

STAT 311 Old final exam problems

Dec 13, 2005

1. Suppose that $X_1, \dots, X_n$ are a random sample of size n from Bernoulli distribution with $P(X_i = 1) = p, 0 < p < 1$. (a) Compute the correlation coefficient of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + X_3$. Are Y_1 and Y_2 independent? (b) Determine the probability distribution of $X_1^2 + \dots + X_n^2$.

2. A person is flipping a fair coin repeatedly until he obtains two heads. Let X be the number of tosses until he obtains two heads. (a) Determine the probability mass function of X . (b) Compute $\mathbb{E}X$

Solution. Note that $X \geq 2$. If p is the probability of obtaining head, $p(2) = p^2, p(3) = \binom{2}{1}p^2(1-p), p(4) = \binom{3}{1}p^2(1-p)^2, \dots$. In general we have $p(x) = \binom{x-1}{1}p^2(1-p)^{x-2}$.

$$\mathbb{E}X = \sum_{x=2}^{\infty} x(x-1)p^2(1-p)^{x-2} = p^2 \sum_{x=2}^{\infty} \frac{d^2}{dq^2} q^x,$$

where $q = 1 - p$.

3. X and Y are uniformly distributed over $0 < x < 1, x < y < x + 1$.
(a) Compute the marginal densities of X and Y .
(b) Are X and Y independent? Justify your answer.
(c) Compute $P(\frac{1}{2} < Y < \frac{3}{2})$.
4. The density of X is given by $f(x) = |x|$ for $-1 < x < 1$ and $f(x) = 0$ otherwise.
(a) Compute the cumulative distribution of X and plot its graph. (b) Determine the density function of $U = X^2$
5. Let X_1 and X_2 be independent Poisson random variables with means λ_1 and λ_2 respectively. Assume further that $\lambda_1 + \lambda_2 = 1$. Hint. The probability mass function of a Poisson random variable with parameter λ is given by $p(x) = e^{-\lambda} \lambda^x / x!, x = 0, 1, 2, \dots$
(a) Derive the moment generating function of X_1 .
(b) Derive the probability mass function of $X_1 + X_2$.
(c) Compute $\mathbb{E}(X_1 | X_1 + X_2 = z)$.