

# Stat 311-Lecture 07 Variance

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1. The variance of random variable is defined as  $\mathbb{V}X = \mathbb{E}(X - \mathbb{E}X)^2$ .
2. Theorem.  $\mathbb{V}X = \mathbb{E}X^2 - (\mathbb{E}X)^2$ .  
 $\mathbb{V}(aX + b) = a^2\mathbb{V}X$ .
3. For  $X \sim \text{Binomial}(n, p)$ ,  $\mathbb{E}X = np$ .

$$\mathbb{E}X(X - 1) = n(n - 1)p^2, \mathbb{V}X = np(1 - p).$$

4.  $X$  is a Poisson random variable with parameter  $\lambda$  if  $p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$ ,  $x = 0, 1, 2, \dots$ . Note that

$$e^\lambda = 1 + \lambda + \frac{\lambda^2}{2!} + \dots$$

$$\mathbb{E}X = \mathbb{V}X = \lambda.$$

5. Problem. Suppose the probability of containing at least one typographical error in a page of a book is 0.005. Assuming typographical errors occur independently from page to page, what is the probability that a 400 page novel will contain exactly one page with errors?

*Solution.* If  $X$  is the number of pages containing at least one error. It is distributed as  $\text{Binomial}(400, 0.005)$ . Then  $P(X = 1) = \binom{400}{1} 0.005 \cdot 0.995^{399} = 0.2707$ . Your calculator may have hard time computing this probability. So we approximate this with  $\text{Poisson}(2)$ .  $P(X = 1) \approx e^{-2} \cdot 2 = 0.2707$ .

6. Iterative procedure for computing Poisson probability function.

$$p(x + 1) = \frac{\lambda}{x + 1} p(x).$$

Continuing the previous problem, what is the probability that a 400 page novel will contain at most 3 pages with errors? *Solution.* We need to compute  $P(X \leq 3) = p(0) + p(1) + p(2) + p(3)$ . We compute  $p(0)$  first.  $p(0) = e^{-2} = 0.1353$ .  $p(1) = 2p(0) = 0.2707$ ,  $p(2) = p(1) = 0.2707$ ,  $p(3) = \frac{2}{3}p(2) = 0.180$ .

7. Problem. The number of cars arriving at a vehicle equipment inspection station follows Poisson with

parameter 10. 50% of arriving cars will have broken equipments. What is the probability exactly 10 cars will arrive during the hour and all have no broken equipment?

*Solution.* Let  $X$  be the number of arriving cars and  $Y$  be the number of cars with broken equipments.  $P(X = 10, Y = 0) = P(Y = 0 | X = 10)P(X = 10) = 0.5^{10} e^{-10} 10^{10} / 10!$ .

What is the probability that  $c \geq 10$  cars arrive, of which 10 have no broken equipment?

*Solution.*

$$\begin{aligned} P(X = c, Y = 10) &= P(Y = 10 | X = c)P(X = c) \\ &= \binom{c}{10} 0.5^c e^{-10c} / c! \\ &= e^{-10} \frac{5^c}{10!(c - 10)!} \end{aligned}$$

What is the probability that 10 cars with no broken equipment arrives during the next hour? (This means more than 10 cars will arrive but at least 10 of them will have no broken equipment.)

$$\sum_{c=10}^{\infty} e^{-10} \frac{5^c}{10!(c - 10)!} = e^{-5} 5^{10} / 10!$$

Probably mass function for  $\text{Poisson}(5)$ .