

Stat312: Midterm Makeup Exam I.

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Answer all questions clearly and **circle your final answer**.

Your answers should be correct up to the **second decimal places**.

One page note and a calculator are allowed.

No textbooks, scrap papers or hand-held computers, PDA are allowed.

This exam booklet consists of **3 problems** and **6 pages**.

Name: _____

Student ID: _____

Pledge: On my honor, I have neither given nor received unauthorized aids on this examination.

Signature: _____

Question	Points
1	
2	
3	
Total	

1. Suppose a random sample $X_1, \dots, X_n$ are coming from a normal distribution with mean μ and variance σ^2 . We are interested in estimating μ and σ^2 .
 - (1) Write the likelihood function (5pts).
 - (2) By maximizing the likelihood function, find the maximum likelihood estimators for μ and σ^2 . Derive everything (5pts).

- (3) Determine if the estimator for σ^2 in (2) is unbiased (5pts, no point given if (2) is not solved).
- (4) What is the moment estimator of σ^2 ? Derive everything (5pts).

2. Suppose we toss 10 identical coins and observed 4 heads. Let p be the probability of getting head when a single coin is tossed.

- (1) Write the likelihood function. Properly define all variables you are using (5pts).
- (2) By maximizing the likelihood function, find the maximum likelihood estimator for p . Derive everything (5pts).

(3) What is the maximum likelihood estimator for the probability of getting tail?

3. The following 10 sample observations on breakdown voltage of a particular circuit under certain condition is given.

62, 50, 53, 57, 41, 53, 55, 61, 59, 64.

- (1) What is the sample mean and the sample variance of the breakdown voltage? (5pts).
- (2) Construct 98% confidence interval for the mean breakdown voltage. What is the statistical assumptions you are making to construct the confidence interval? You may use the following R output (5pts).

```
> qnorm(1:10/100)
> [1] -2.326348 -2.053749 -1.880794 -1.750686 -1.644854
> [6] -1.554774 -1.475791 -1.405072 -1.340755 -1.281552
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