

Stat312: Sample Midterm II.

Moo K. Chung
mchung@stat.wisc.edu

March 25, 2003

1. Consider the sample of fat content of 10 randomly selected hot dogs: 25, 21, 22, 17, 29, 25, 16, 20, 19, 22. Suppose that these are from a normal population. We want to test if the fat content of hot dogs is 23.

- (a) State the null and alternate hypotheses (5 points).
(b) What is your test statistic and a rejection rule at 95% level? (5 points)
(c) Is the fat content of hot dogs 23? Explain your result (5 points).
(d) Compute the probability of type I error when you do the hypothesis test using the test statistic in (b). Explain your result (5 points).
(e) Define the P -value and compute the P -value for using the test statistic in (b) (5 points).

```
>X<-c(25,21,22,17,29,25,16,20,19,22)
>sum(X)
[1] 216
> var(X)
[1] 15.6
> qnorm(c(0.025,0.05))
[1] -1.96 -1.64
> qt(0.025,c(9,10))
[1] -2.26 -2.23
> pt(c(1.0,1.1,1.2,1.3,1.4,1.5),9)
[1] 0.83 0.85 0.87 0.89 0.90 0.92
```

Solution. (a) Let μ be the population mean of the fat content in hot dogs. Then $H_0 : \mu = 23$ and $H_1 : \mu \neq 23$. (b) Since the population variance is unknown, the test statistic should be the t statistic given by $T = (\bar{X} - \mu)/(S/\sqrt{10})$. Note that $t_{0.025,9} = 2.26$. So we reject H_0 if $|T| > 2.26$. (c) $s^2 = 15.6$. The t -value is $t = (\bar{x} - 23)/(s/\sqrt{10}) = -1.12$. Since the value is not in the rejection region, we do not reject H_0 . So we may assume that

the fat content of hot dogs is 23. (d) By definition $\alpha = P(\text{reject } H_0 | H_0 \text{ is true}) = P(|T| > t_{0.025,9}) = 0.05$. You should answer this question without computing anything. (e) The P -value is the smallest level of significance at which H_0 would be rejected. For given t -value $t = 1.12$, we reject H_0 if $|t| > t_{\alpha/2,9}$ at α level. So $P\text{-value} = P(|T| > 1.12) = 2P(T > 1.12) = 2(1 - P(T \leq 1.12)) = 2(1 - 0.85) = 0.3$

2. 4 out of 10 doctors knew the generic name for the drug methadone. We want to test if fewer than half of all doctors know the generic name for methadone.

- (a) State the null and alternate hypotheses. Define population parameters clearly (5 points).
(b) What is your test statistic? Explain your result (10 points).
(c) What is the distribution of the test statistic in (b) under the null hypothesis? Explain your result (5 points).
(d) Perform the hypothesis test using a significance level of 0.2 (10 points).

```
> pbinom(0:10,10,0.5)
[1] 0.00 0.01 0.055 0.17 0.38
[6] 0.62 0.83 0.95 0.99 1.00
[11] 1.00
```

Solution (a) Let p be the proportion of doctors who knew the generic name. Then $H_0 : p = 0.5$ vs. $H_1 : p < 0.5$. (b) Since the sample size is too small, you can not use Z statistic. Let X_i be a Bernoulli random variable defined as $X_i = 1$ if the i -th doctor know and $X_i = 0$ if the i -th doctor does not know the generic name. Note that $P(X_i = 1) = p, P(X_i = 0) = 1 - p$. The point estimator for p would be $\hat{p} = \sum_{i=1}^{10} X_i / 10 = \bar{X}$. We will take $\sum_{i=1}^{10} X_i$ rather than $\bar{X}$ as the test

statistic because it will be much easier to figure out the distribution of the test statistic. (c) If you answered $\bar{X}$ in (b), you may have some difficulty answering this question. Note that $\sum_{i=1}^{10} X_i \sim \text{Binomial}(10, p)$. We went through binomial distributions a couple of times during the lectures. Also review STAT311. Under H_0 , $\sum_{i=1}^{10} X_i \sim \text{Binomial}(10, 0.5)$. (d) We reject H_0 if $\sum_{i=1}^{10} x_i$ is small. Note that $P(\sum_{i=1}^{10} X_i \leq 3) = 0.17$ while $P(\sum_{i=1}^{10} X_i \leq 4) = 0.38$. So we reject H_0 at level 0.2 if $\sum_{i=1}^{10} x_i \leq 3$. Since we have $\sum_{i=1}^{10} x_i = 4$, we do not reject H_0 .

3. There are two coins. You threw the first coin 100 times and observed 40 heads. When you threw the second coin 110 times, you observed 50 heads. You want to test if the two coins give the same number of heads.
 - (a) State the null and alternate hypotheses (5 points)?
 - (b) What is your test statistic and its distribution? (5 points)
 - (c) Do they give the same number of heads? Test it at level 0.3 (10 points).
 - (d) What is a 68% confidence interval for the difference between the probabilities of getting heads (10 points)?

```
> pnorm(1:10/10)
[1] 0.54 0.58 0.62 0.66 0.69
[6] 0.73 0.76 0.79 0.82 0.84
```

Solution. (a) Let p_1 and p_2 be the probabilities of getting heads for the first and the second coins respectively. Then $H_0 : P_1 = p_2$ vs. $H_1 : p_1 \neq p_2$. (b) Let X be the number of heads when you throw the first coin $n = 100$ times and Y be the number of heads when you throw the second coin $m = 110$ times. The point estimator for $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2 = X/n - Y/m$. Then the test statistic is

$$Z = \frac{X/n - Y/m}{\sqrt{p(1-p)(1/n + 1/m)}} \sim N(0, 1),$$

where $p = p_1 = p_2$. Since p is unknown, you estimate it by pooling the samples: $\hat{p} = (x + y)/(m + n)$. (c) $\hat{p} = (40 + 50)/(100 + 110) = 0.43$ and $z = -0.60$. Since we reject H_0 if $|z|$ is large, the P -value would be $P(|Z| > 0.6) = 2P(Z > 0.6) = 2(1 - P(Z \leq 0.6)) = 2(1 - 0.73) = 0.54$.

Since the P -value is larger than 0.3, we do not reject H_0 at level 0.3. (d) Although `qnorm` values are not given, we can estimate $z_{0.32/2}$ from the R output. Note that $P(Z < z_{0.32/2}) = 1 - P(Z > z_{0.32/2}) = 1 - 0.32/2 = 0.84$. So $z_{0.32/2} = 1$. A $100(1 - \alpha)\%$ confidence interval is

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \sqrt{\hat{p}_1(1 - \hat{p}_1)/100 + \hat{p}_2(1 - \hat{p}_2)/110}.$$

With $\hat{p}_1 = 40/100$ and $\hat{p}_2 = 50/110$, we get $[-0.12, 0.01]$.

4. Bonus doggy problem: name the title of a movie where there is a dog carrying a human body part in its mouth(1 point). *Solution.* Akira Kurosawa's *Yojimbo*. He is a Japanese movie director. Many of his Samurai movies had influenced Hollywood western movies. Self study problem: Compare Kurosawa's *Yojimbo* (1961) and Clint Eastwood's spaghetti western *A Fistful of Dollars* (1964). In *Yojimbo*, there is a somewhat morbid but funny scene about a dog carrying a human hand in its mouth at the beginning of the movie.