

# Stat312: Midterm Exam I.

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Answer all questions clearly and **circle your final answer**.

Your answers should be correct up to the **second decimal places**.

One page note and a calculator are allowed.

No textbooks, scrap papers or hand-held computers, PDA are allowed.

This exam booklet consists of **3 problems** and **6 pages**.

**Name:** \_\_\_\_\_

**Student ID:** \_\_\_\_\_

Pledge: On my honor, I have neither given nor received unauthorized  
aids on this examination.

**Signature:** \_\_\_\_\_

| Question | Points |
|----------|--------|
| 1        |        |
| 2        |        |
| 3        |        |
| Total    |        |

1. Suppose a random sample  $X_1, \dots, X_n$  are coming from a normal distribution with mean 0 and variance  $\sigma^2$ . We are interested in estimating  $\sigma^2$ .
  - (1) Write the likelihood function (5pts).
  - (2) By maximizing the likelihood function, find the maximum likelihood estimator for  $\sigma^2$ . Derive everything (5pts).

- (3) Determine if the estimator in (2) is unbiased (5pts, no point given if (2) is not solved).
- (4) By matching moments, estimate  $\sigma^2$  (5pts).

2. Consider a random sample  $X_1, X_2, X_3$  coming from distribution

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0.$$

- (1) Write the loglikelihood function (5pts).
- (2) By maximizing the loglikelihood function, find the maximum likelihood estimator for  $\theta$ . Derive everything (5pts).

(3) What is the maximum likelihood estimator of  $P(X_1 > 1)$ .

3. 49 students take a midterm exam in STAT 312. If  $x_i$  is the grade for the  $i$ -th student, we have  $\sum_{i=1}^{49} x_i = 3675$  and  $\sum_{i=1}^{49} x_i^2 = 275,625$ .

- (1) What is the sample variance? (5pts).
- (2) Construct 6% confidence interval for the average grade. You may use the following R output (5pts).

```
> qnorm(1:10/100)
> [1] -2.326348 -2.053749 -1.880794 -1.750686 -1.644854
> [6] -1.554774 -1.475791 -1.405072 -1.340755 -1.281552
```