

Medical Image Analysis

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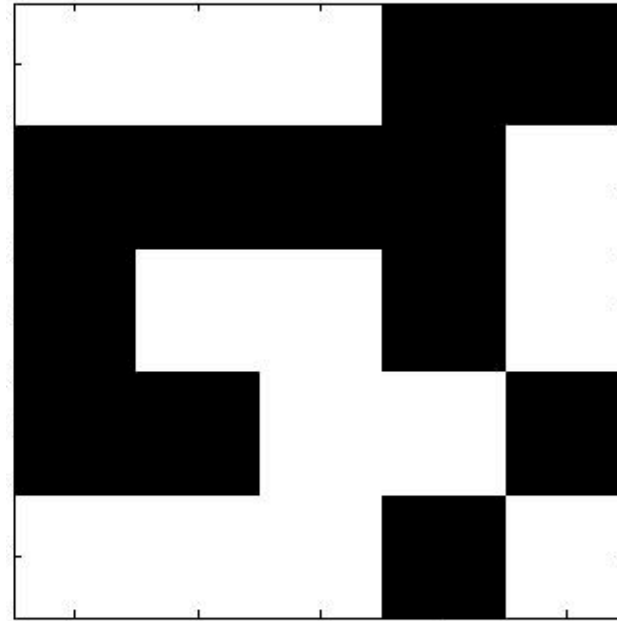
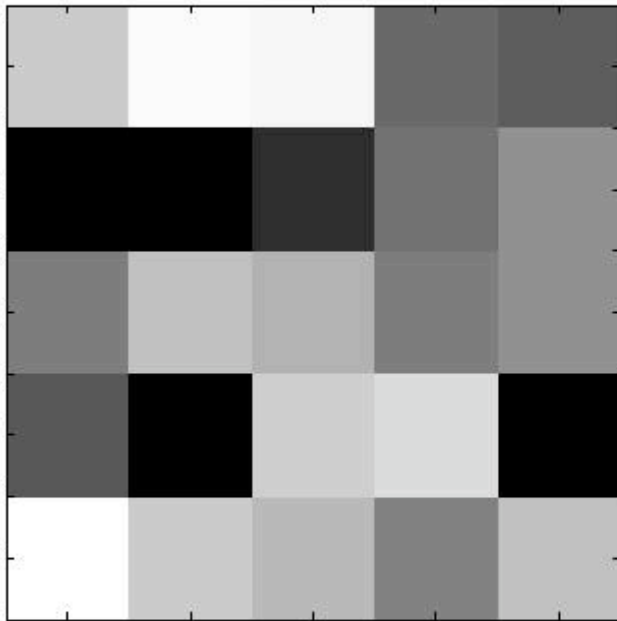
Lecture 13.

Image Segmentation

March 27, 2007

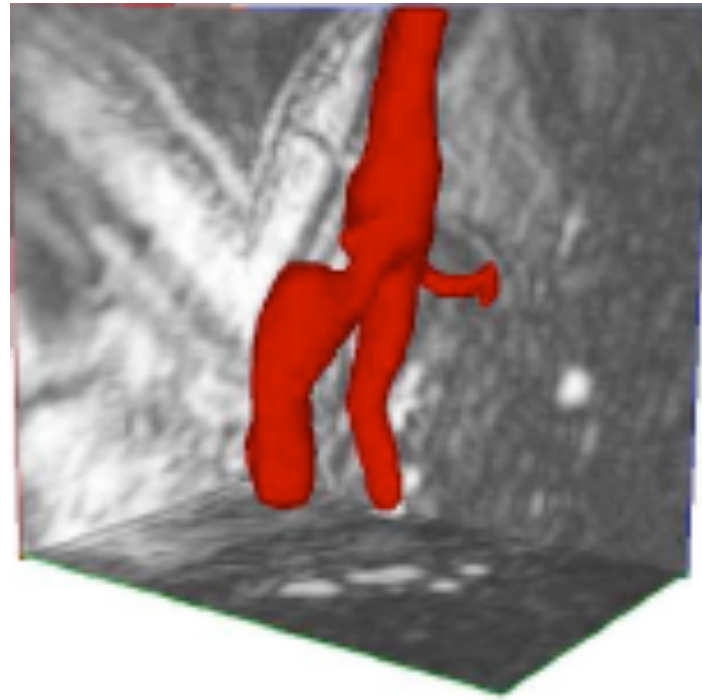
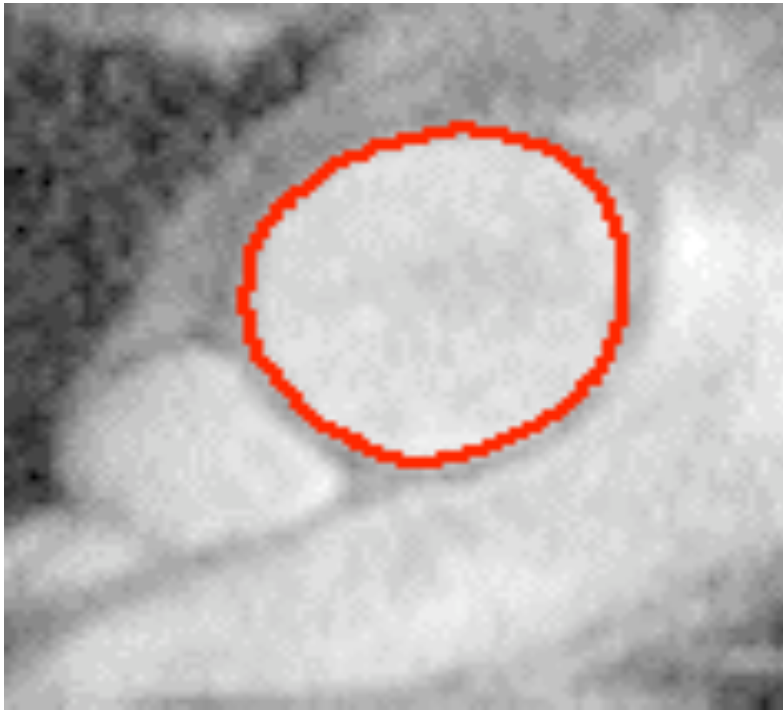
Segmentation

- **Segmentation** is the partition of a digital image into multiple regions according to some criterion.



Source: C. Phillips

Example: CT image segmentation



Example: MRI

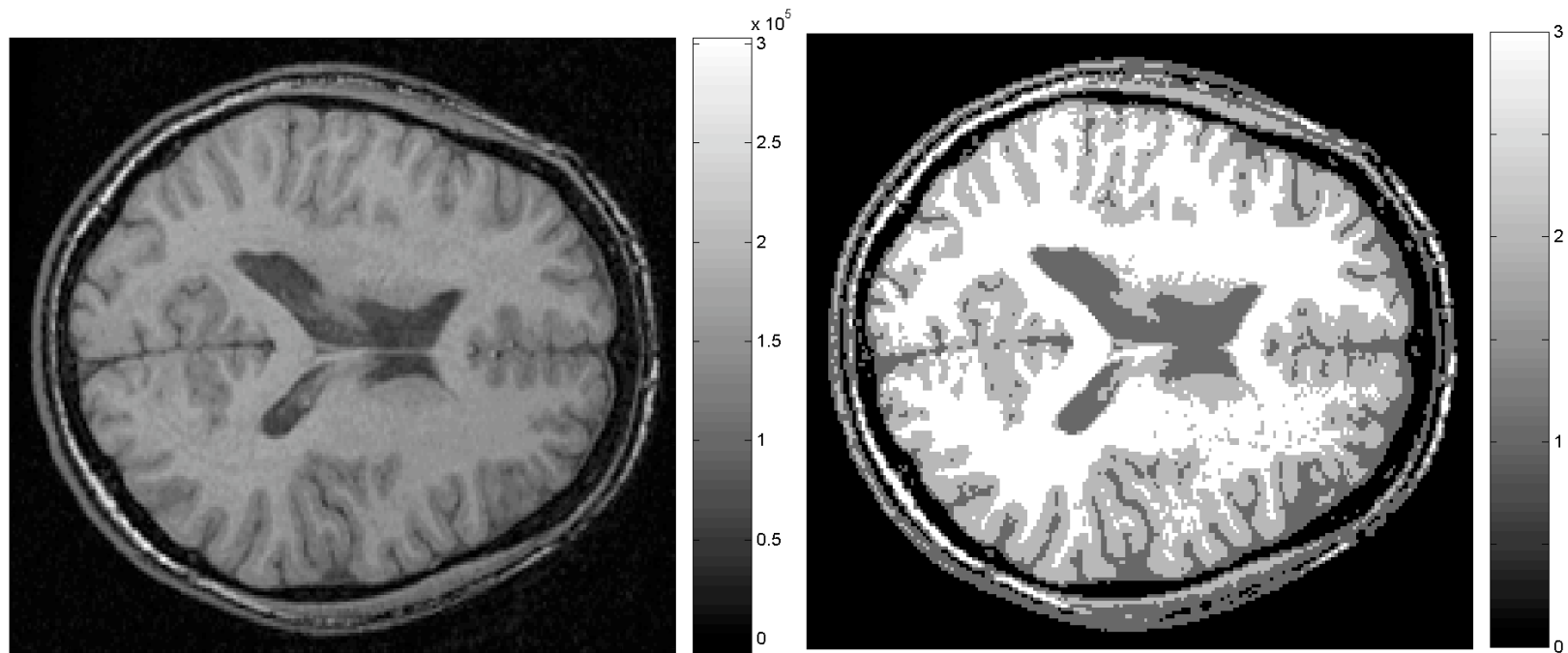


Image Segmentation Methodology

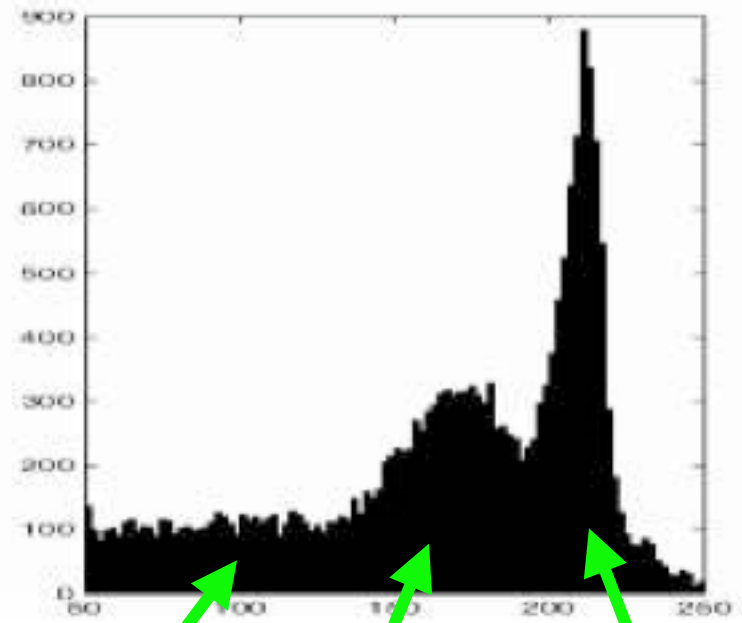
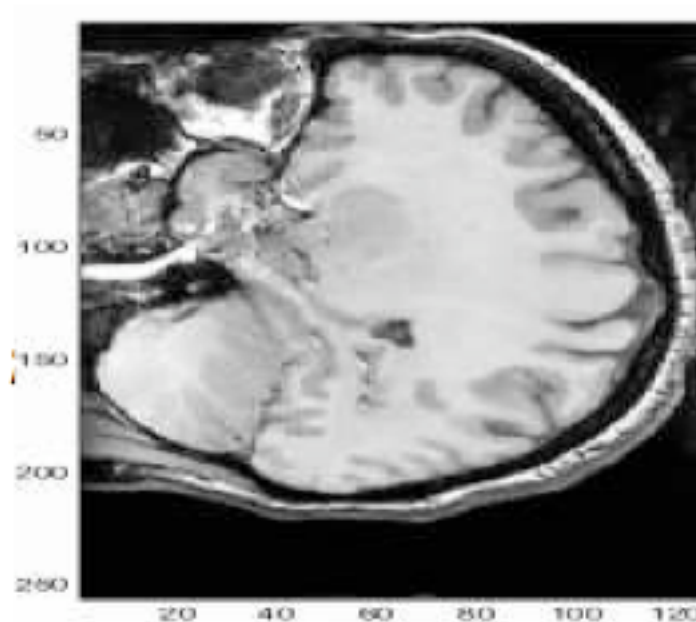
- Intensity histogram based approach

Gaussian mixture modeling

- Shape based approach (PDE based)

Active contour, deformable surface algorithms, level set

Image intensity histogram



CSF gray matter white matter

Gaussian mixture model

$$f(y) = pf_1(y) + (1 - p)f_2(y)$$

$$f_1(y) \approx N(\mu_1, \sigma_1^2)$$

$$f_2(y) \approx N(\mu_2, \sigma_2^2)$$

p = mixing proportion

Parameters are estimated by MLE

Maximum likelihood function

$$L = \prod_{i=1}^n [pf_1(y_i) + (1-p)f_2(y_i)]$$

$$\log L = \sum_{i=1}^n \log [pf_1(y_i) + (1-p)f_2(y_i)]$$

Solve $\frac{\partial \log L}{\partial p} = 0$ numerically

Heuristic idea of EM algorithm

- Expectation maximization (EM) algorithm: iterative method for maximizing difficult likelihood problems
- Instead of maximizing the difficult likelihood directly, we maximize easier likelihood by introducing *latent variables*.

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EM algorithm

Observed variable Y

Missing variable X: Bernoulli(p)

Complete variable $Z=(X,Y)$

The joint density of Z

$$f(x, y) = [pf_1(y)]^x [qf_2(y)]^{1-x}$$

Conditional density of X given Y

$$f(x|y) = \frac{[pf_1(y)]^x [qf_2(y)]^{1-x}}{pf_1(y) + qf_2(y)}$$

EM algorithm

Conditional expectation

$$\mathbb{E}(X|y, p) = \frac{pf_1(y)}{pf_1(y) + qf_2(y)}$$

E-step: Construction of Q-function:

$$\begin{aligned} Q(p) &= E[\log L(X, Y) | Y] \\ &= \sum_{i=1}^n \frac{p_0 f_1(y_i)}{p_0 f_1(y_i) + q_0 f_2(y_i)} \log \frac{p}{1-p} \\ &\quad + n \log(1-p). \end{aligned}$$

EM-algorithm

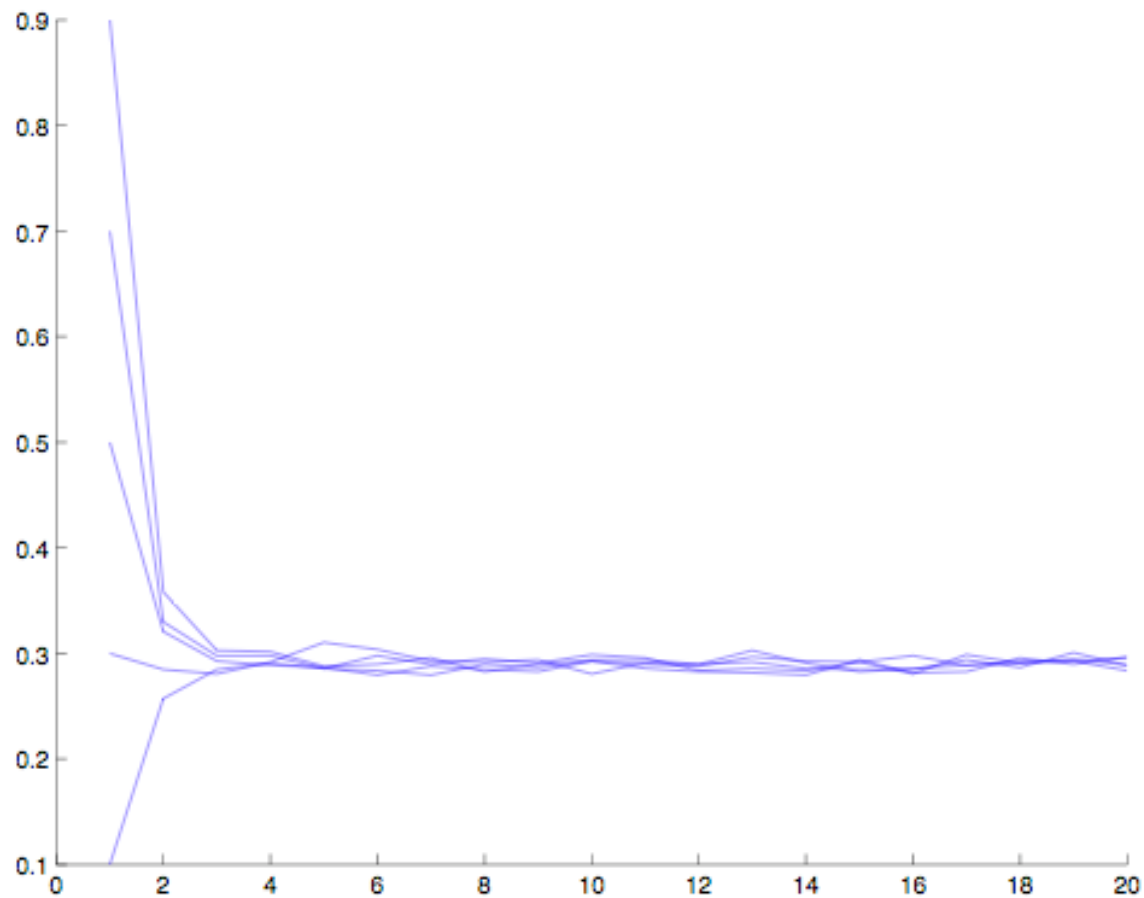
- M-step: $\frac{\partial Q}{\partial p} = 0$

$$p = \frac{1}{n} \sum_{i=1}^n \frac{p_0 f_1(y_i)}{p_0 f_1(y_i) + q_0 f_2(y_i)}$$

Iteration: pick any initial value between 0 and 1

$$\hat{p}_{j+1} = \frac{1}{n} \sum_{i=1}^n \frac{\hat{p}_j f_1(y_i)}{\hat{p}_j f_1(y_i) + (1 - \hat{p}_j) f_2(y_i)}$$

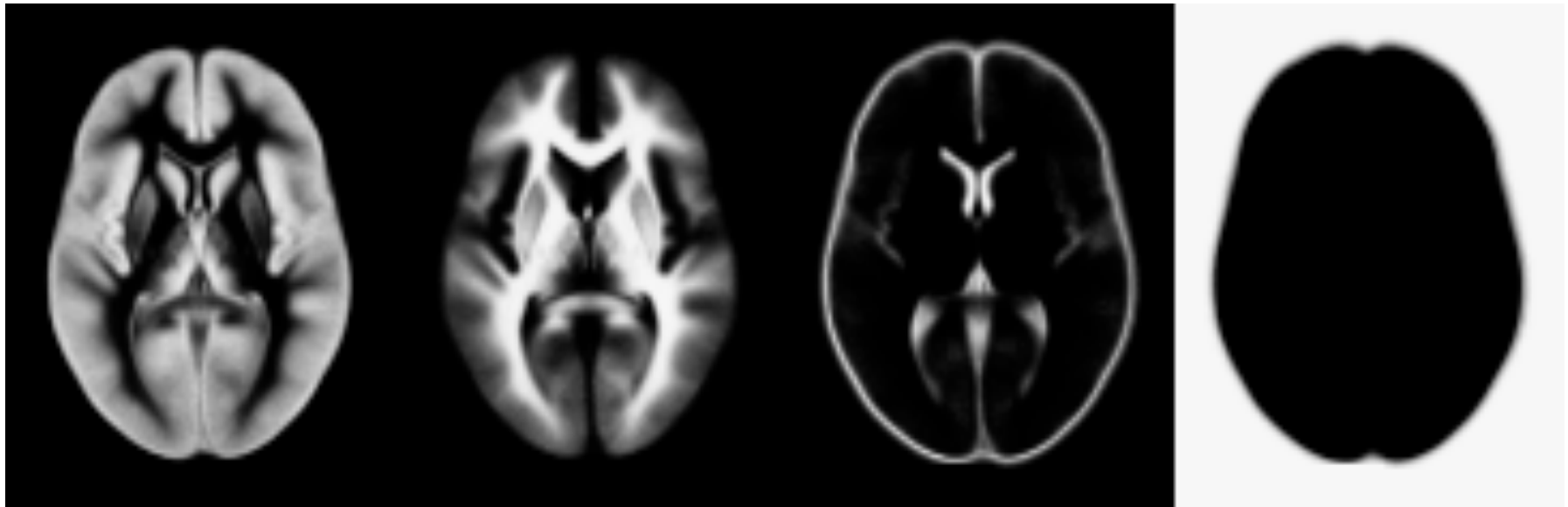
Convergence of EM algorithm



Bayesian framework

- Once we obtained all parameters of the Gaussian mixture model, we can compute the posterior tissue probability map

Prior tissue probability maps



ICBM Tissue Probabilistic Atlases

Source: C. Phillips

Bayes' theorem

- Posterior probability can be obtained from a prior probability

$$P(class | intensity) = \frac{P(intensity | class)P(class)}{\sum_{class} P(intensity | class)P(class)}$$

Likelihood

Prior map

: the probability obtaining image intensity given class.
This can be obtained from our Gaussian mixture model

Snake (active contour)

Active Contour (snake): Kass et al. in 1987

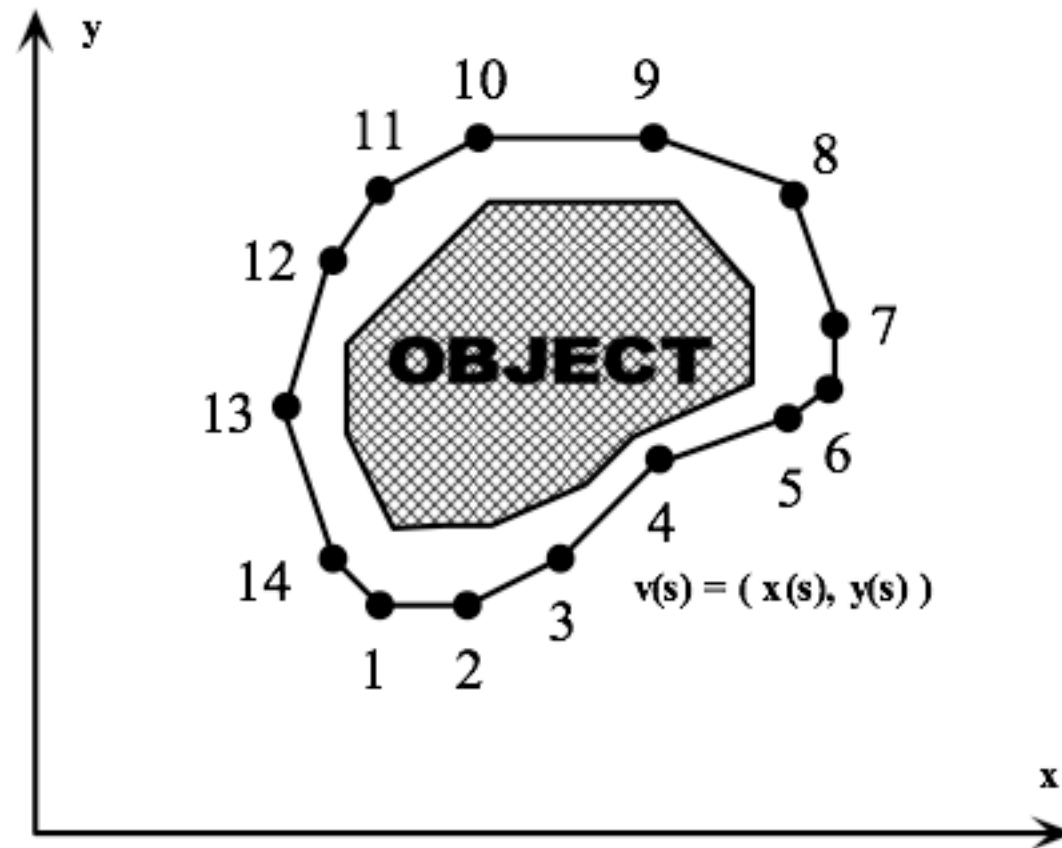
An ordered control points $p_1, p_2, \dots, p_n$ that will capture the boundary of an anatomical object

We move these control points such that they minimizes a certain energy

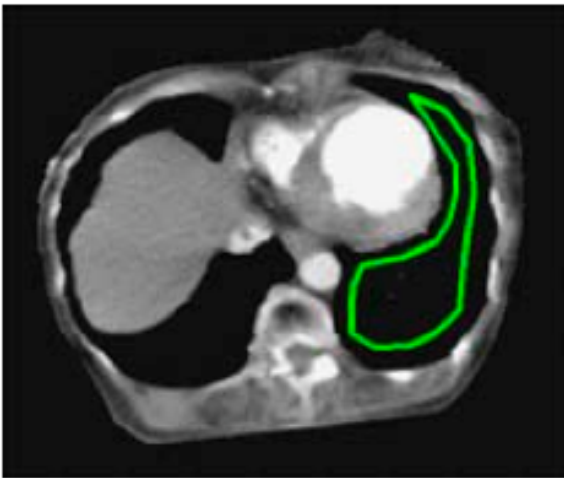
Snake movement



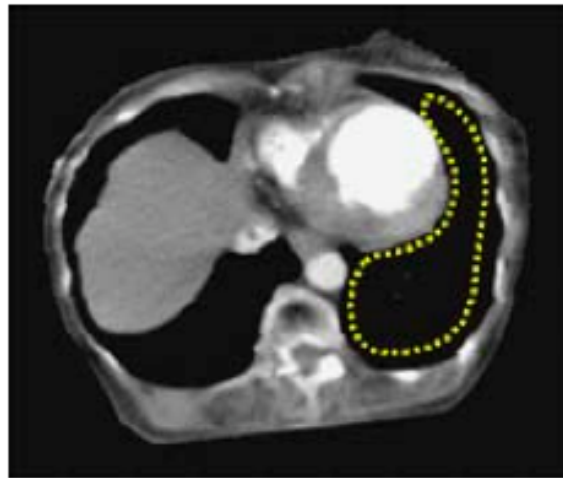
Basic form of active contour



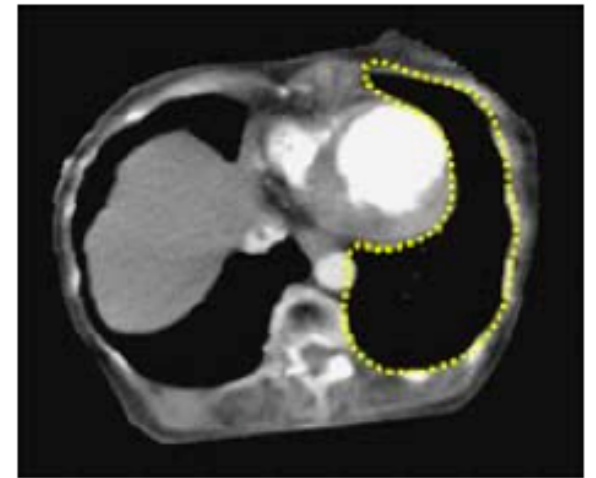
Active contour in action



Initialization

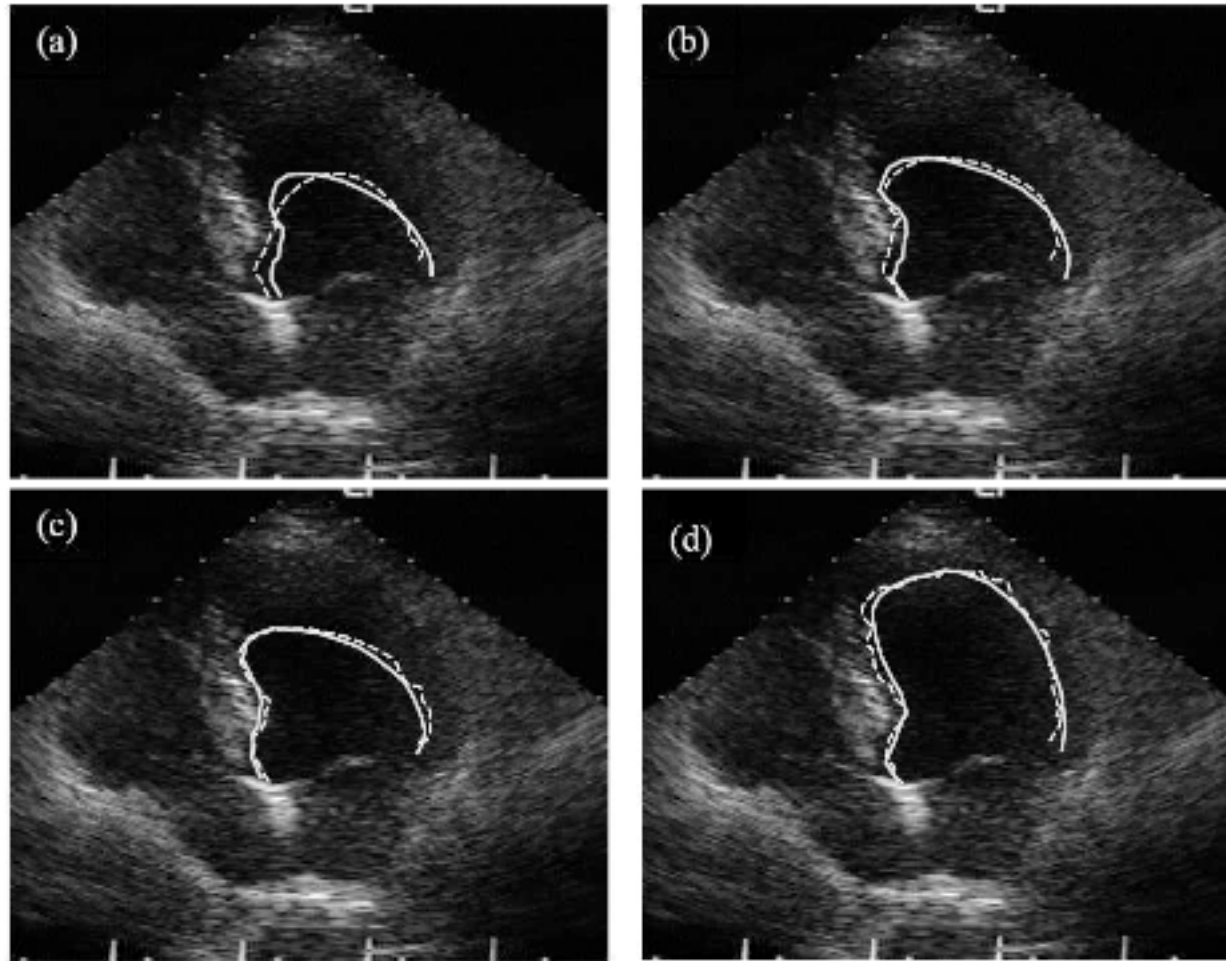


5th iteration



15th iteration

Ultrasound image example



Source: hamarneh and Gustavsson

Minimization of energy

$$E(C) = \underbrace{\alpha \int |X'(p)|^2}_{\text{Internal Energy}} + \underbrace{\beta \int |X''(p)| - \lambda \int |\nabla I(X(p))|}_{\text{External Energy}}$$

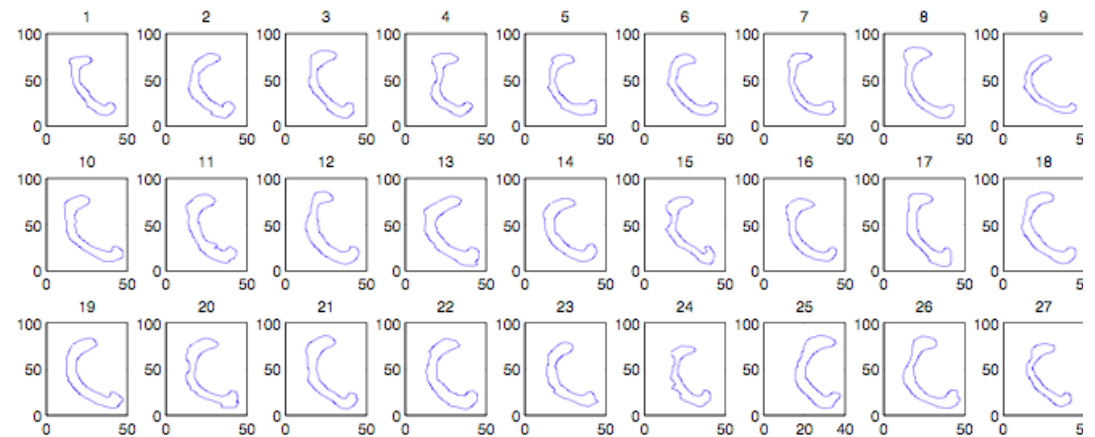
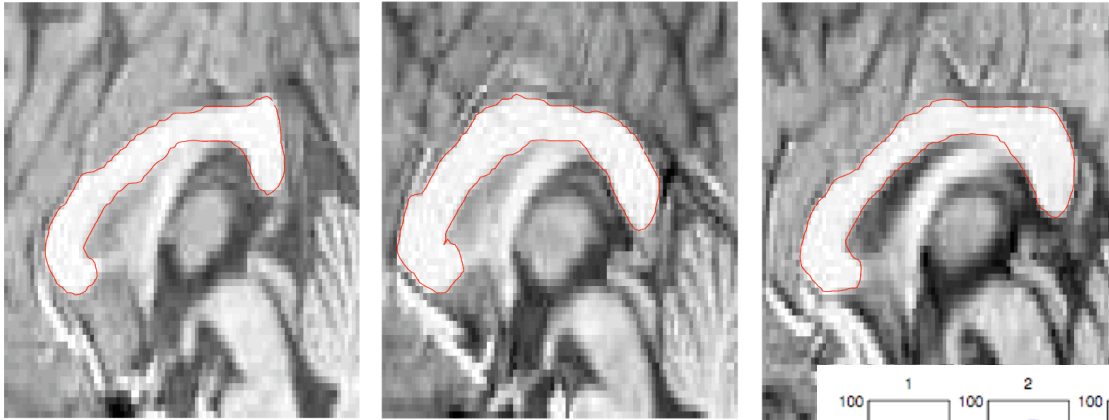
Euler-Lagrange equation:

$$-\frac{d}{dp}(\alpha X') + \frac{d^2}{dp^2}(\beta X'') - \lambda \Delta I(X) = 0.$$

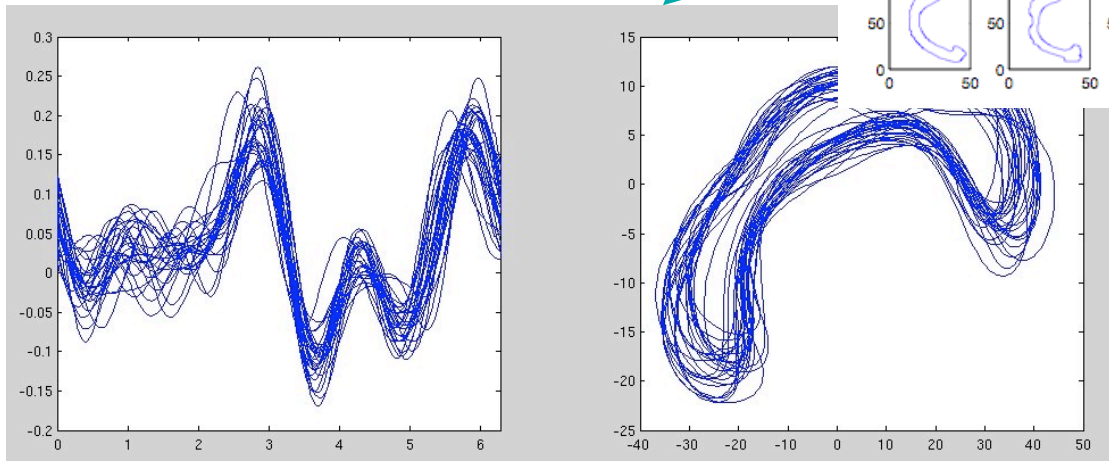
The equation is solved numerically
by the finite difference scheme.

Application: corpus callosum shape classification

Shubing Wang



Parameterization
normalization



Classification
Data analysis

Lecture 14 Topics

**Time Series Analysis
and fMRI data**