

Stat 311-Lecture 05, Random Variables

Sept 20, 2005

1. Given a sample space S , a *random variable* X is a function that maps an element in S to a real number, i.e. $X : S \rightarrow \mathbb{R}$. A random variable is *discrete* if the number of elements in a sample space is *countable*. The probability (mass) function p of X is

$$p(a) = P(X = a).$$

Note $\sum_{a \in S} p(a) = \sum_{a \in S} P(X = a) = 1$.

2. *Bernoulli random variable* X is a Bernoulli random variable with parameter p if $P(X = 1) = p$ and $P(X = 0) = 1 - p$. It will be denoted by $X \sim \text{Bernoulli}(p)$. The distribution can be written as $p(x) = (1 - p)^{1-x}p^x$.

3. *Problem.* Toss a coin until the first head is obtained. Let X be the number of tosses. Find the probability distribution of X .

Solution. $P(X = x) = (1 - p)^{x-1}p$. This is called the geometric distribution.

4. Toss a coin n times and let X be the number of heads. If X_i is the number of heads in the i -th toss, $X = X_1 + X_2 + \dots + X_n$. This is called the *Binomial random variable* with parameter n and p . It will be denoted by $X \sim \text{Binomial}(n, p)$. The distribution is given by

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

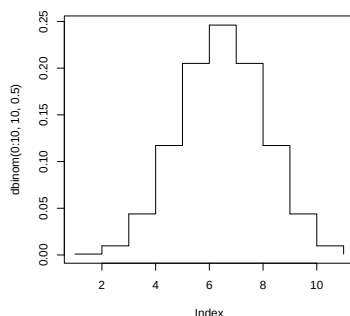


Figure 1: Probability mass function of $\text{Binomial}(10, 0.5)$

5. Two factory workers produced 5 items each. When there are 4 defective products found, what is the probability that the first worker is responsible for the half of the defection? Assume the probability of producing a defective item is p .

Solution. Let X_i be the number of defective products for the i -th worker. Then $X_1, X_2 \sim \text{Binomial}(5, p)$. Note that $X_1 + X_2 \sim \text{Binomial}(10, p)$. Hence

$$\begin{aligned} & P(X_1 = 2 | X_1 + X_2 = 4) \\ &= \frac{P(X_1 = 2, X_2 = 2)}{P(X_1 + X_2 = 4)} \\ &= \frac{P(X_1 = 2)P(X_2 = 2)}{P(X_1 + X_2 = 4)} \\ &= \frac{\binom{5}{2}p^2(1-p)^3 \binom{5}{2}p^2(1-p)^3}{\binom{10}{4}p^4(1-p)^6} = \frac{\binom{5}{2}\binom{5}{2}}{\binom{10}{4}} = 0.48. \end{aligned}$$

Homework 2. Due October 6 (Thursday) 8:00am. Solve the following 8 problems in Chapter 3: Problems 6, 20, 32, 53, 67, 82, 84. Theoretical Exercises 2.