

Stat 311-Lecture 10 Normal Distribution

Oct 20, 2005

1. The *expected value* of a continuous random variable X with pdf $f(x)$ is

$$\mathbb{E}X = \int_{-\infty}^{\infty} xf(x) dx.$$

$$\mathbb{E}h(X) = \int_{-\infty}^{\infty} h(x)f(x) dx.$$

2. The *cumulative distribution function* F of a continuous random variable X is defined as

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(x) dx.$$

Given F , we can recover the density function by $f(x) = \frac{dF(x)}{dx}$ by the fundamental theorem of calculus.

3. *Example.* $f(x) = 2x$ for $0 < x < 1$ and $f(x) = 0$ otherwise. The cdf is given by $F(x) = x^2$ for $0 < x < 1$. $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 1$. By differentiating F , we recover the density function.

4. *Problem.* For $X \sim \text{Unif}(0, 1)$, what is the probability density of $Y = 2X$? *Answer.* $Y \sim \text{Unif}(0, 2)$.

5. A continuous random variable Z is called a standard normal if

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}.$$

We denote $Z \sim N(0, 1)$. $\mathbb{E}Z = 0$, $\mathbb{V}Z = 1$. The cdf of the normal standard normal is

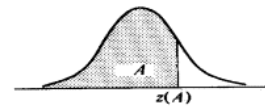
$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2}} e^{-x^2/2} dx.$$

Let $X = \sigma Z + \mu$. Then X is a normal distribution with parameters μ and σ^2 . Its density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}.$$

We denote this distribution $X \sim N(\mu, \sigma^2)$.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Figure 1: Table for the cdf of standard normal $N(0, 1)$

6. *Problem.* The length of time required to complete a college test is found to be normally distributed with mean 50 minutes and standard deviation 12 minutes. (a) When should the test be terminated if we wish to allow sufficient time for 90% of the students to complete the test? (b) What proportion of students will finish the test between 30 and 60 minutes? *Solution.* Let X be the length of time to complete the test. Then $Z = \frac{X-50}{12} \sim N(0, 1)$, the standard normal distribution. Need to find c such that $P(X < c) = P(Z < \frac{c-50}{12}) = 0.9$. $\frac{c-50}{12} = 1.28$. So at least $c = 65.36$ minutes should be given. $P(30 < X < 60) = P(-1.67 < Z < 0.83) = 0.75$.