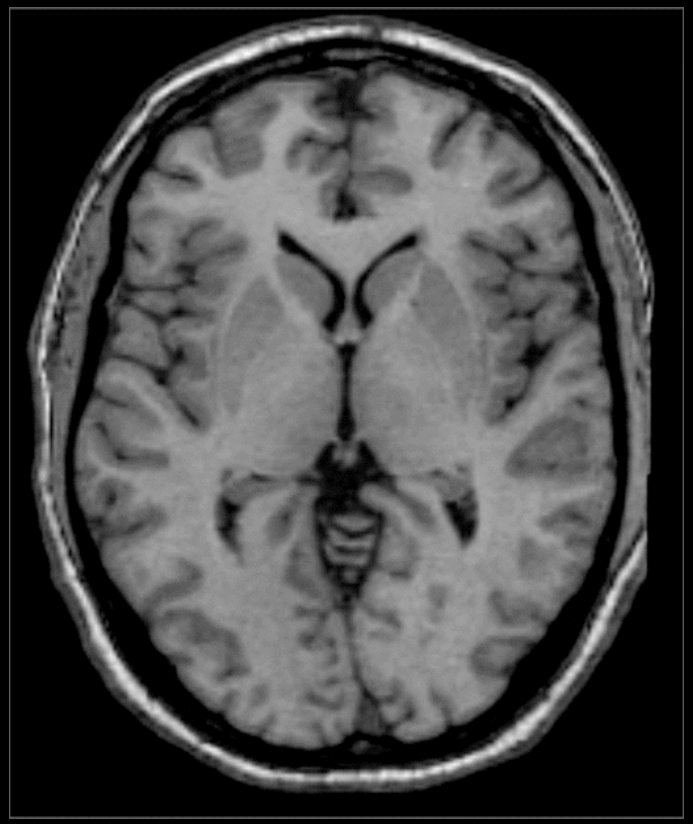


Corrected



Source: Jason Lerch, MNI

Tissue segmentation: neural network classifier

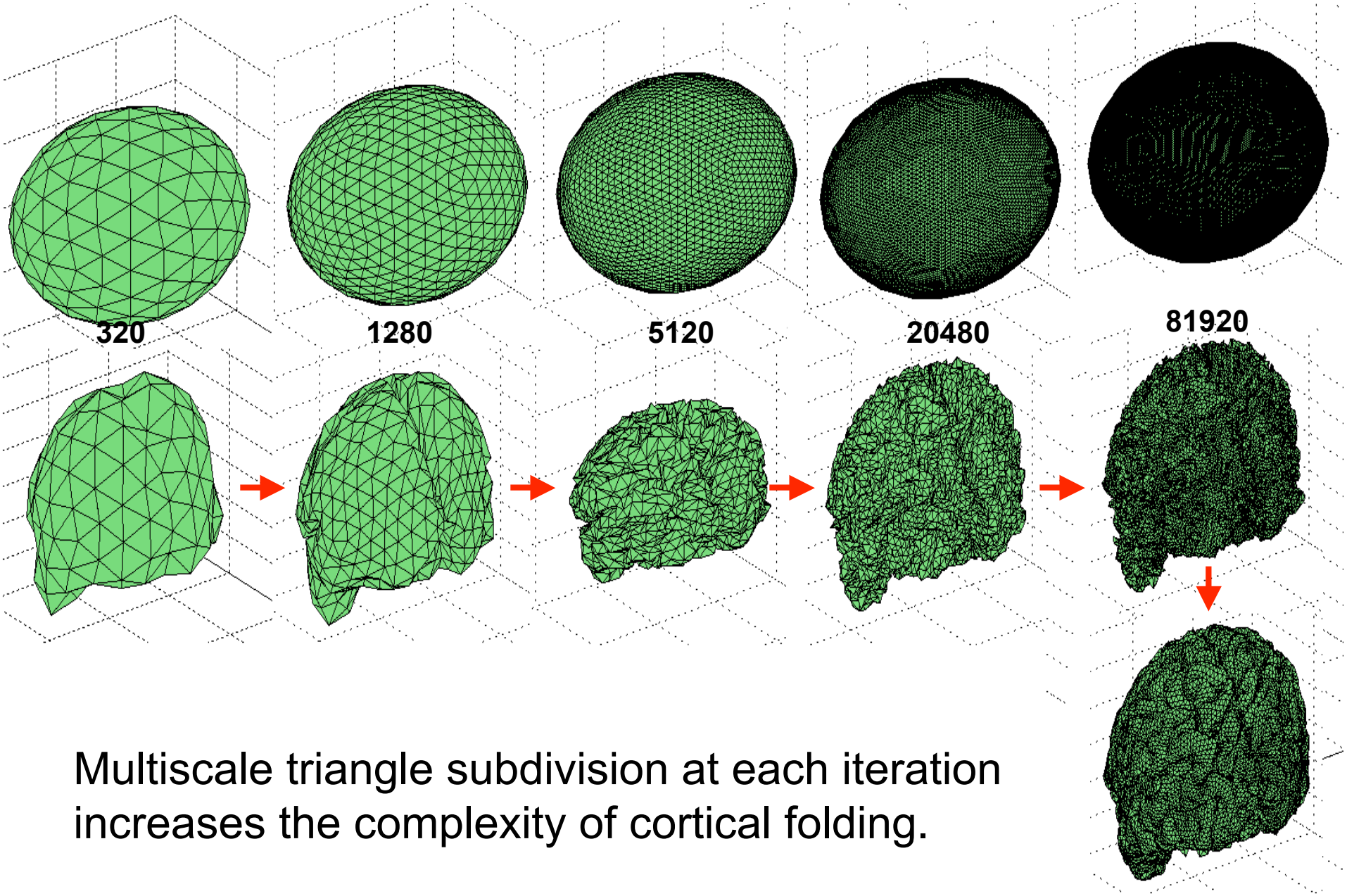


Original image



3 disjoint classes
Assigned values: 1,2 and 3

Deformable surface algorithm McDonalds *et al*. (2001) NeuroImage



Multiscale triangle subdivision at each iteration increases the complexity of cortical folding.

Two available cortical thickness analysis software

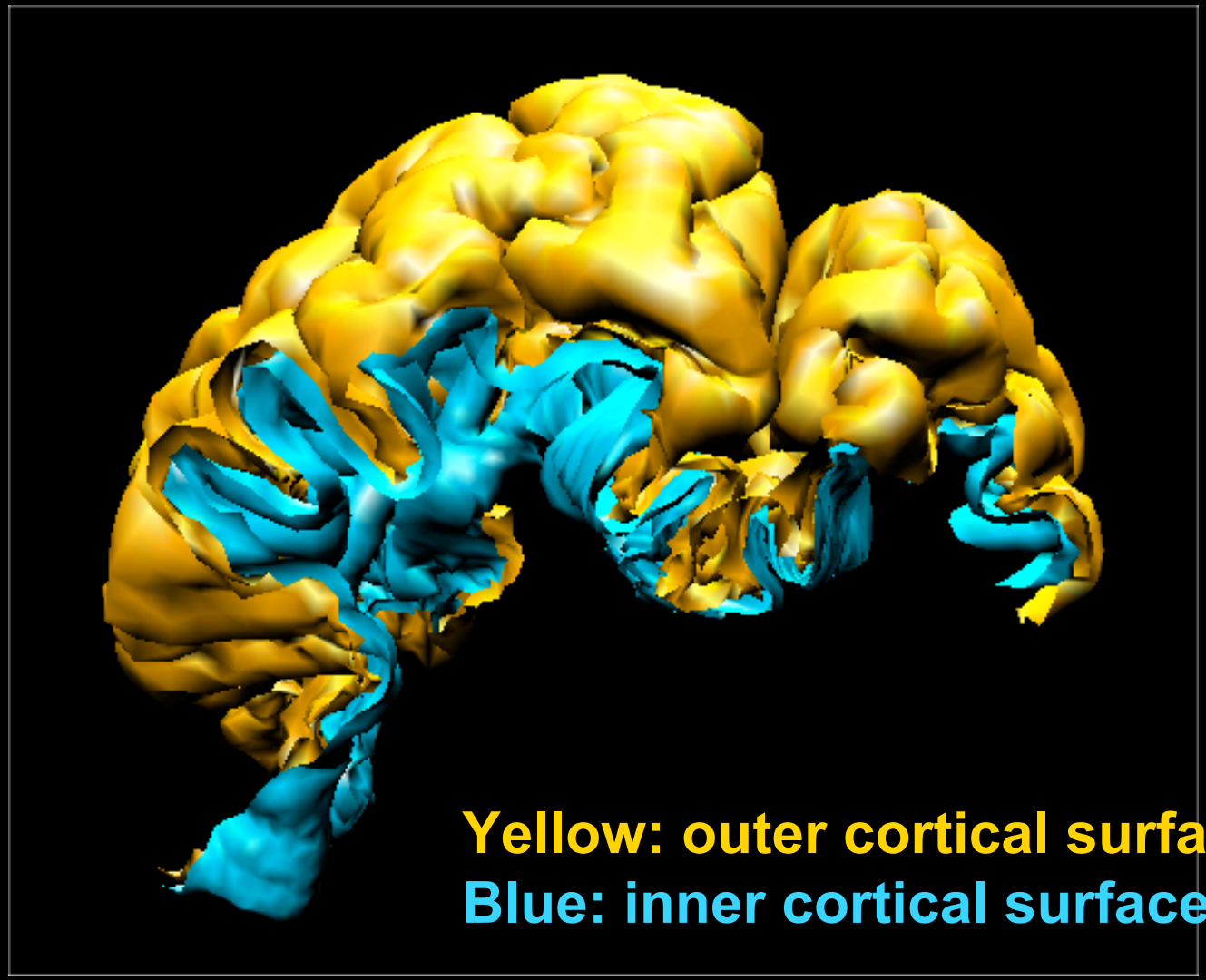
FreeSurfer: Bruce Fischl

http://surfer.nmr.mgh.havard.edu

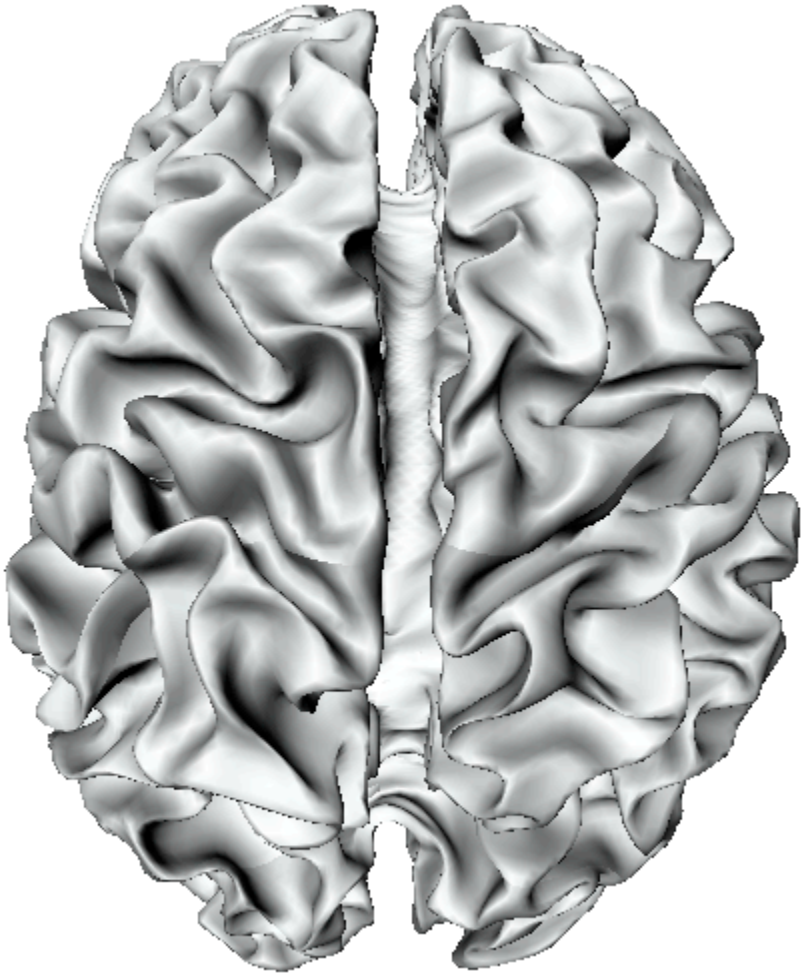
BrainVisa: J.F. Mangin

http://brainvisa.info

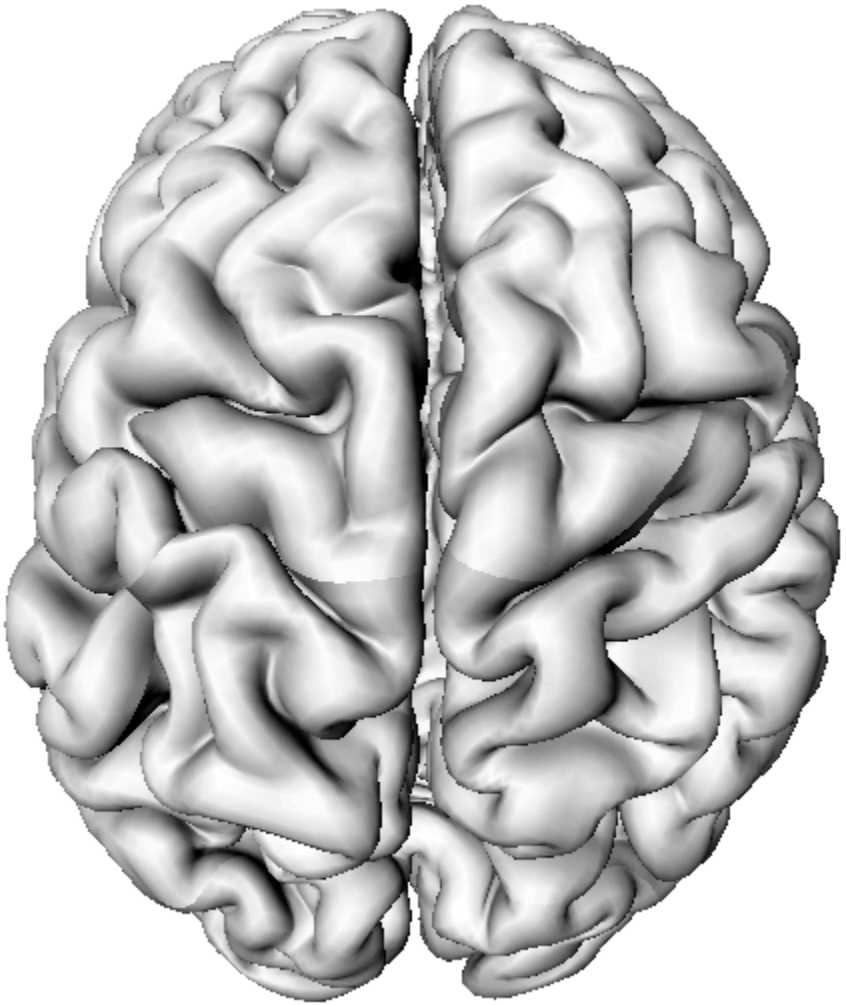
Final result gives two cortical surfaces



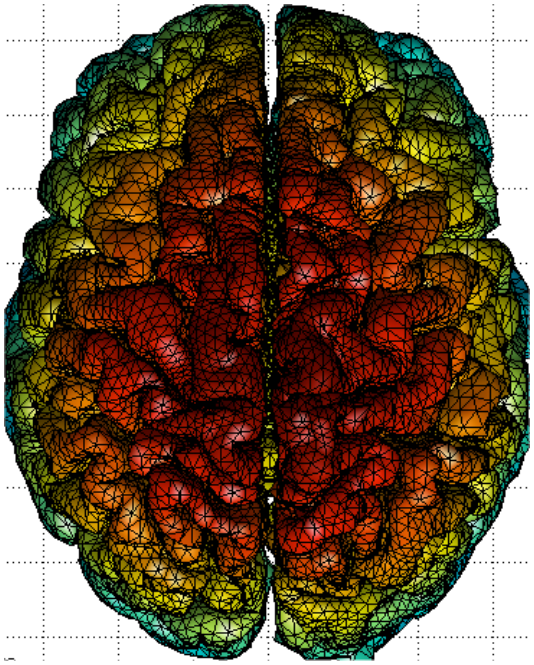
Final surface extraction result



Inner surface

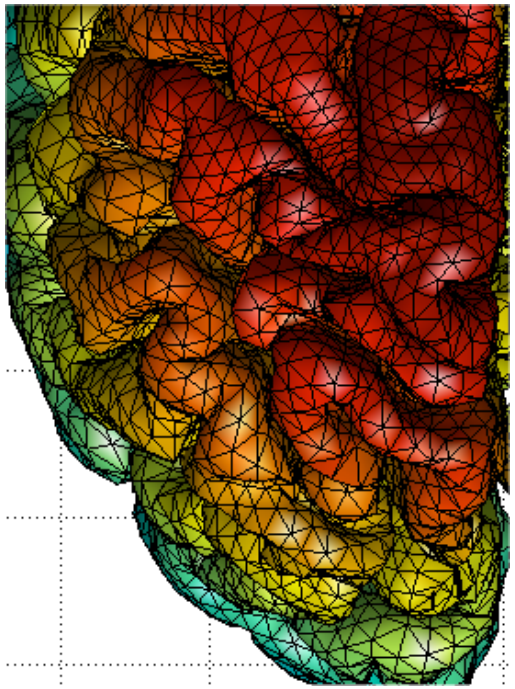


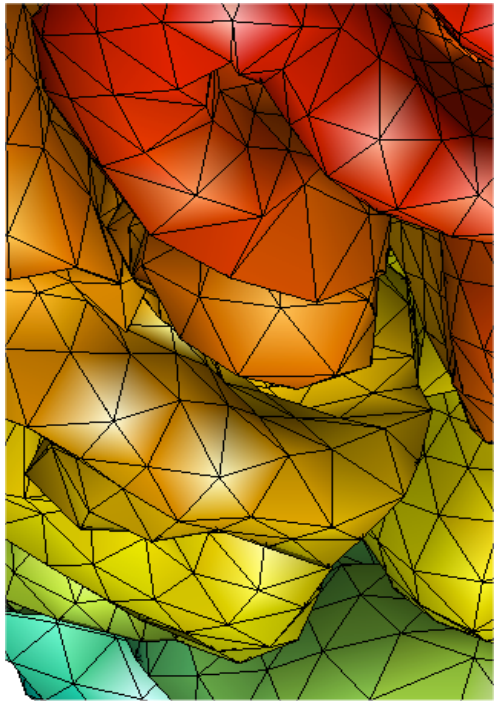
Outer surface



Polygonal mesh data structure

Basis of most surface rendering tools for 3D computer games: as 3D Max Studio, Maya





Data structure for polygonal mesh

Coordinates for subject 1

Vertex	1	2	3	4	5	6	….	40962
x	57.1876	41.0450	-53.1115	-38.1080	1.8440	-0.2458		
y	21.6388	-56.3448	29.8912	-65.5394	22.9715	9.4176		
z	2.9667	21.1399	-5.5088	23.6724	21.5146	16.9014		
Thickness	5.0	4.9	3.0	2.1	3.4	4.5		

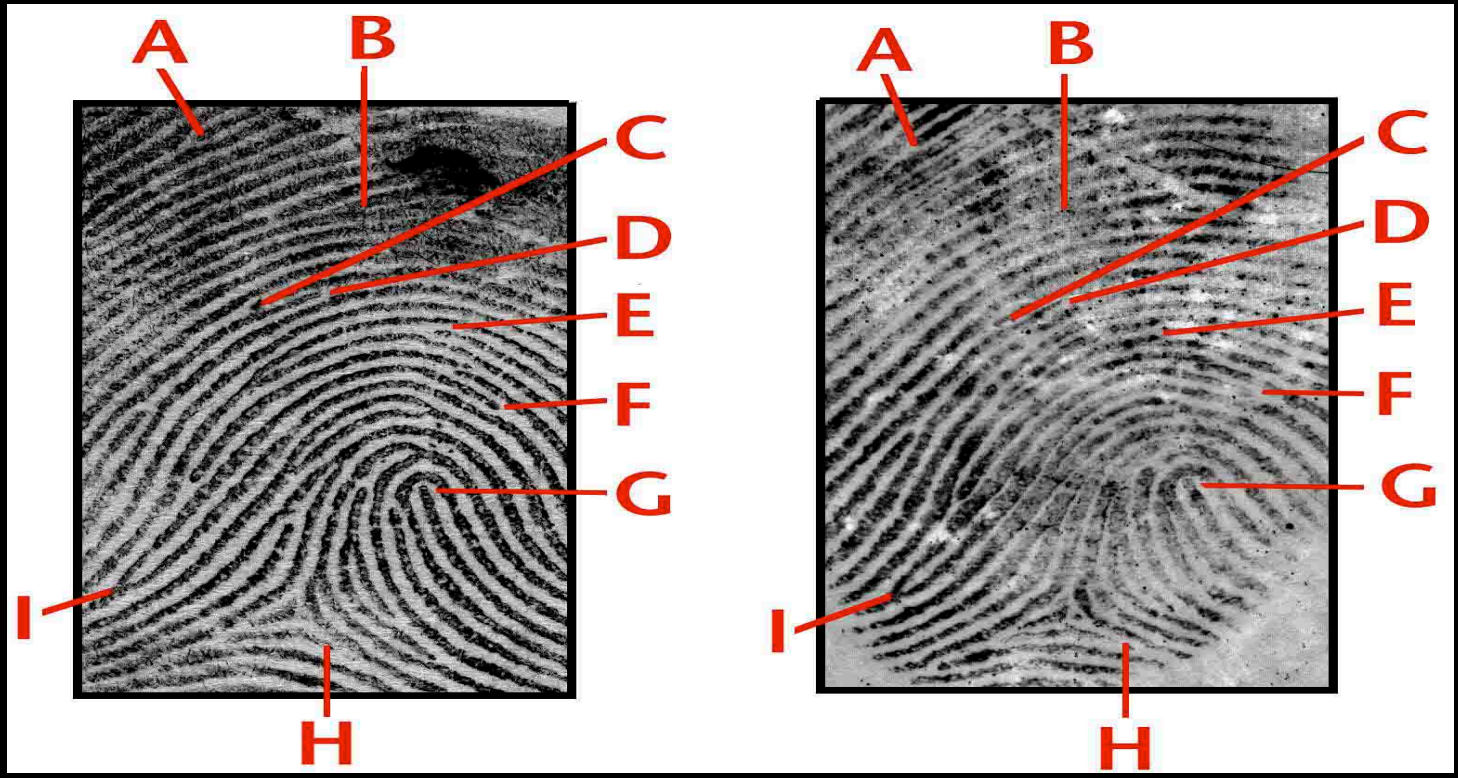
Coordinates for subject 2

Vertex	1	2	3	4	5	6	….	40962
x	53.4240	41.0552	-61.4073	-43.2099	1.6256	-3.9101		
y	22.5535	-56.7731	20.9221	-65.9948	22.7979	29.7043		
z	7.1866	22.4754	-0.1368	21.3962	20.2838	-10.8959		
Thickness	5.5	3.4	2.7	5.1	3.7	4.5		

Corresponding vertices have approximate anatomical homology.

Surface Registration

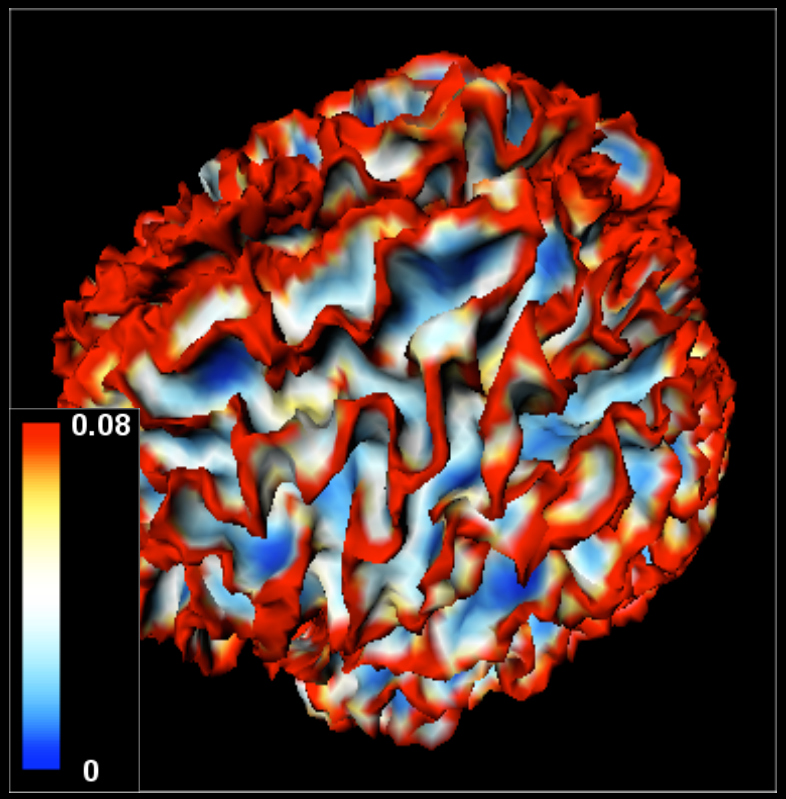
In order to compare cortical thickness across subjects, it Is necessary to find a mapping between homologous anatomical regions.



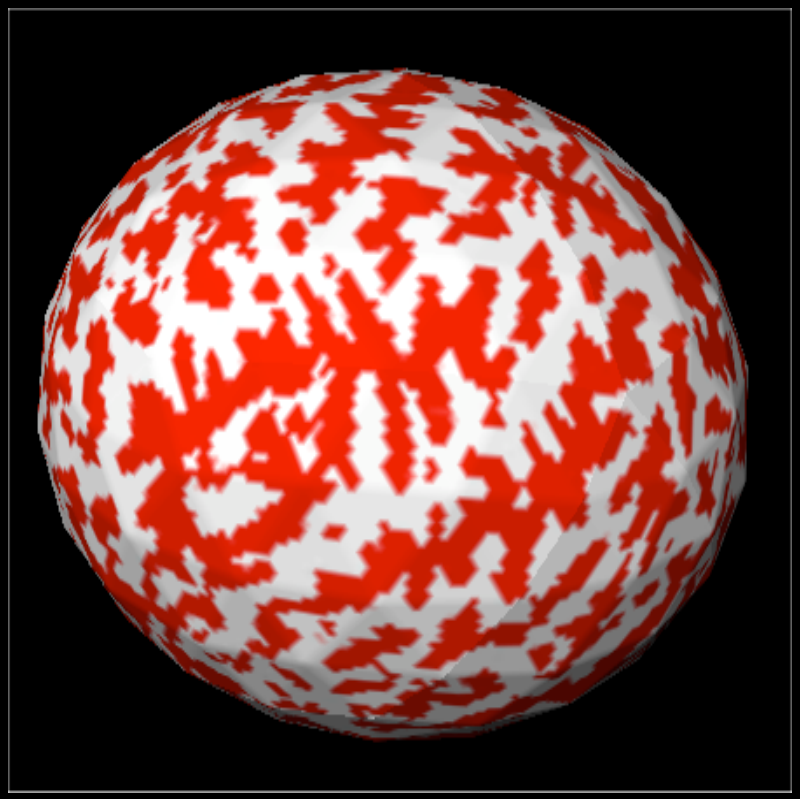
Sulcal pattern alignment= finger print of brain

Compute cortical curvature and map curvature to unit sphere

3D problem

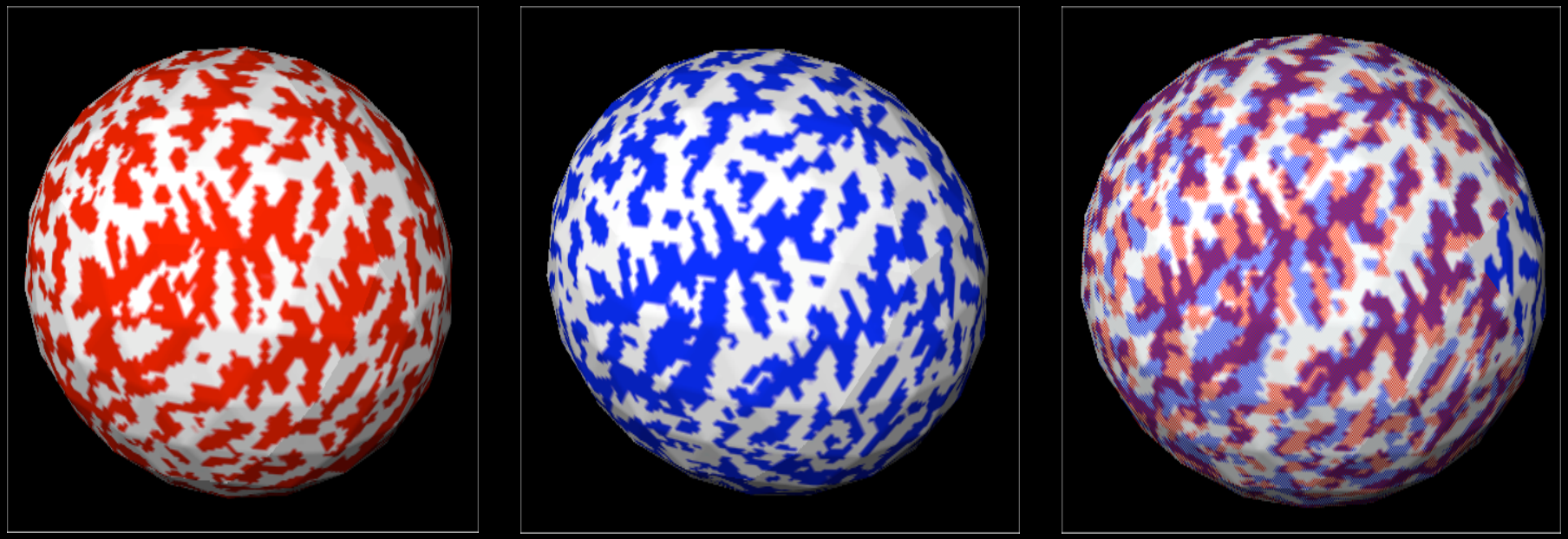


2D problem



Unit sphere gives a natural coordinate system (spherical coordinates).

Sulcal pattern matching

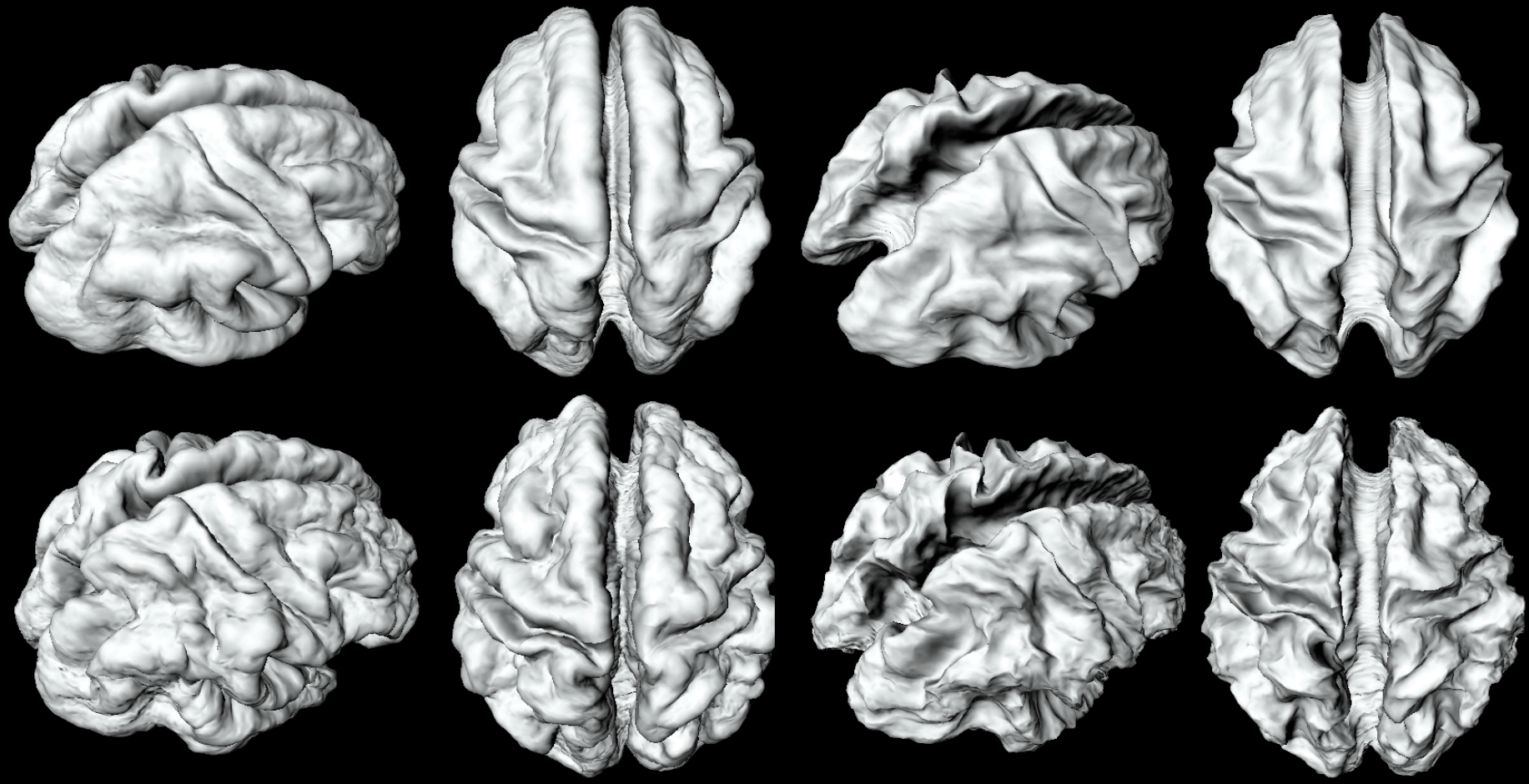


Misalignment

Sulcal pattern matching by minimizing
objective function = curvature difference - smoothness of deformation

Average Template construction

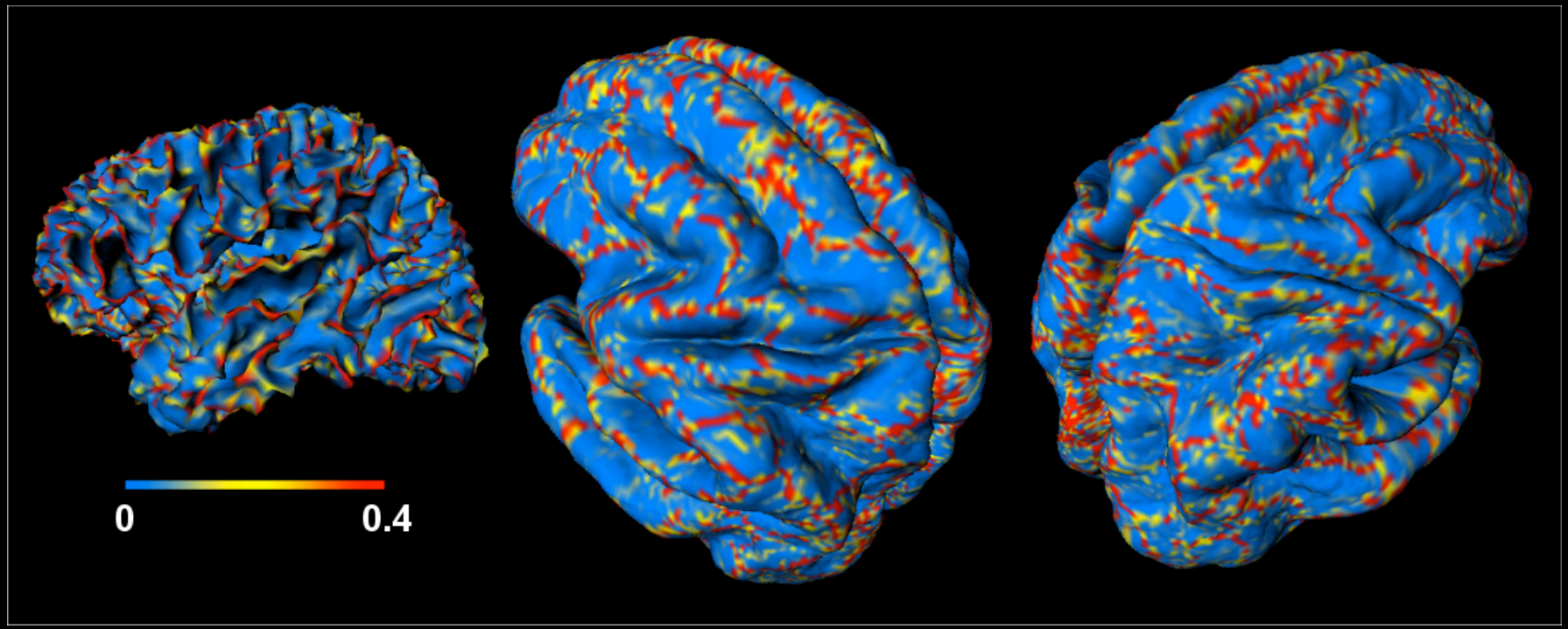
Averaged corresponding coordinates of individual surface



Top: Old technique Chung et al. (2003)

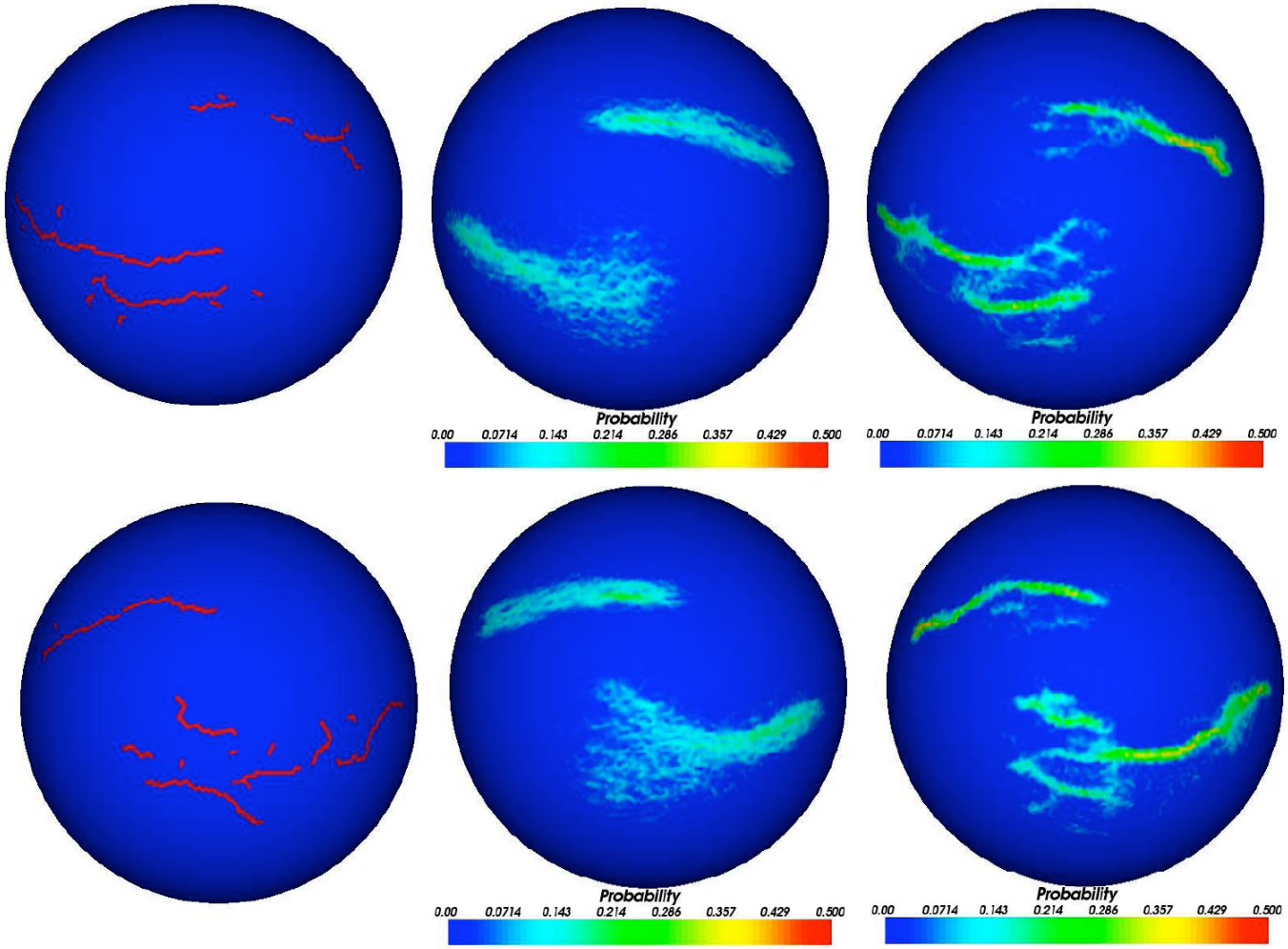
Bottom: New technique Chung et al. (2005)

Demonstrating the alignment of sulci on average template



Principle curvature maps projected on the average template

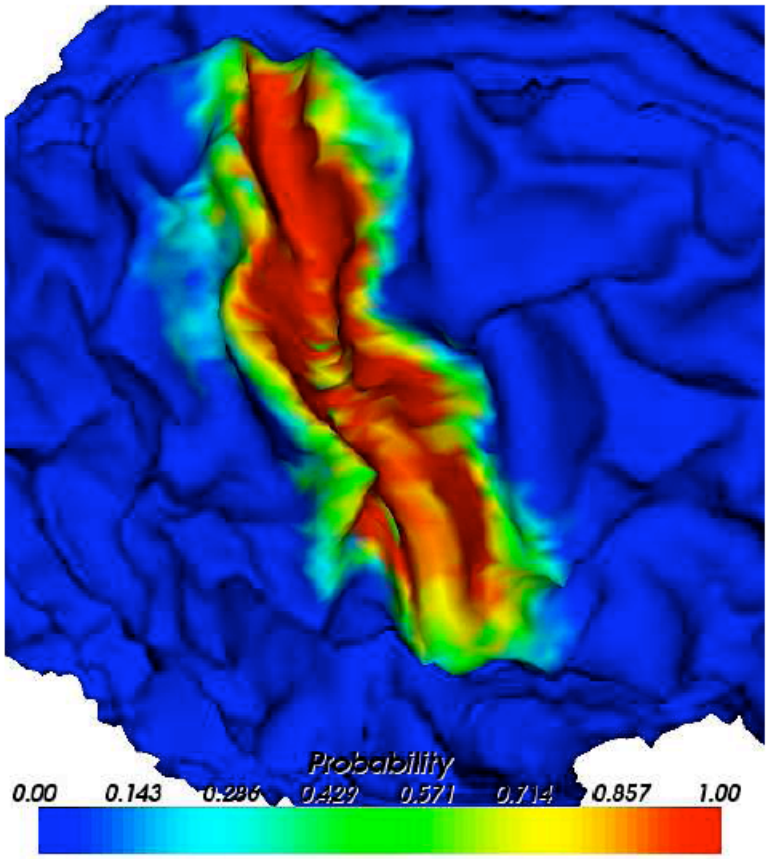
Validation of surface registration based on 149 subjects

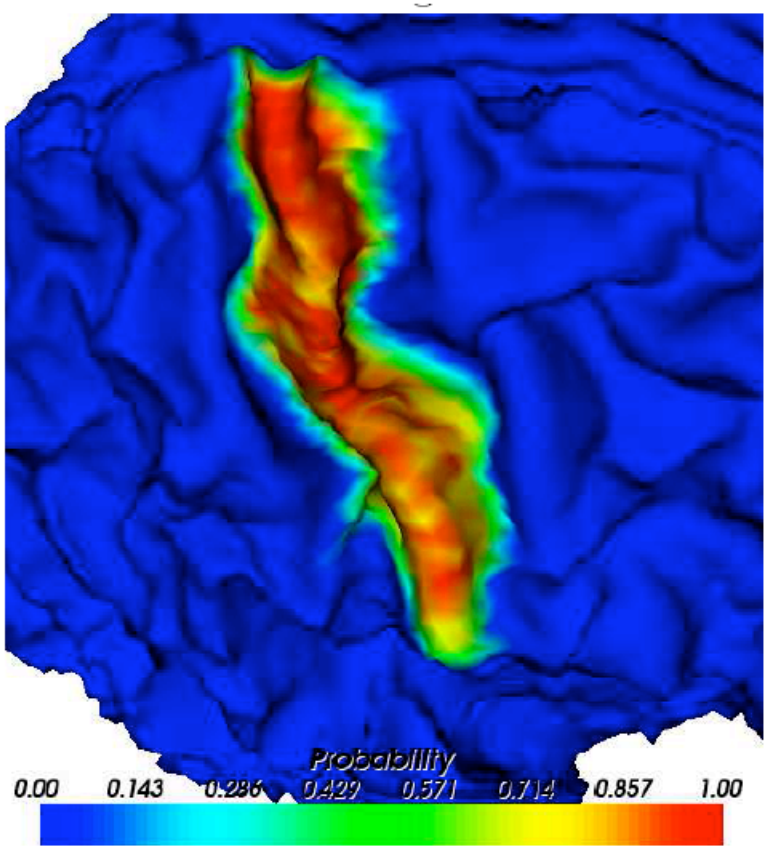


Central and temporal sulci

Traces of sulci provided by Cathia et al. (2003)

Right central sulcus matching probability





3D volume registration

2D surface registration

Alternate approach:

spherical harmonic (SPHARM) correspondence

- Surface registration is given implicitly by matching the coefficients of basis functions of deformation.

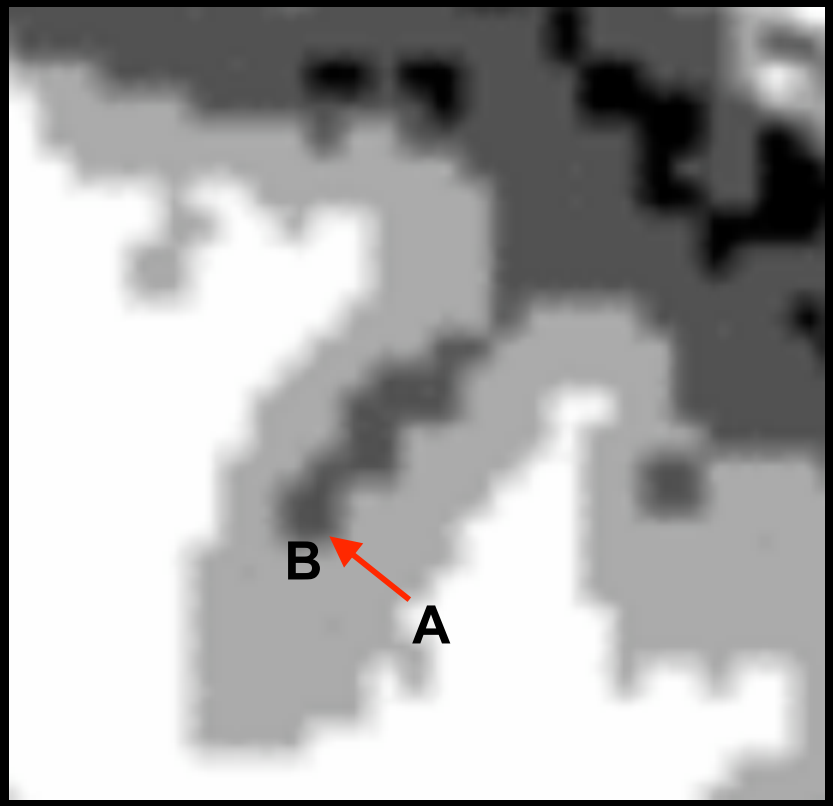
- Since the deformation is given in terms of smooth basis function, we do not need to worry about increasing the smoothness of deformation.

- This is shown to be optimal in the least squares fashion.

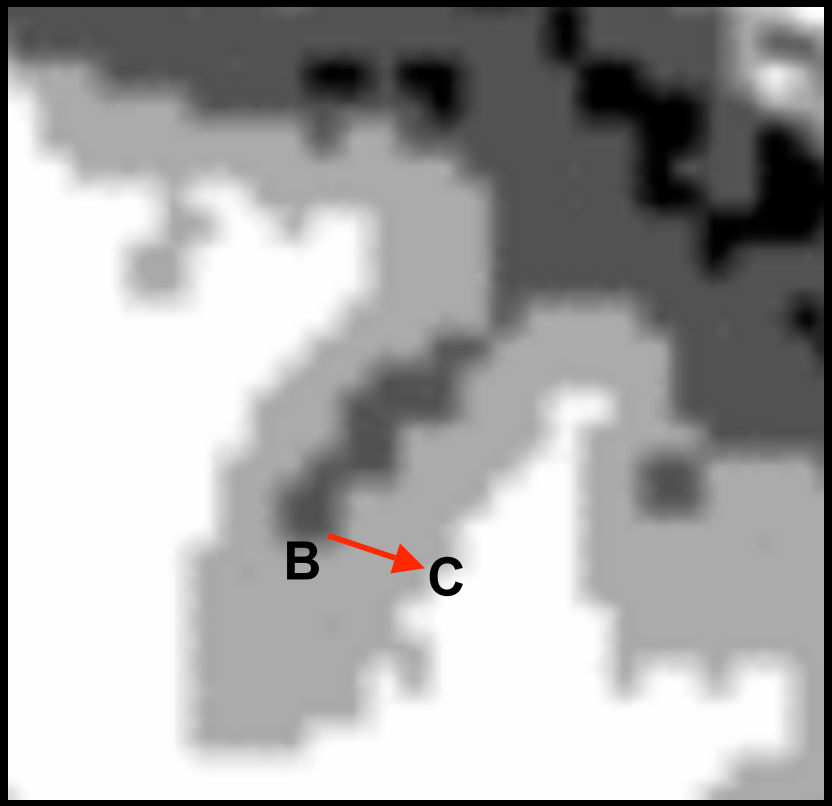
- SPHARM will be discussed in lecture 6

Measuring cortical thickness

thickness of gray matter is used as an anatomical index.

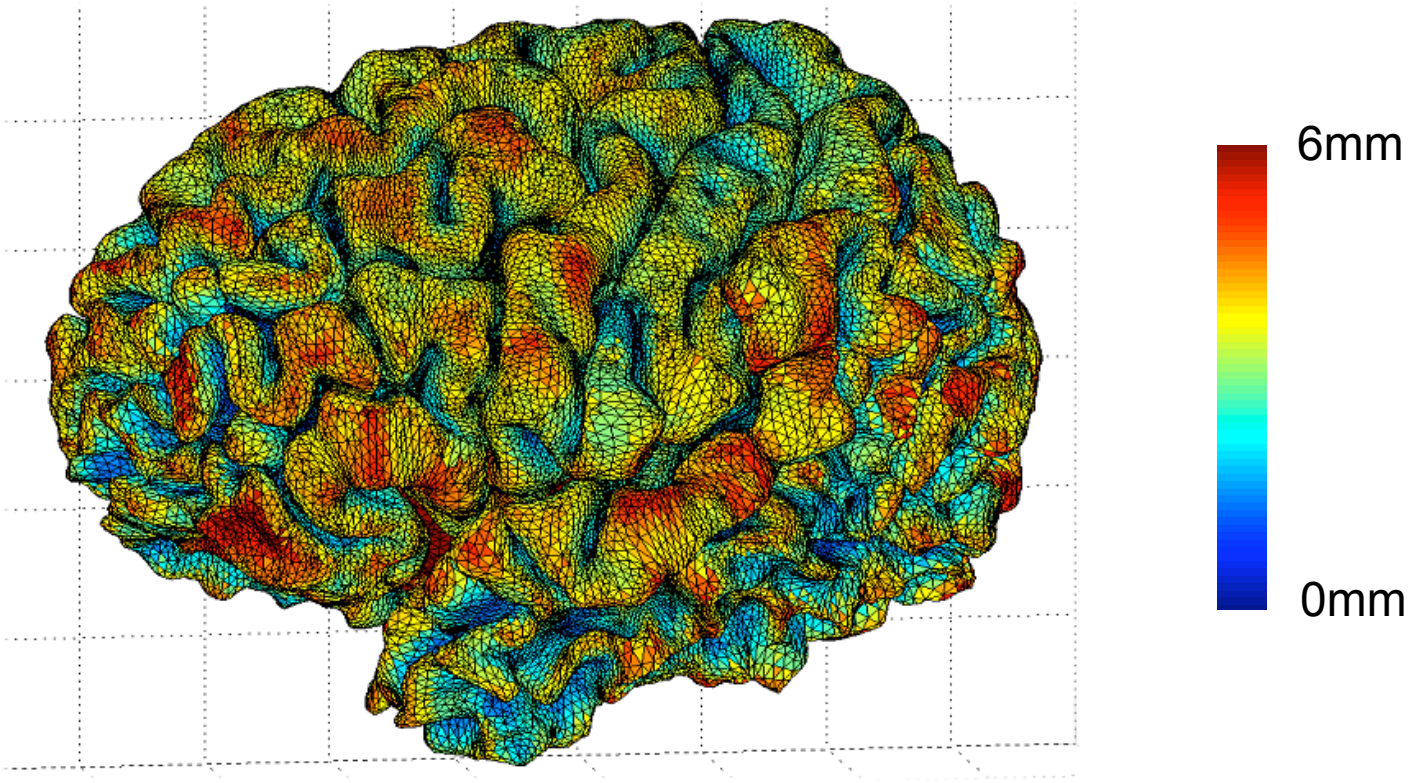


orthogonal projection from A to B



orthogonal projection from B to C

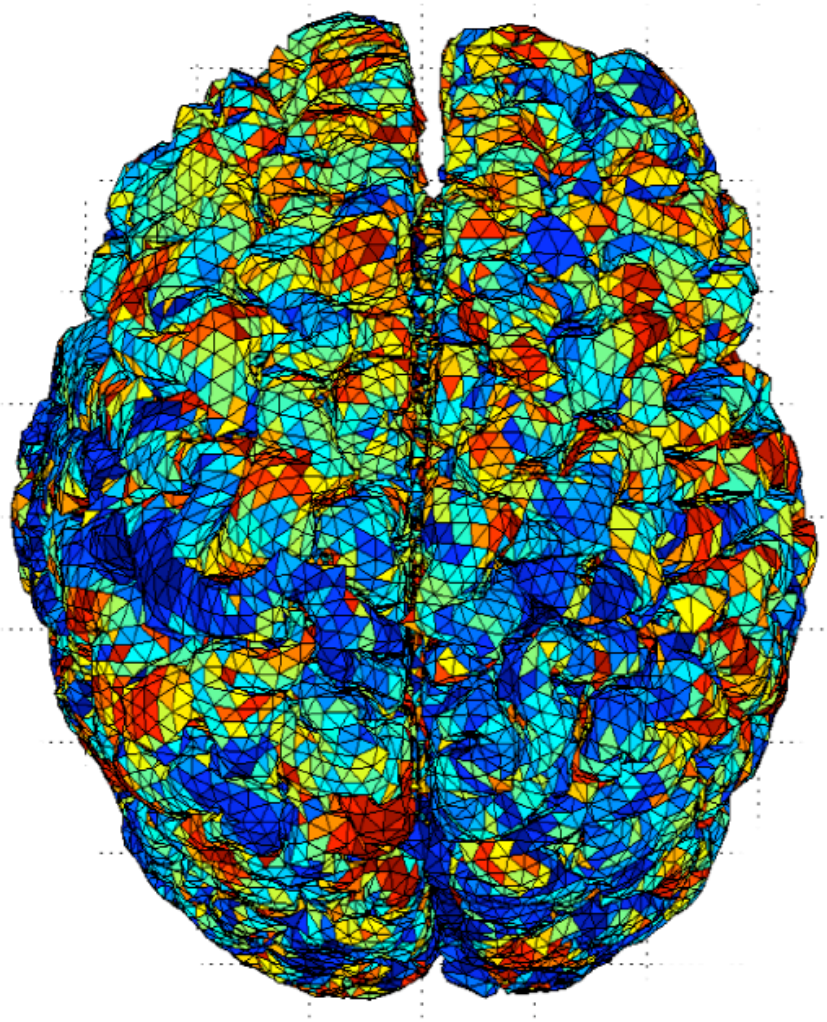
Cortical thickness measurements

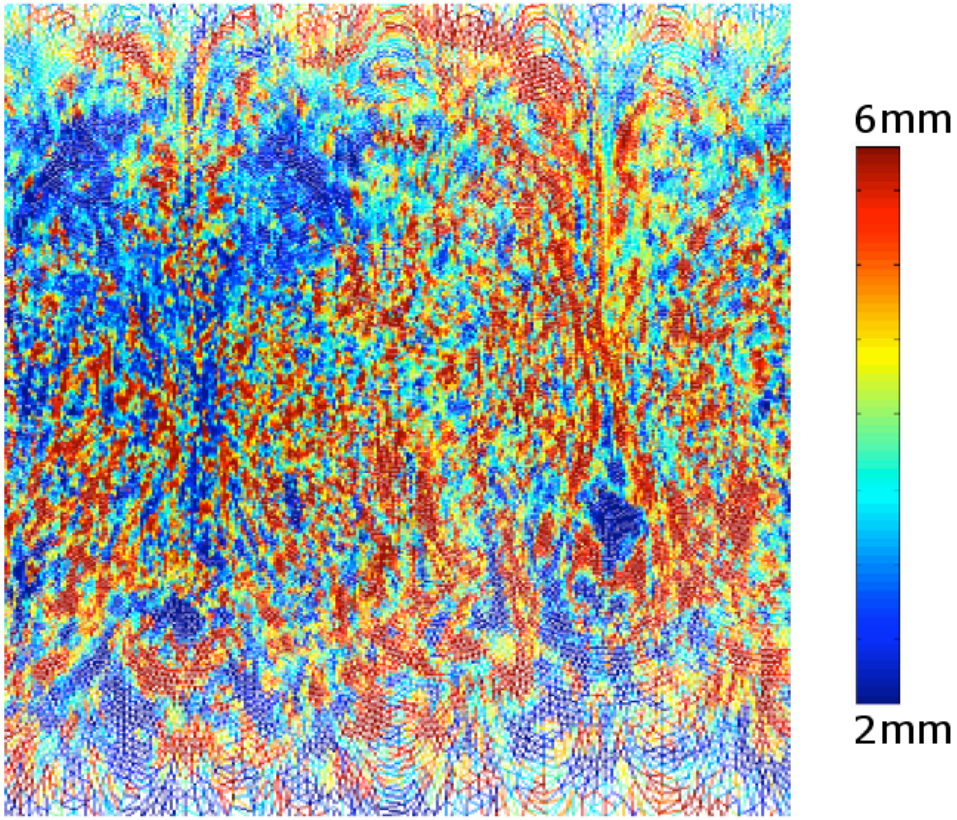


$$Y(p) = \theta(p) + \epsilon(p), p \in \partial\Omega$$

where θ is the mean thickness and ϵ is a zero mean Gaussian random field.

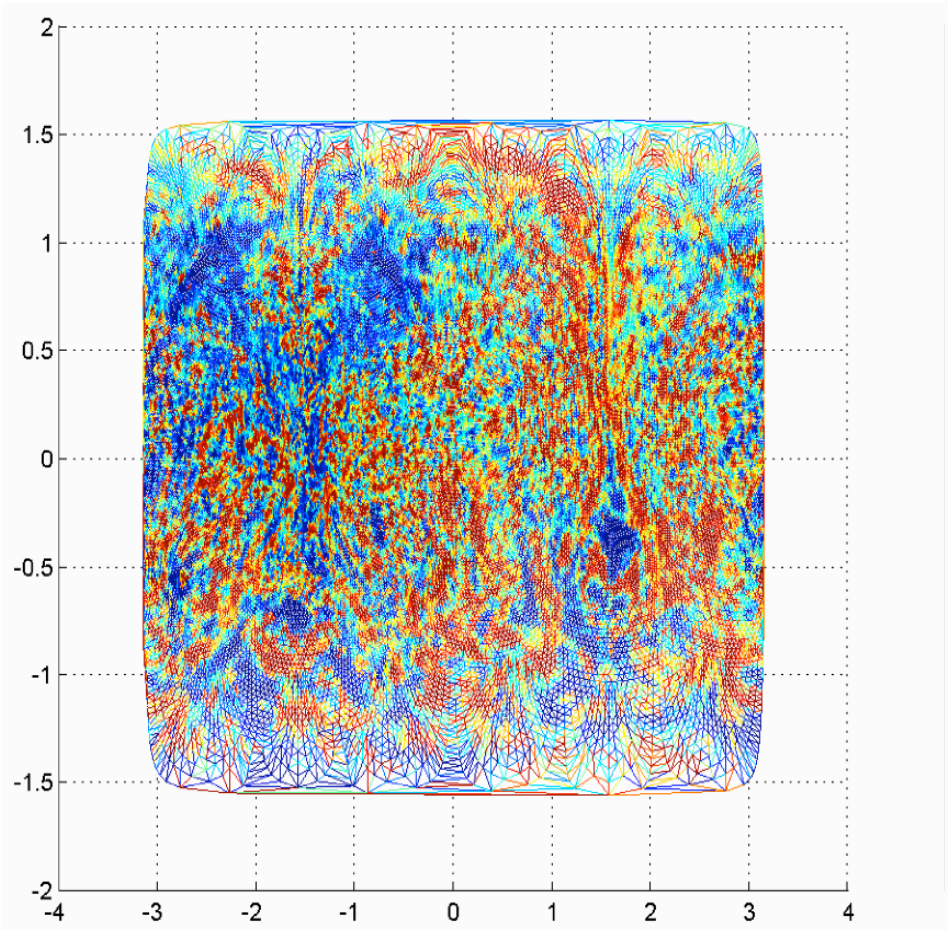
Noisy thickness measures from triangle mesh



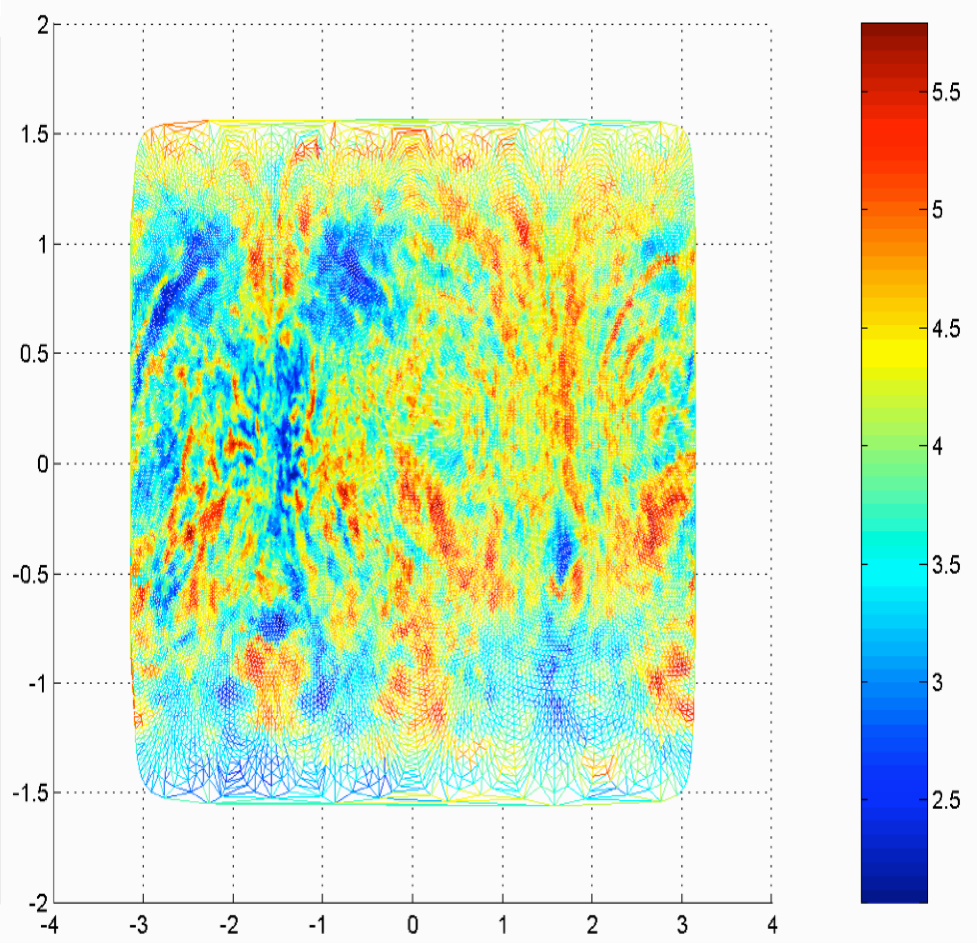


Flattened map:
Spherical projection
(visualization only)

Flat map of cortical thickness



Original thickness



Heat kernel smoothing

Why do we smooth images before statistical analysis?

1. To increase the signal-to-noise ratio (SNR).

2. Random field based multiple comparison correction requires very smooth Gaussian random field assumption.

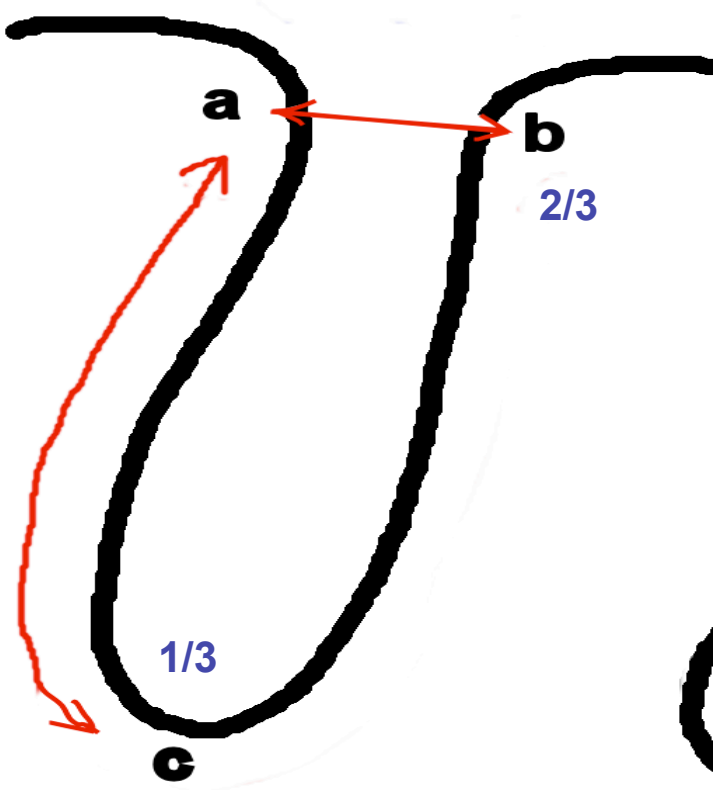
Why Gaussian kernel smoothing?

1. It is a standard technique in imaging.
2. Computationally fast.
3. Computationally easy to implement.

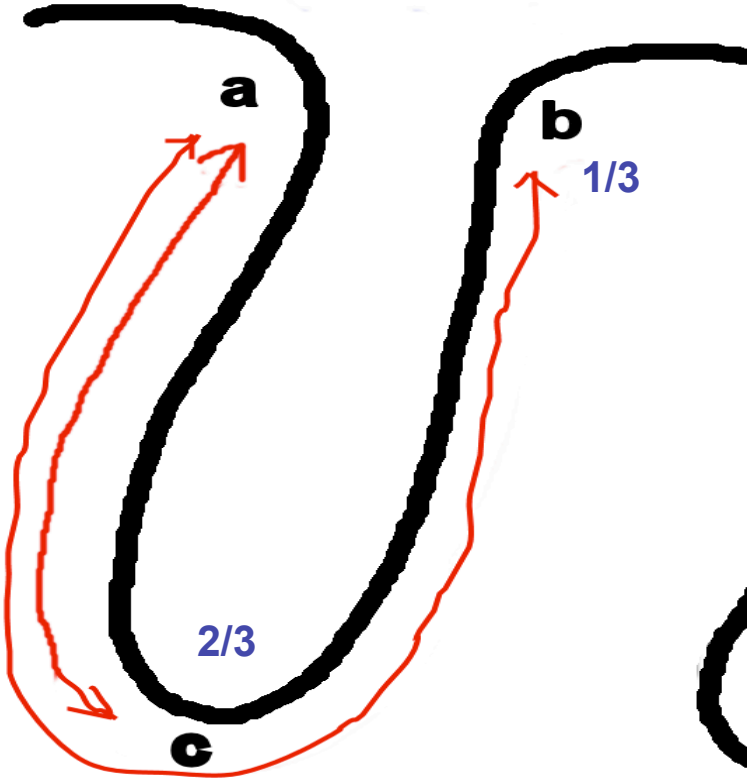
How to perform “kernel smoothing” on anatomical boundary?

Difficulty of performing smoothing along boundary

Due to curved geometry, the shortest distance between two points is not a straight line. So we may incorrectly assign less weights to the closer measurements.



Improper kernel weighting



Proper kernel weighting

Smoothing on Anatomical Boundary

Gaussian kernel smoothing

$$F(x,t) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} e^{-(x-y)^2/4t} f(y)dy$$



Isotropic diffusion

$$\frac{\partial F}{\partial t} = \Delta F = \frac{\partial^2 F}{\partial^2 x_1} + \cdots + \frac{\partial^2 F}{\partial^2 x_n}, \quad F(x,0) = f(x)$$

Heat kernel smoothing

Gaussian kernel smoothing is a special case of heat kernel smoothing. It is the solution of the isotropic diffusion equation.

$$K_\sigma * Y(p) = \int_{\partial\Omega} K_\sigma(p,q) Y(q)\, d\mu(q)$$

Estimate heat kernel K and simply perform integral convolution on surface.

Mathematical detail will be discussed later.

Iterated heat kernel identity

Theorem 4 *Heat kernel smoothing with large bandwidth can be decomposed into multiple kernel smoothing with smaller bandwidth via*

$$K_\sigma^{(k)} * f = \underbrace{K_\sigma * \cdots * K_\sigma}_{k \text{ times}} * f = K_{\sqrt{k}\sigma} * f.$$

Example.
Single Gaussian kernel smoothing with the bandwidth of 10 mm FWHM.
= 100 repeated applications of Guassian kernel smoothing with the bandwidth of 1 mm FWHM.

1st order approximation of heat kernel

$$\widetilde{W}_\sigma(p, q_i) = \frac{\exp\left[-\frac{d^2(p,q_i)}{2\sigma^2}\right]}{\sum_{j=0}^{m} \exp\left[-\frac{d^2(p,q_j)}{2\sigma^2}\right]}$$

Heat kernel smoothing is performed iteratively with smaller FWHM.

Algorithm 1

For $i = 1$ *to* n *do*

Find a set of neighboring vertices $N(q_i)$ *of* q_i.

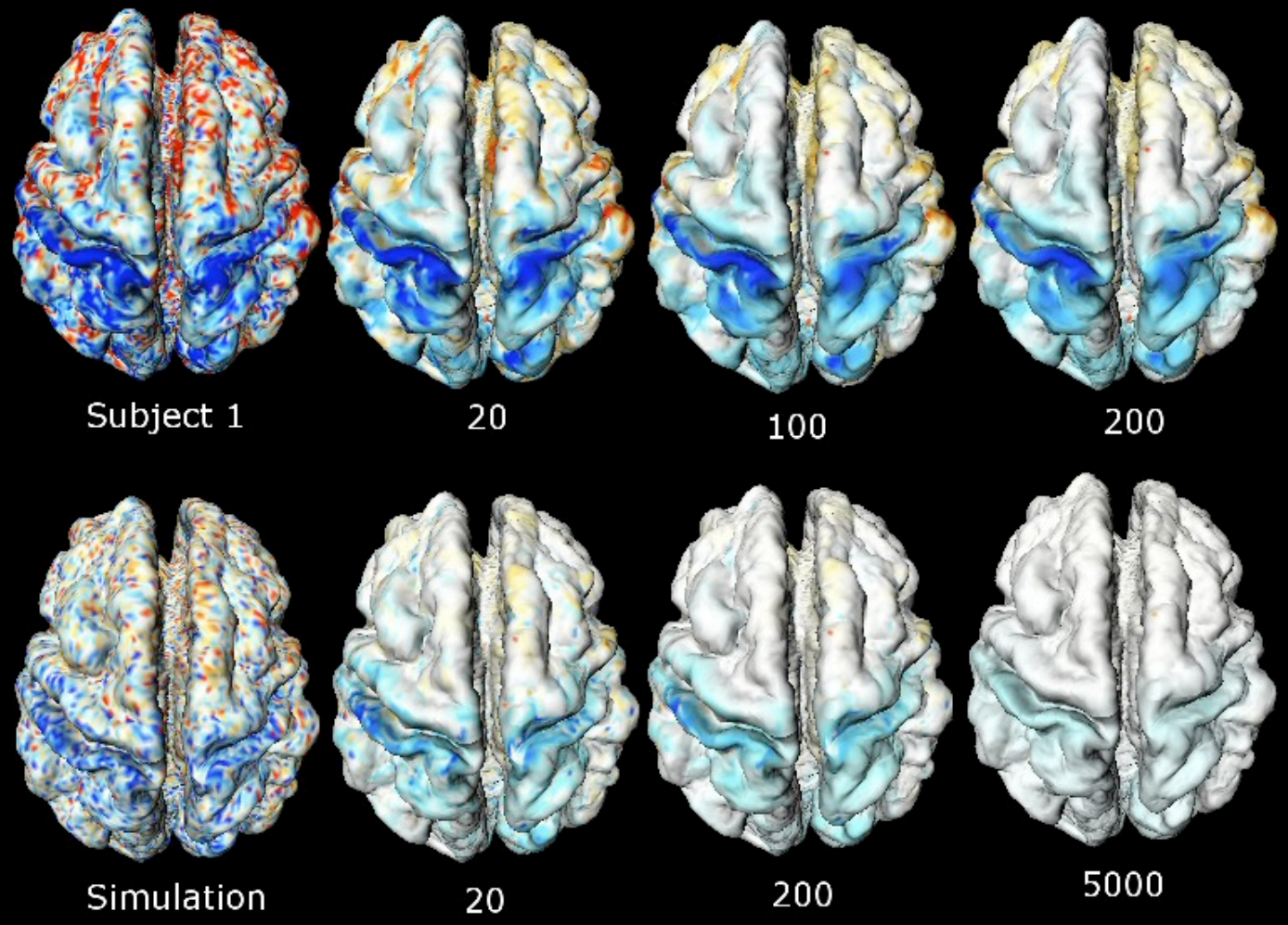
Compute the weighted average and store $Z(q_i) \leftarrow W_\sigma * Y(q_i)$.

End.

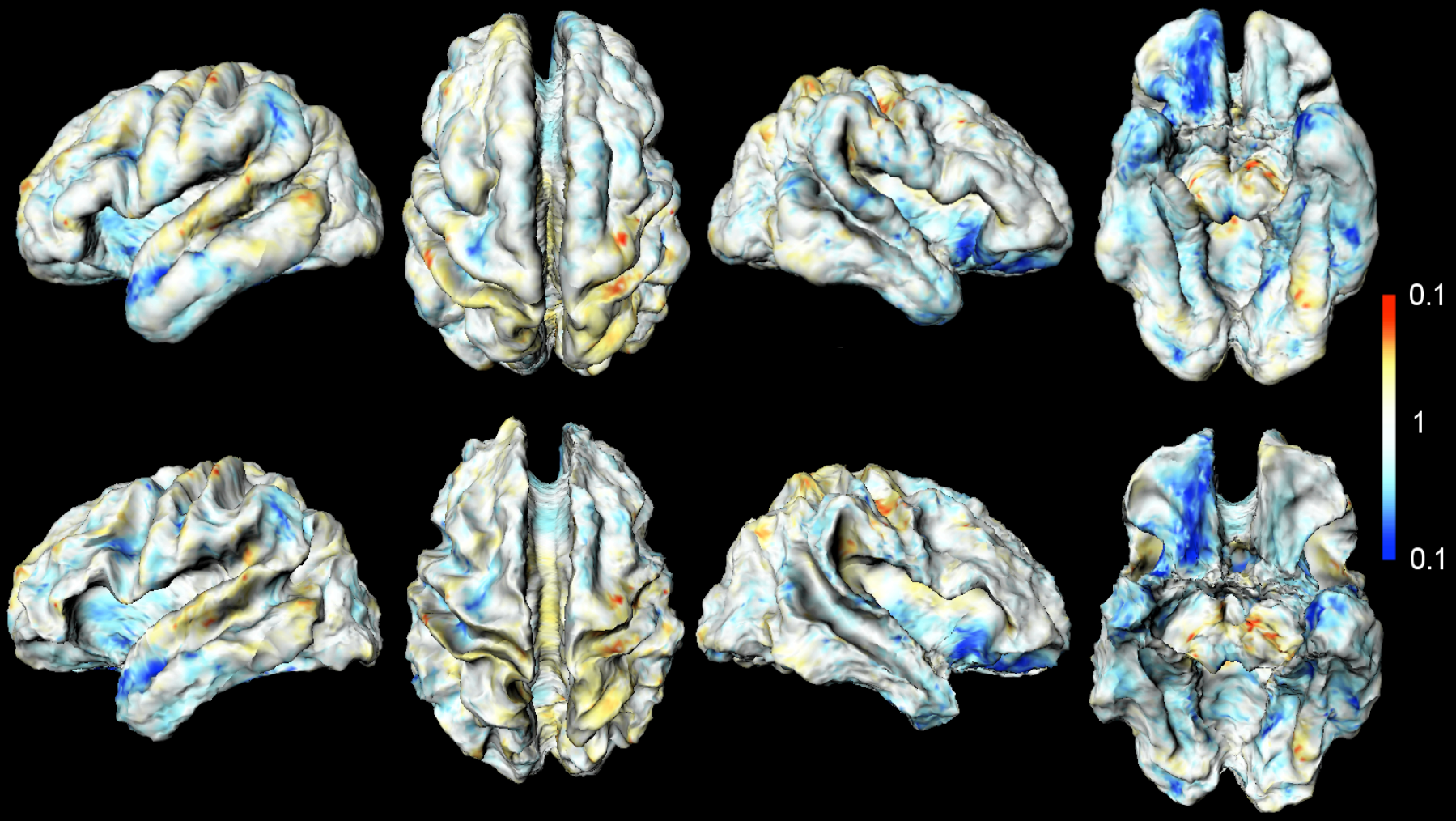
Update $Y \leftarrow Z$.

Repeat this procedures k*-times.*

Heat kernel smoothing on cortical thickness



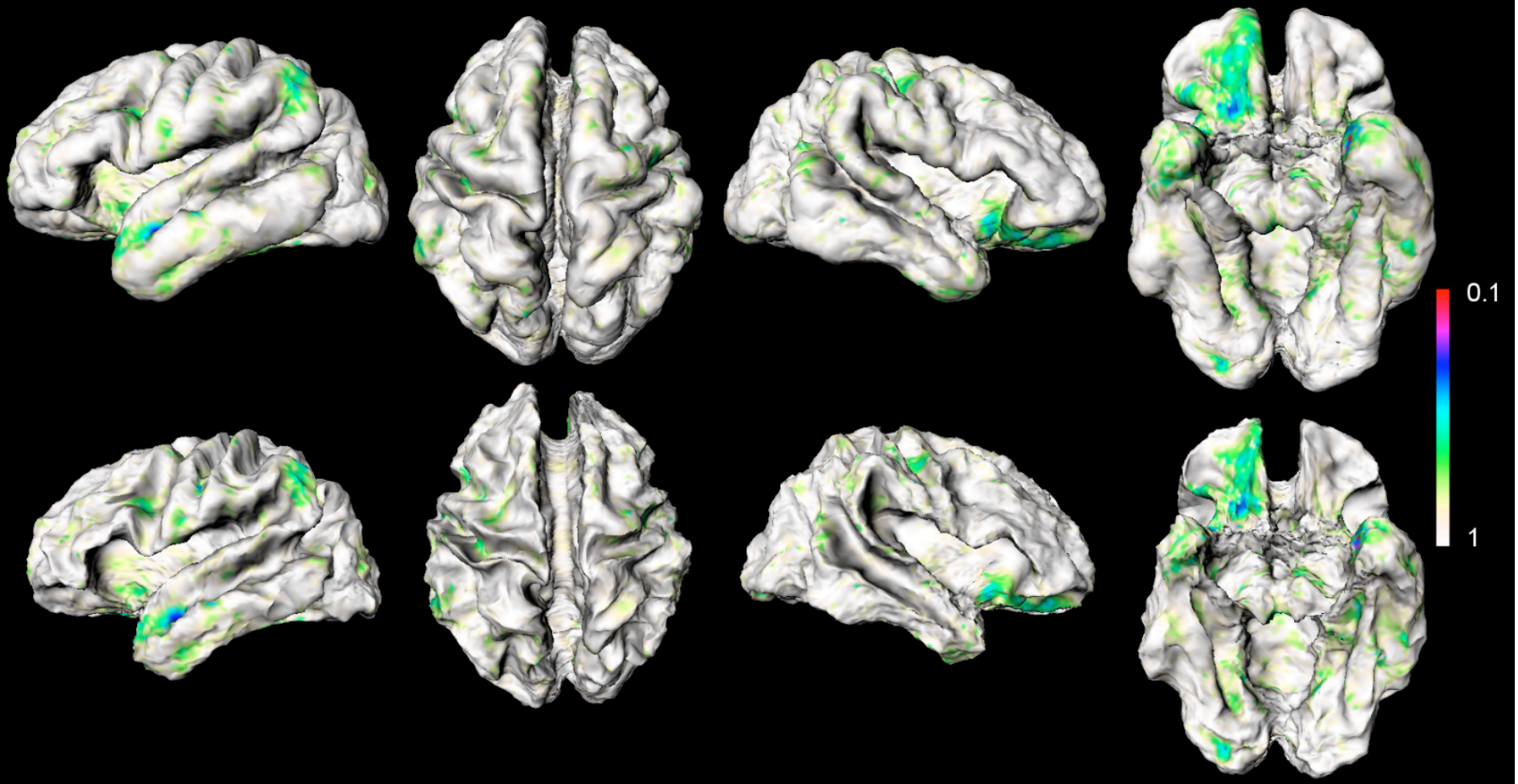
corrected *p*-value map for *t*-test in autism



Decrease: left superior temporal sulcus, left occipital-temporal gyrus, right orbital prefrontal

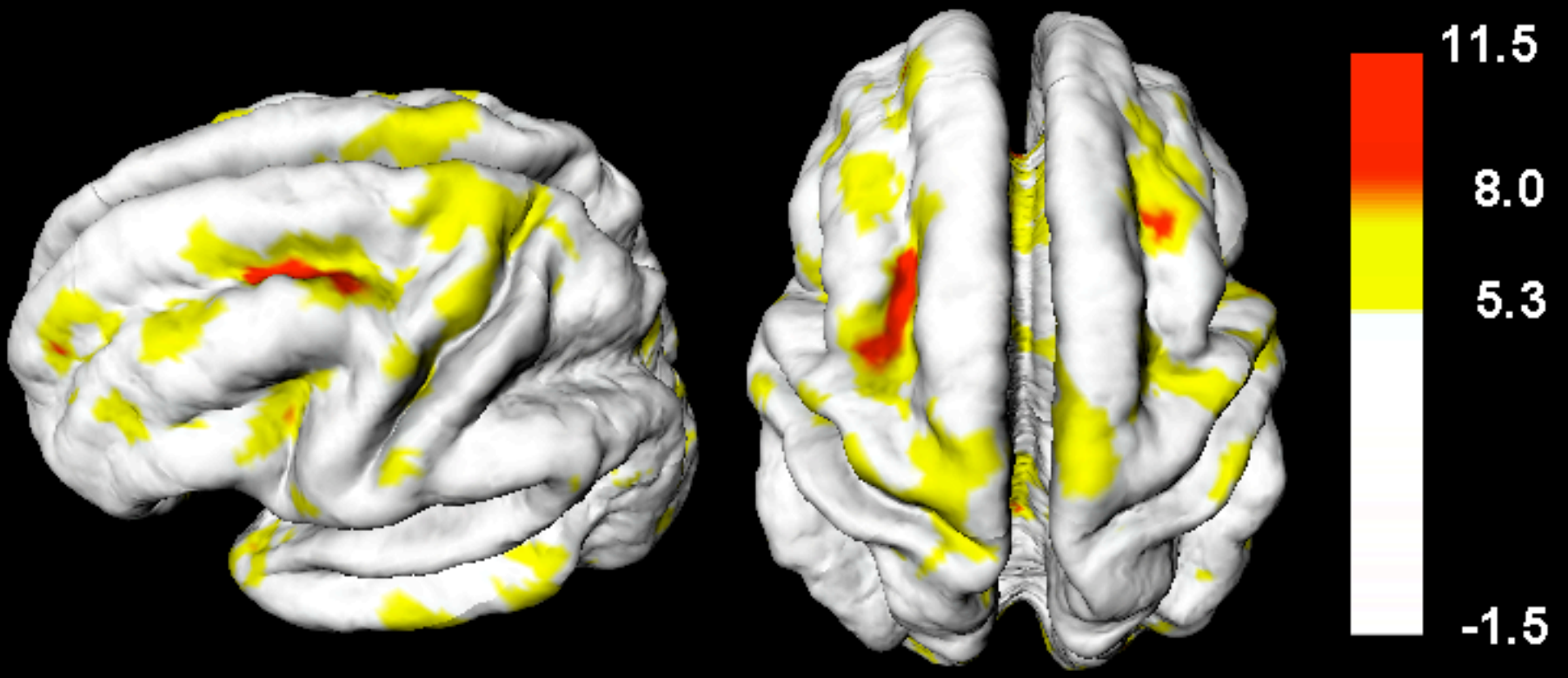
Increase: left superior temporal gyrus, left middle temporal gyrus, left and right postcentral sulci

corrected *p*-value map for *F* test accounting for age effect



Decrease: **left superior temporal sulcus**
left occipital-temporal gyrus
right orbital prefrontal

Cortical thickness change *t* map between age 12 and 16.



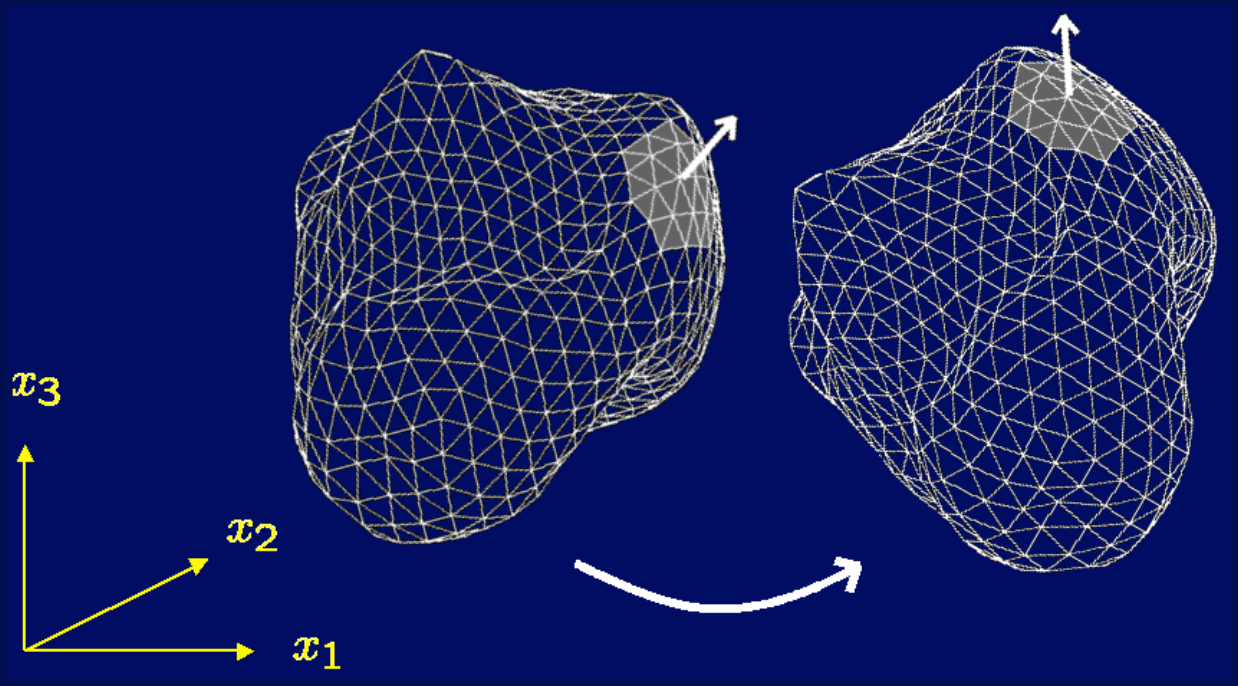
Other surface measures

- cortical thickness, curvatures, surface area, tissue density.
- fractal dimension = measure of complexity of anatomical shape.
- surfaces of most anatomical boundaries: corpus callosum, hippocampus, amygdala, liver, mandible.

Curvature estimation via Surface Parameterization

Global: tensor splines, SPHARM
Local: quadratic surface fitting

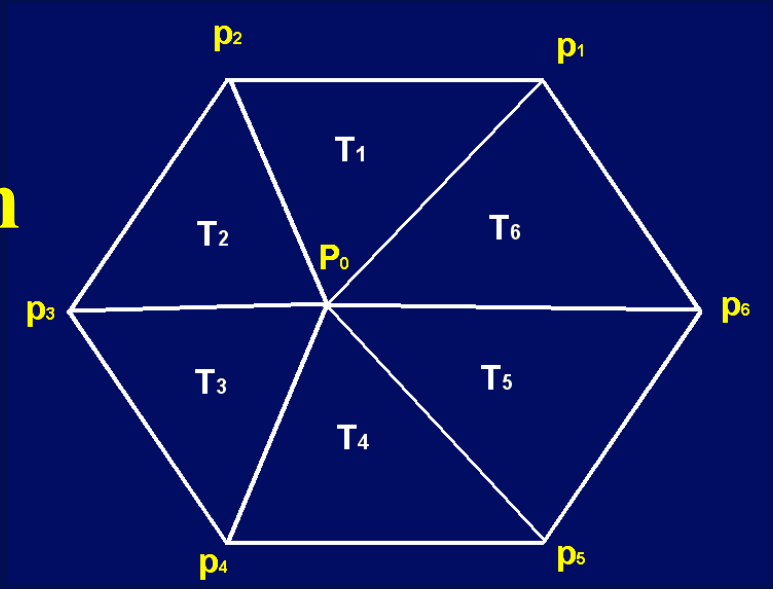
$$X(u^1, u^2) = \begin{pmatrix} x_1(u^1, u^2) \\ x_2(u^1, u^2) \\ x_3(u^1, u^2) \end{pmatrix}$$



Find the best fitting tangent plane via PCA

$$s(u^1, u^2) = \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_1^2 + 2\beta_4 u_1 u_2 + \beta_5 u_2^2 + \cdots$$

Polynomial Regression on irregular triangular mesh



$$Y = \mathbb{X}\beta$$

$$\begin{pmatrix} u_1^3 \\ u_2^3 \\ \vdots \\ u_m^3 \end{pmatrix} = \begin{pmatrix} u_1^1 & u_1^2 & (u_1^1)^2 & u_1^1 u_1^2 & (u_1^2)^2 \\ u_2^1 & u_2^2 & (u_2^1)^2 & u_2^1 u_2^2 & (u_2^2)^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ u_m^1 & u_m^2 & (u_m^1)^2 & u_m^1 u_m^2 & (u_m^2)^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{pmatrix}$$

$$\hat{\beta} = (\mathbb{X}'\mathbb{X})^{-}\mathbb{X}'Y$$

Riemannian Metric Tensors

The first fundamental form

$$g_{ij} = \langle \frac{\partial X}{\partial u^i}, \frac{\partial X}{\partial u^j} \rangle$$

The second fundamental form

$$l_{ij} = \langle \mathbf{n}, \frac{\partial^2 X}{\partial u^i \partial u^j} \rangle$$

Mean curvature

$$K_M = \mathrm{tr}(g^{-1} l)/2$$

Gaussian curvature

$$K_G = \det(g^{-1} l) = \det(l)/\det(g)$$

Laplace-Beltrami operator

$$\Delta_X F = \frac{1}{|g|^{1/2}} \sum_{i,j=1}^{2} \frac{\partial}{\partial u^i} \left(|g|^{1/2} g^{ij} \frac{\partial F}{\partial u^j} \right)$$

$$s(u^1,u^2)=\beta_1 u_1+\beta_2 u_2+\beta_3 u_1^2+2\beta_4 u_1 u_2+\beta_5 u_2^2+\cdots$$

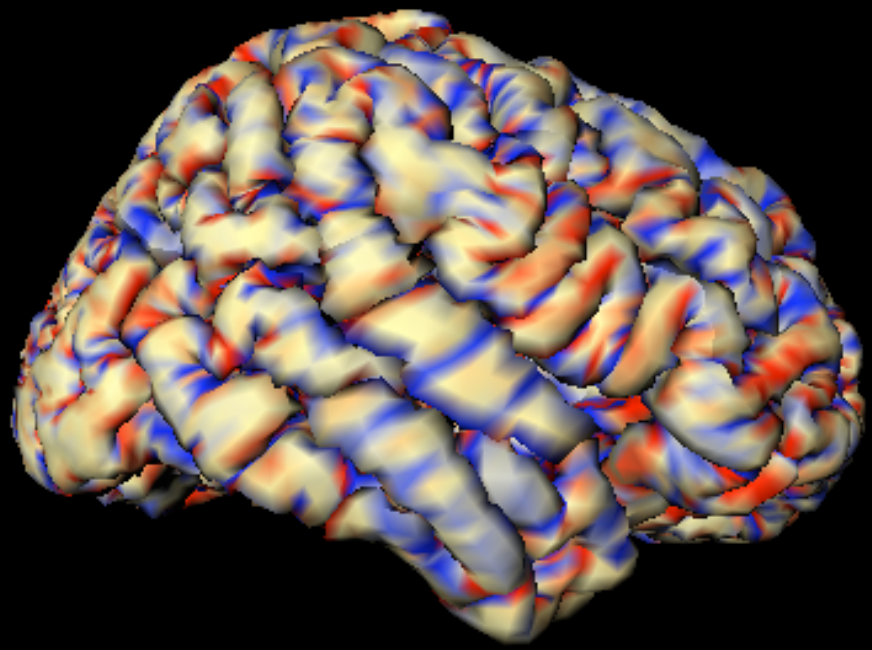


Why quadratic surface ?

$$g=\begin{pmatrix}1+\beta_1^2 & \beta_1\beta_2\\ \beta_1\beta_2 & 1+\beta_2^2\end{pmatrix} \qquad l=\begin{pmatrix}\beta_3 & \beta_4\\ \beta_4 & \beta_5\end{pmatrix}$$

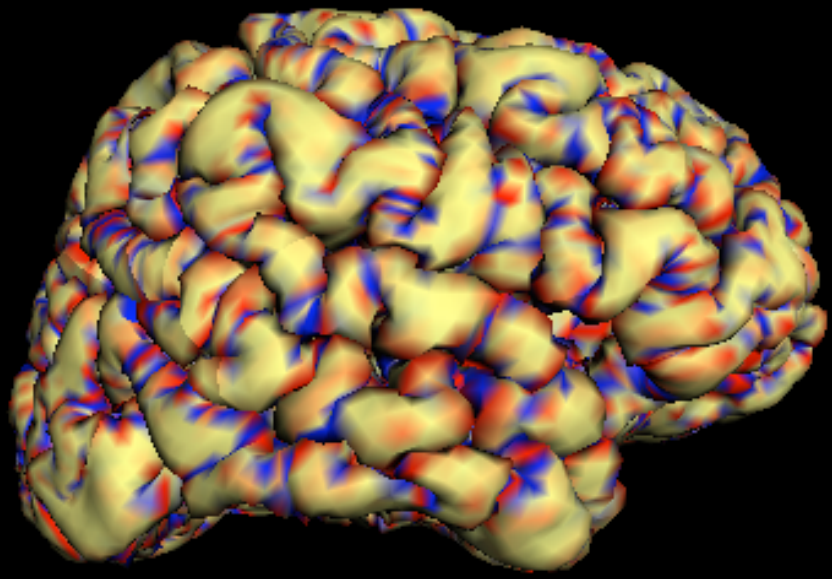
$$K_M=\frac{\mathrm{tr}(g^{-1}l)}{2}=\frac{\beta_3(1+\beta_2^2)+\beta_5(1+\beta_1^2)-2\beta_1\beta_2\beta_4}{2+4(\beta_1^2+\beta_2^2)}$$

Mean Curvature



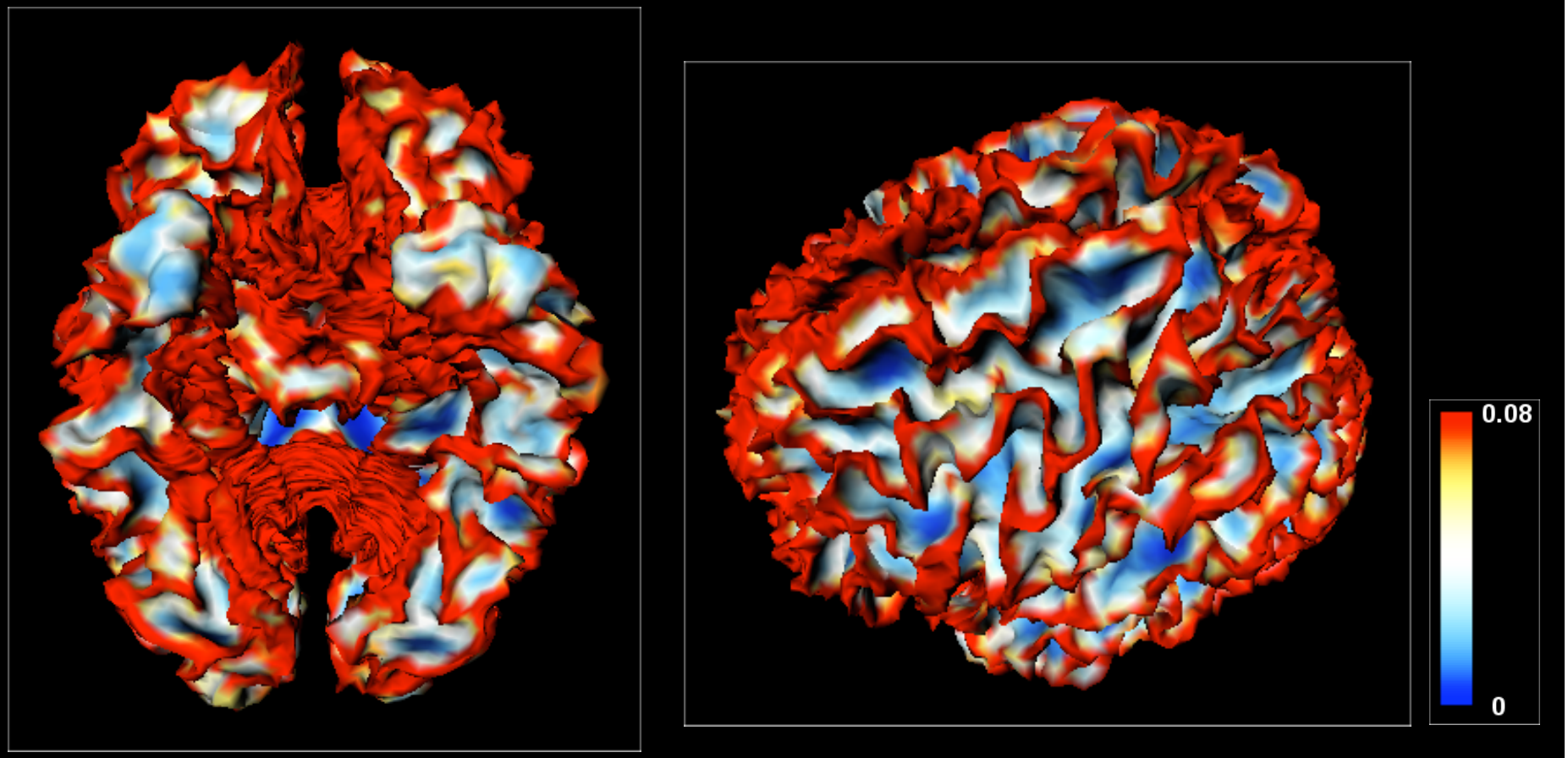


Gaussian Curvature



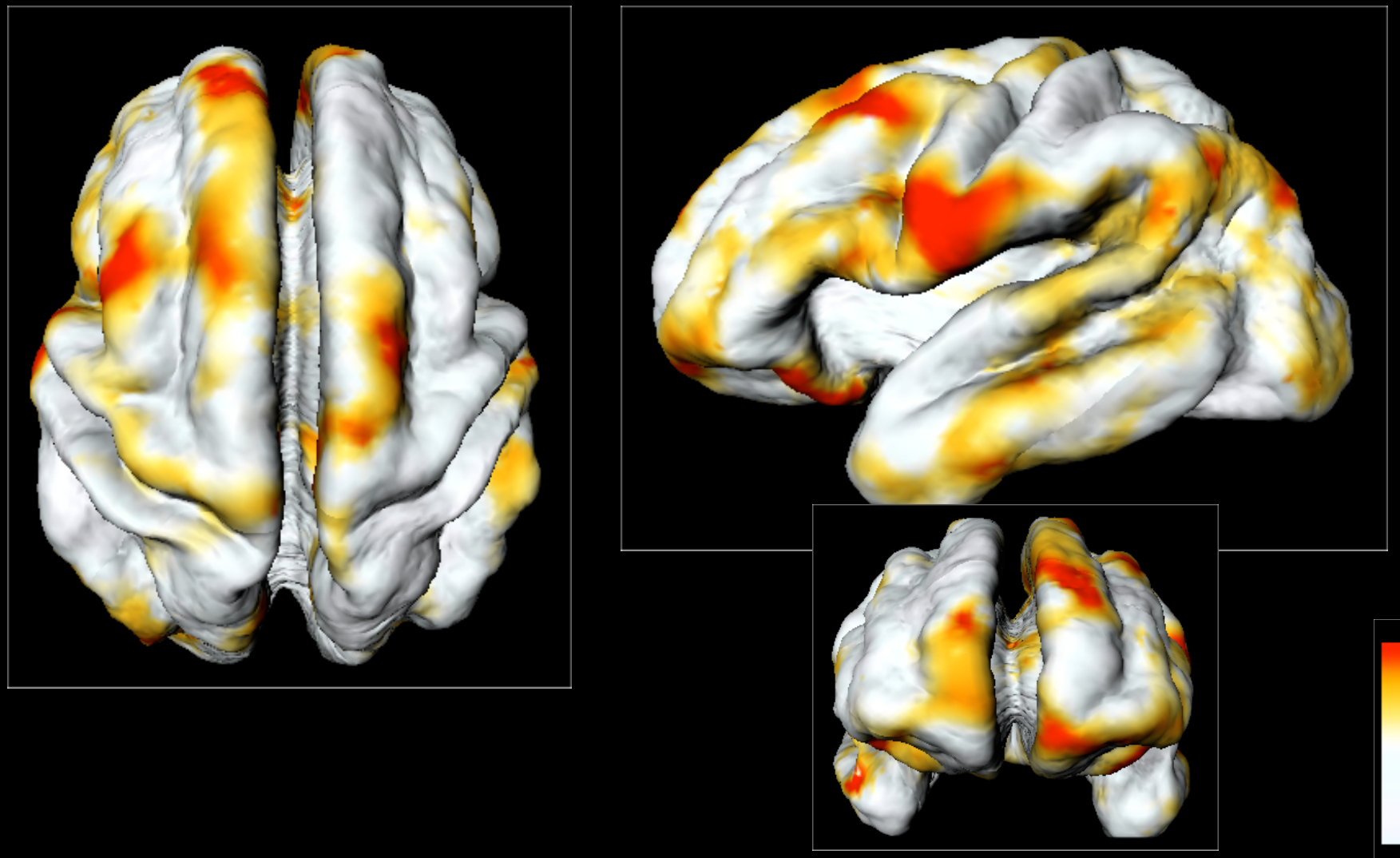


Bending Energy for 14 year old subject



Bending energy or thin-plate spline energy can be used to measure the curvature of the surface.
Between ages 12 and 16, it increases both locally and globally.

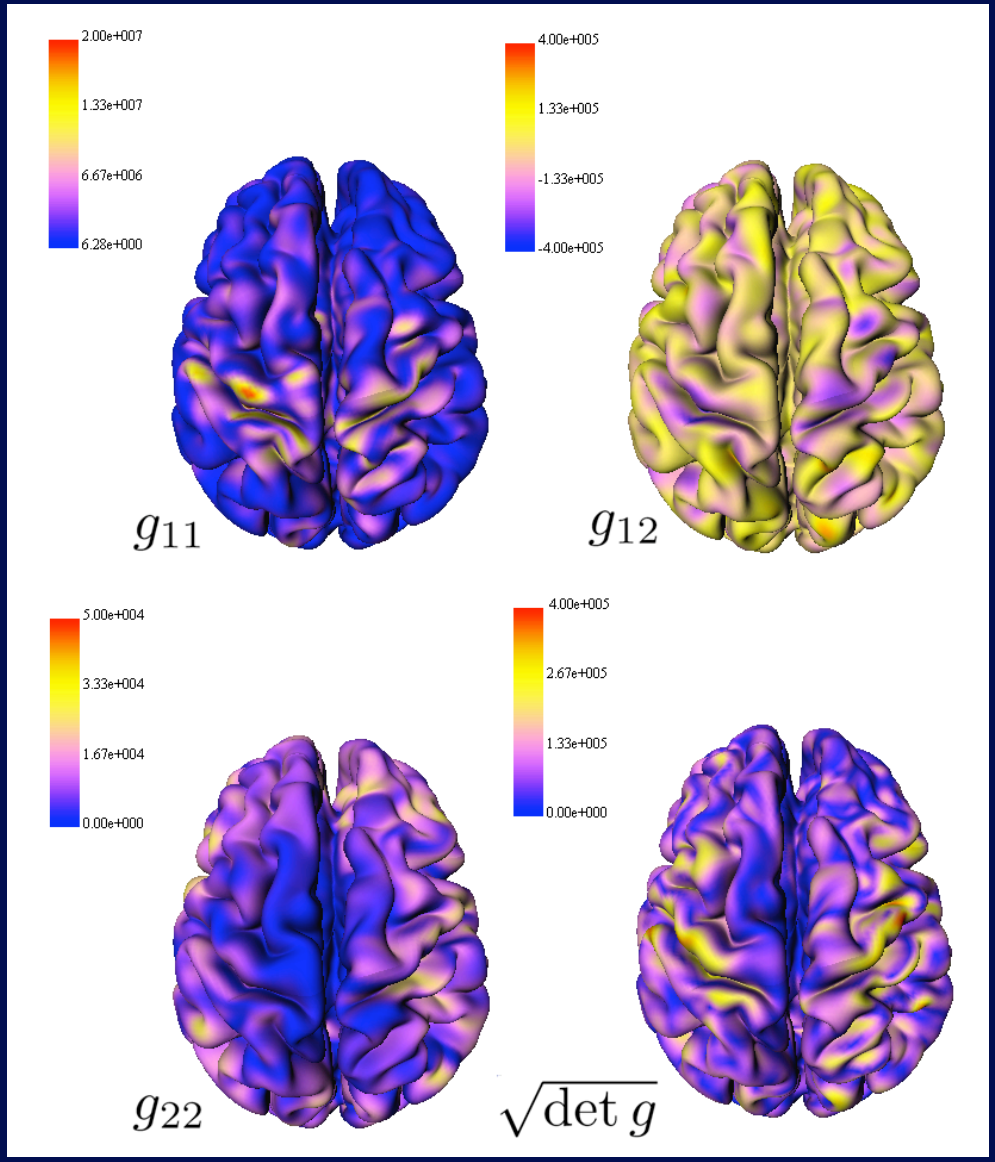
Curvature change *t* map between age 12 and 16



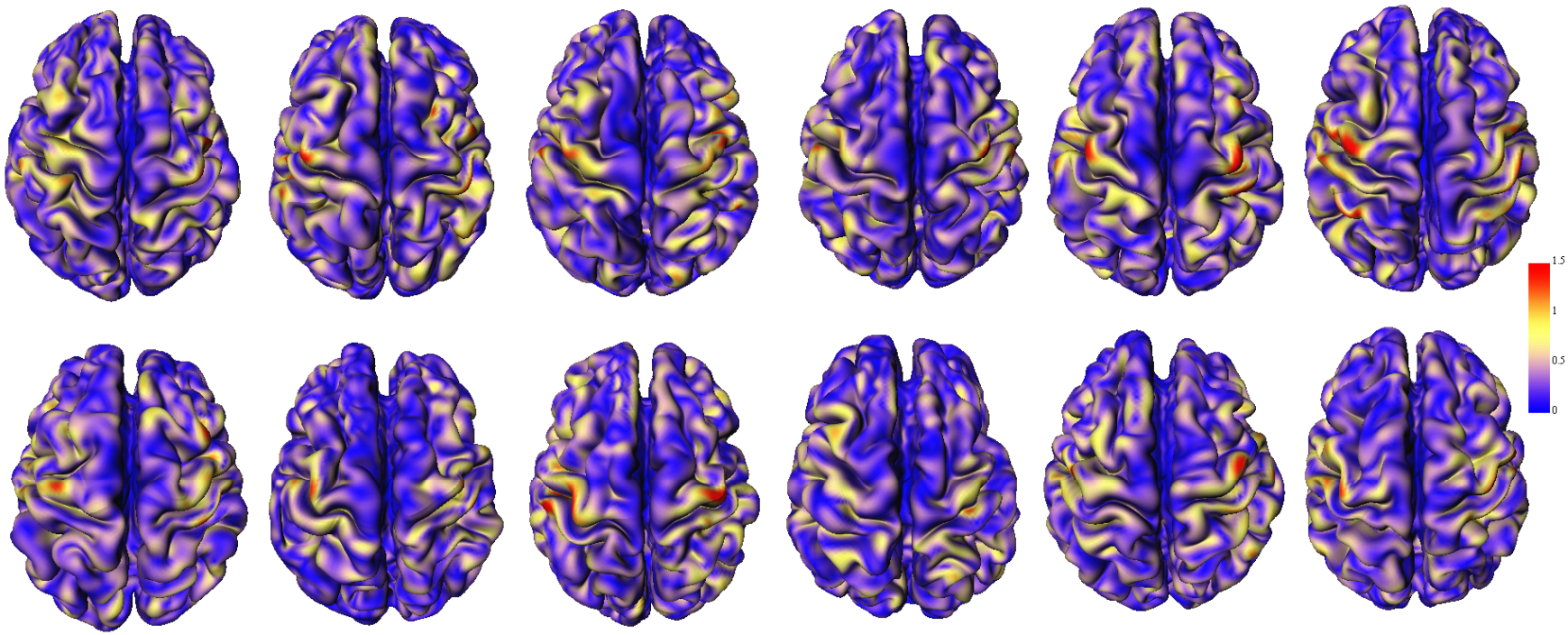
Surface area expansion/shrinking

Local surface area element:

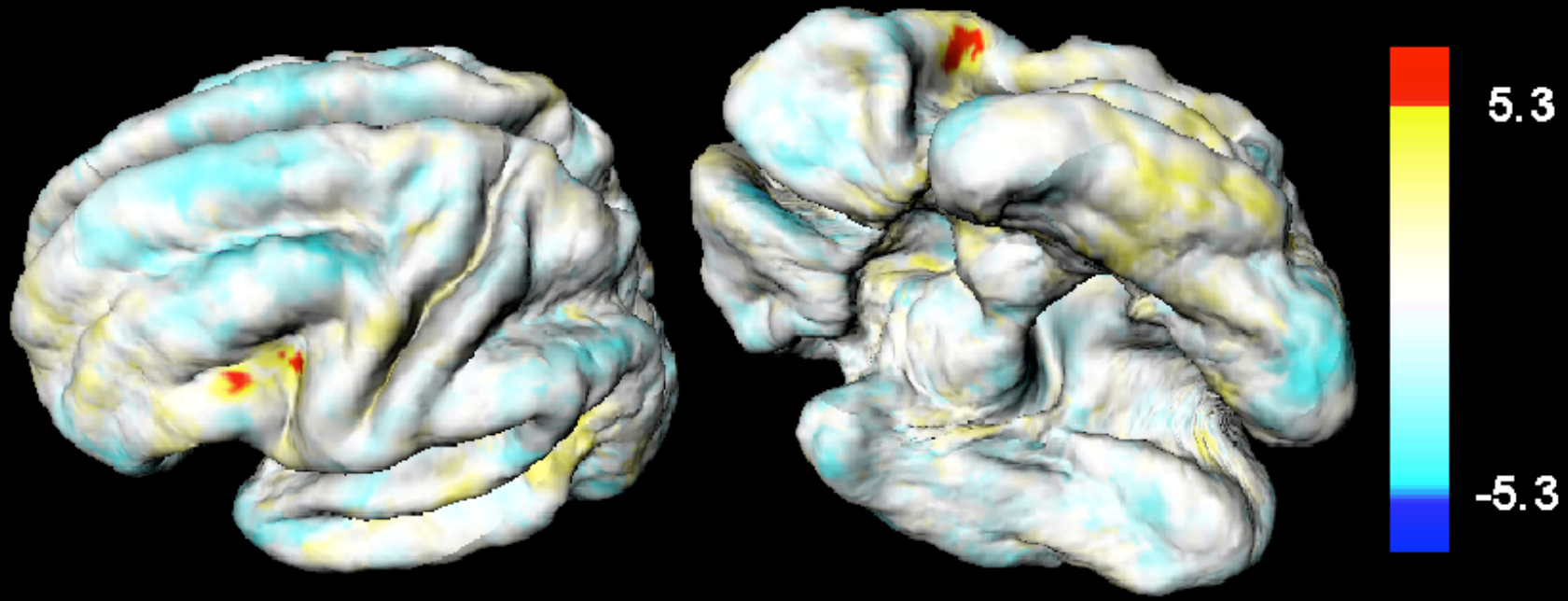
$$\sqrt{|g|} = \sqrt{1 + \beta_1^2 + \beta_2^2}$$



Local area expansion with respect to a template (it ranges between 0 and 1.3)



Surface area change *t* map



dilatation rate between age 12 and 16

min = - 57 % mean = - 0.02 % max = 65 %

Lecture 6 Topics

Spherical harmonic (SPHARM) representation