

Stat 311-Lecture 13 Sum of random variables

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1. For random variables X and Y and their joint distribution $f(x, y)$, the probability of

$$P(X \leq Y) = \int \int_{x \leq y} f(x, y) \, dx dy.$$

If X and Y are independent $Unif(0, 1)$, the joint distribution is a uniform random variable in the unit square.

2. Given the distributions of X and Y , we are interested in investigating the distribution of $X + Y$ for various applications. Assuming the independence of X and Y , the cdf of $X + Y$ is

$$F_{X+Y}(z) = \int_{-\infty}^{\infty} f_Y(y) F_X(z - y) \, dy.$$

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3. We already know that if independent random variables $X \sim \text{Binomial}(n, p)$ and $Y \sim \text{Binomial}(m, p)$,

$$X + Y \sim \text{Binomial}(n + m, p).$$

4. Problem. If independent random variables $X \sim Unif(0, 1)$ and $Y \sim Unif(0, 1)$, find the density of $X + Y$.
5. Problem. The lifetime (in hours) of an electronic component is a random variable with density function given by

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0.$$

Two of these components operate independently in a piece of equipment. Let X_i be the life times of the i -th component. Find the probability density of $X_1 + X_2$. Ans. $f_{X+Y}(z) = \lambda^2 e^{-\lambda z}, z \geq 0$. This is *gamma*(2, λ).

6. For $X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2)$,

$$\alpha X + \beta Y \sim N(\alpha \mu_X + \beta \mu_Y, \alpha^2 \sigma_X^2 + \beta^2 \sigma_Y^2).$$

7. For $X \sim \text{Poisson}(\lambda_X), Y \sim \text{Poisson}(\lambda_Y), X + Y \sim \text{Poisson}(\lambda_X + \lambda_Y)$.