

Medical Image Analysis

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Lecture 08.

Logistic Discriminant Analysis
in Medical Imaging

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Motivation/Data

Based on 8 elderly controls (EC) and 7 mild cognition impairment (MCI) subjects, we perform the logistic discriminant analysis (LDA) on cortical thickness and cortical surface area to see if these two imaging biomarkers can be used to discriminate MCI from EC.

Data source: Sterling Johnson

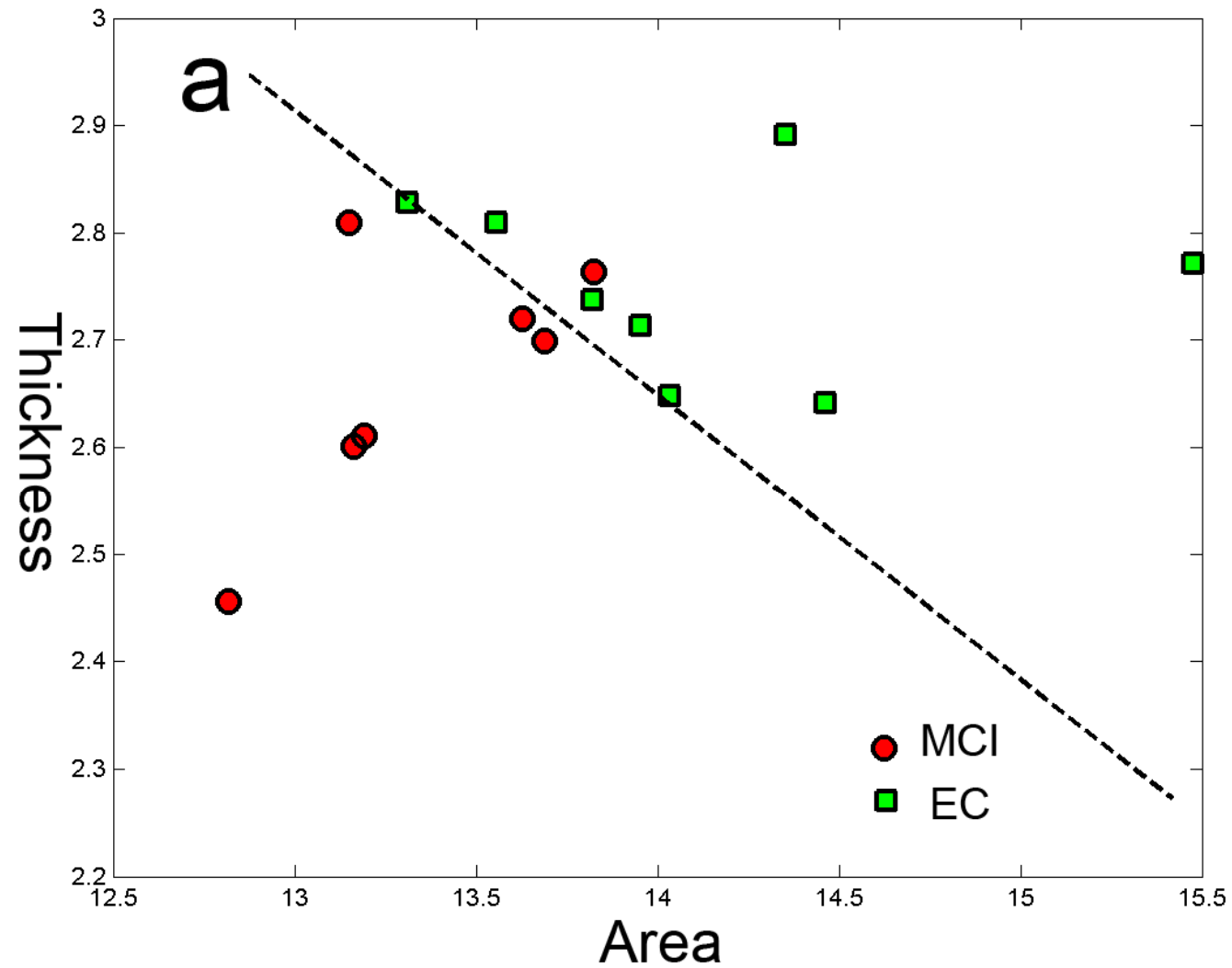
Previous work on AD and cortical thickness

- Cortical thickness has been shown to characterize cortical atrophy in AD patients quite well (Lerch et al., NeuroImage, 2005).
- Question: It is unclear if cortical thickness will be an important biomarkers of discriminating EC vs. MCI.

Group gender age education memory thickness surface area

0	2	63	16	47	2.7718	15.4768
0	2	66	16	48	2.7132	13.953
0	1	70	18	43	2.8094	13.5577
0	1	75	20	48	2.7371	13.821
0	2	75	16	59	2.6476	14.0353
0	2	64	13	42	2.8913	14.354
0	1	69	20	56	2.8284	13.3125
0	1	81	16	41	2.6412	14.463
1	2	75	12	29	2.7204	13.628
1	1	62	20	39	2.6992	13.6875
1	1	68	17	34	2.7632	13.823
1	2	77	14	26	2.6106	13.1918
1	1	80	18	28	2.4556	12.8165
1	1	78	20	37	2.6012	13.164
1	2	64	18	31	2.8094	13.1511

Data: Group, Thickness, Area



Response variable: discrete

- Group variable

$$Y_i \sim \text{Bernoulli}(\pi_i)$$

- $Y = 0$ (EC), 1 (MCI)

$$\pi_i = \tilde{P}(Y_i = 1)$$

Inconsistency of liner model

$$Y_i = \mathbf{x}'_i \beta + \epsilon_i,$$

$$\mathbf{x}'_i = (1, x_{i1}, \dots, x_{ip}) \quad \beta' = (\beta_0, \dots, \beta_p)$$

Inconsistency: $\mathbb{E}Y_j = \pi_i = \mathbf{x}'_i \beta$

The expectation has to be linked to predictors somehow

General setting for GLM

$$g(E Y) = z \cdot \lambda + x \cdot \beta$$

g is a link function

$$Y = g^{-1}(z \cdot \lambda + x \cdot \beta) + \varepsilon$$

Logistic regression function

$$\pi_i = g(x_i) = \frac{\exp(\mathbf{x}'_i \beta_i)}{1 + \exp(\mathbf{x}'_i \beta_i)}$$

Logit function - this is the link function we will be using

$$\text{logit}(\pi_i) = \log \frac{\pi_i}{1 - \pi_i} = \mathbf{x}'_i \beta_i$$

Likelihood function

$$L(\beta|y_1, \dots, y_n) = \prod_{i=1}^n \pi_i^{y_i} (1-\pi_i)^{1-y_i} = \prod_{i=1}^n \left[\frac{\exp(\mathbf{x}_i' \beta_i)}{1 + \exp(\mathbf{x}_i' \beta_i)} \right]^{y_i} \prod_{i=1}^n \left[\frac{1}{1 + \exp(\mathbf{x}_i' \beta)} \right]^{1-y_i}$$

The parameters are estimated by maximizing the likelihood function. **See /theories for detail.**

MATLAB command for doing GLM with logit link

```
>> beta=glmfit([area thick], [group ones(n,1)], 'binomial')
```

beta =

188.7976

-7.8718

-29.6755

$$G(\text{group}) = 188.7976 - 7.8718 \cdot \text{area} - 29.6755 \cdot \text{thick}$$

```
>> beta=glmfit([education memory], [group ones(n,1)],  
'binomial')
```

```
beta =
```

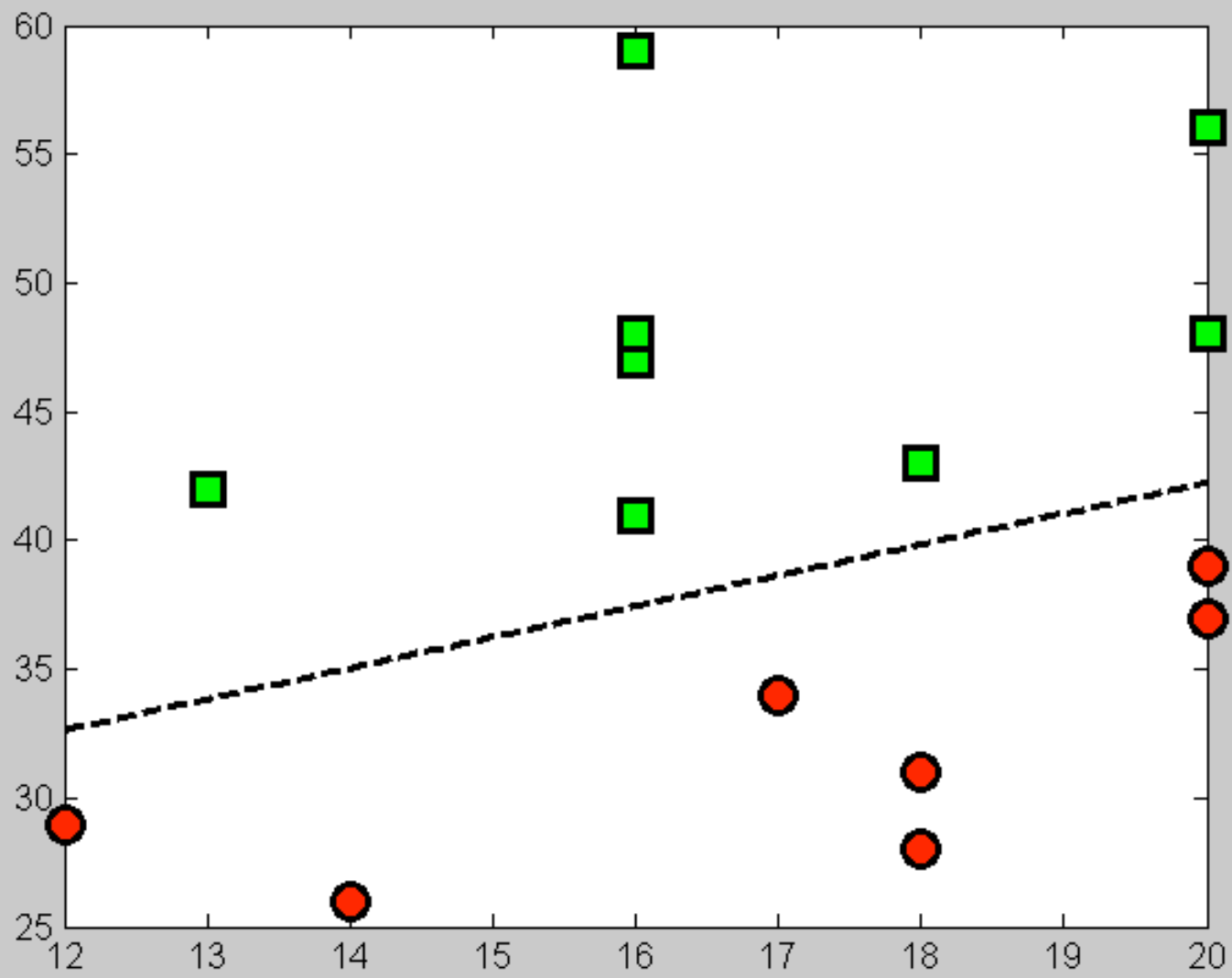
```
55.8206
```

```
3.6825
```

```
-3.0639
```

$$G(\text{group}) = 55.8206 + 3.6825 \cdot \text{education} - 3.0639 \cdot \text{memory}$$

memory



education

Inference on parameters

$$\text{logit}(\pi_i) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p.$$

$$\beta^{(1)} = (\beta_0, \dots, \beta_q)' \quad \beta^{(2)} = (\beta_{q+1}, \dots, \beta_p)'.$$

$$H_0 : \beta^{(2)} = 0.$$

Loglikelihood ratio test

$$2[\log L(\hat{\pi}^{(p)}) - \log L(\hat{\pi}^{(q)})] = D_q - D_p \sim \chi_{p-q}^2.$$

Logistic discriminant

Classification rule (decision rule)

$$P(Y_i = 1) > P(Y_i = 0)$$

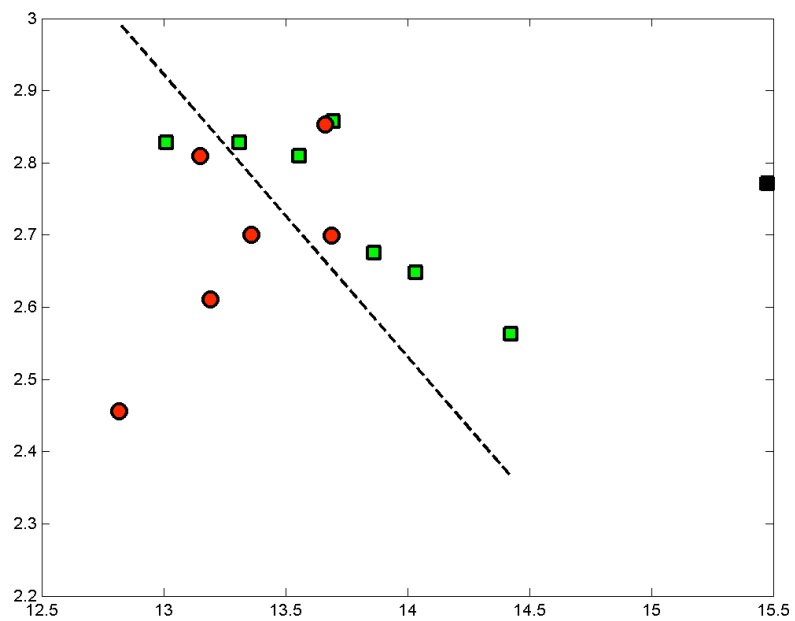
$$\pi_i > 1/2$$

---> group 1

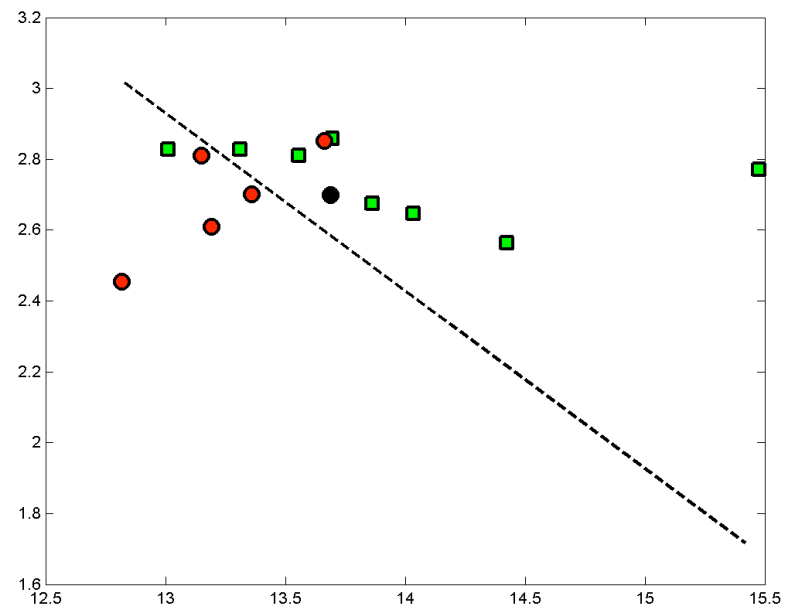
$$\mathbf{x}_i' \boldsymbol{\beta}_i > 0$$

Performance of classifier

- Error rate = over all probability of making wrong decision
- Simplest way to estimate it is to use the concept of cross-validation.
- Leave one out cross-validation

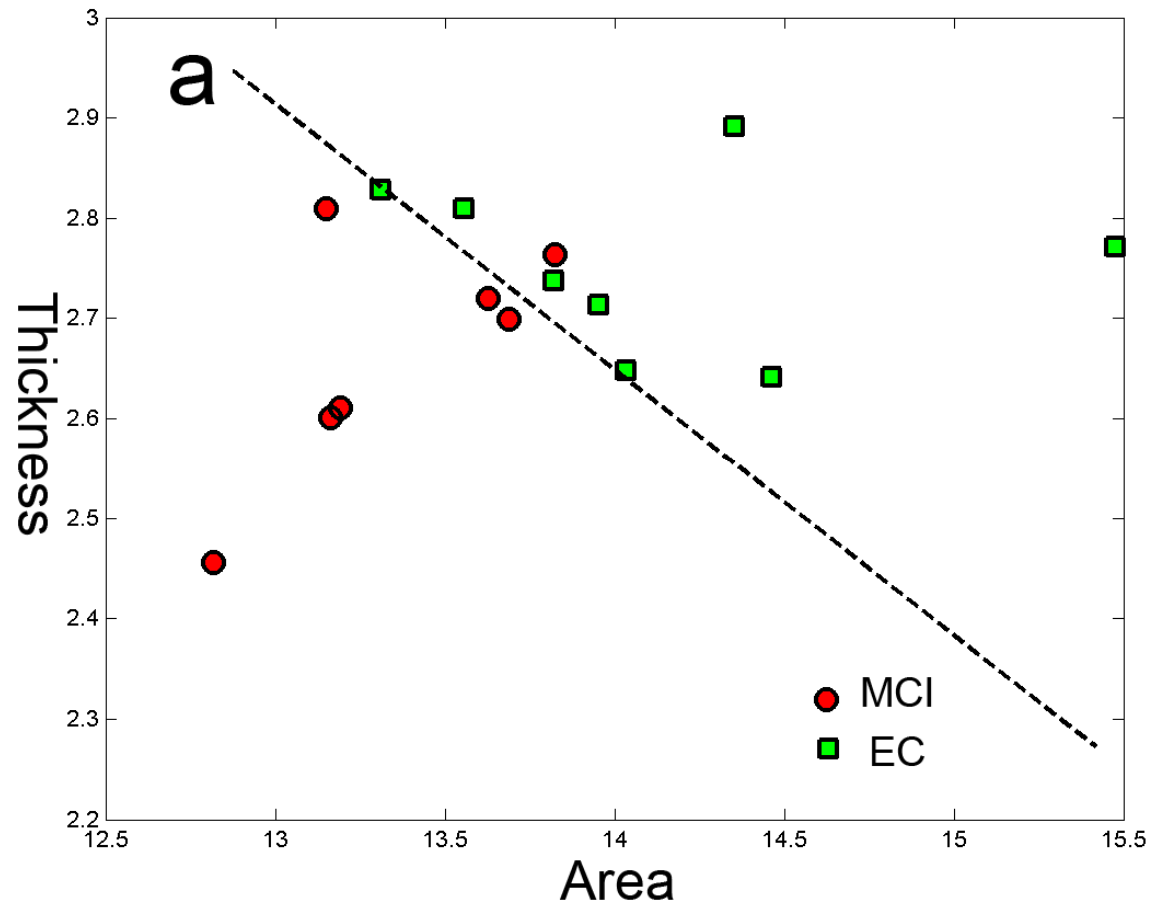


Correct decision



Incorrect decision

Classification boundary

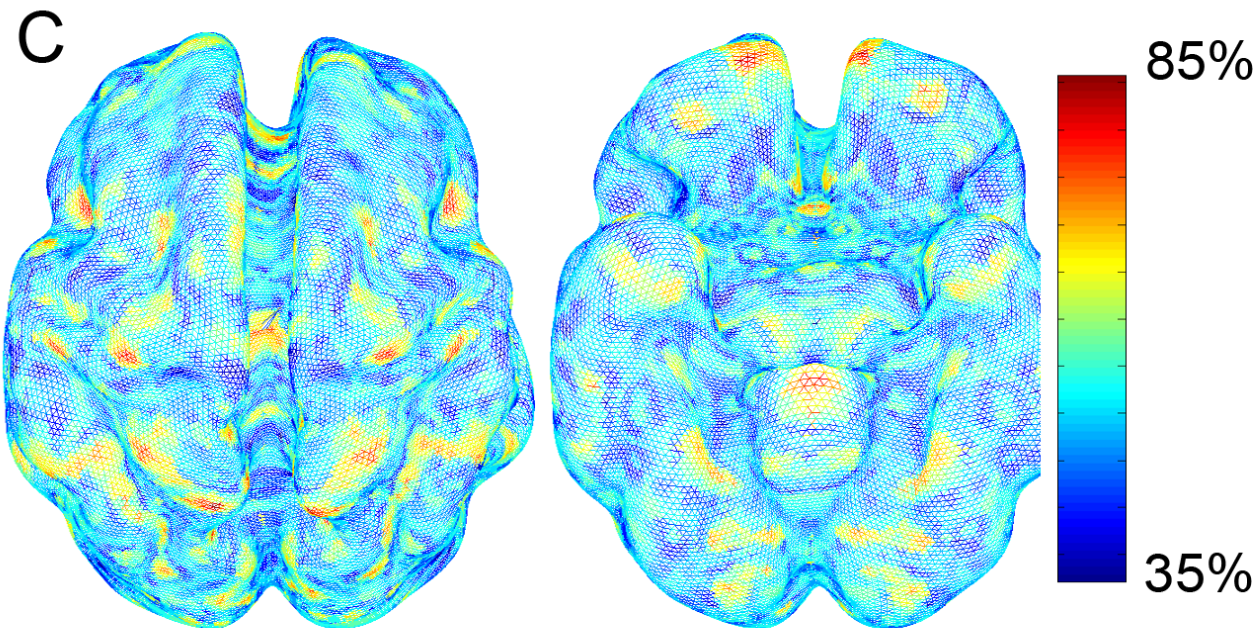


Error rate: thickness --> 47%

Thickness + area --> 20%

area --> 20%

Correct diagnostic rate (1-error rate)
computed at each point on surface



Lecture 9 Topics

Gaussian Kernel Smoothing