

Stat312: Final Exam

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December 17, 2004

Answer all questions clearly and **circle your final answer**.
Your answers should be correct up to the **second decimal places**.
One page note and a calculator are allowed.
No textbooks, scrap papers or hand-held computers are allowed.
All variables that are introduced should be defined.
This exam booklet consists of **five problems** and **nine pages**.

Name: _____
Student ID: _____

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: _____

Question	Points
1	
2	
3	
4	
5	
Total	

You may use the following R output in answering the appropriate exam problems.

```
> qnorm(c(0.99,0.975,0.95,0.925,0.9))  
[1] 2.33, 1.96, 1.64, 1.44, 1.28
```

```
> qt(c(0.99,0.975,0.95,0.925,0.9),10)  
[1] 2.76 2.23 1.81 1.56 1.37
```

```
> qt(c(0.99,0.975,0.95,0.925,0.9),9)  
[1] 2.82 2.26 1.83 1.57 1.38
```

```
> qt(c(0.99,0.975,0.95,0.925,0.9),8)  
[1] 2.90 2.31 1.86 1.59 1.40
```

1. We wish to fit data $(x_1, y_1), \dots, (x_n, y_n)$ with linear model $Y_j = 1 + \beta x_j + \epsilon_j$ with $\epsilon_j \sim N(0, \sigma^2)$. Derive everything.
 - (a) Find the maximum likelihood estimator for β (10pts).

- (b) Estimate β by the least squares method (10pts).
- (c) Determine the distribution of the estimator given in (b) (10pts).

2. Let $X_1, \dots, X_m \sim N(\mu_1, \sigma^2)$ and $Y_1, \dots, Y_m \sim N(\mu_2, \sigma^2)$. Further two samples are independent. Estimate μ_1, μ_2 and σ^2 via the maximum likelihood method (15pts).

3. A manufacturer of automatic washers offers a particular model in one of two colors: white and green. Of the 10 washers sold, 7 were white and 3 were green. Would you conclude that customers have a preference for white color? Test it at $\alpha = 0.05$. Also compute the p -value corresponding to the test procedure (15 pts).

4. Suppose that 20 out of 50 females and 10 out of 40 males are observed to be depressed in a psychological test. Does this data indicate the proportion of depression is different for the two groups? Test it at $\alpha = 0.05$. State your hypothesis and a test statistic and its distribution under the null hypothesis (10pts).

5. 10 students took two midterm exams and the scores are listed here.

Midterm 1|| 80 75 60 90 99 60 55 85 65 70

Midterm 2|| 70 60 70 72 95 66 60 80 70 60

You may use the following R-output for answering the questions.

```
> x<-c(80, 75, 60, 90, 99, 60, 55, 85, 65, 70)
> y<-c(70, 60, 70, 72, 95, 66, 60, 80, 70, 60)
> summary(lm(y~x))
```

```
Call: lm(formula = y ~ x)
```

Residuals:

Min	1Q	Median	3Q	Max
-10.908	-6.312	1.758	4.354	10.836

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	29.4827	13.2261	2.229	0.0564 .
x	0.5523	0.1760	3.139	0.0138 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.647 on 8 degrees of freedom Multiple
R-Squared: 0.5519, Adjusted R-squared: 0.4959 F-statistic: 9.854
on 1 and 8 DF, p-value: 0.01383

- (a) Test if the two midterm scores are statistically independent at $\alpha = 0.05$. What statistical assumptions are you making to perform the test? (5pts)

- (b) Test if the second midterm exam is easier than the first midterm exam at $\alpha = 0.05$ What statistical assumptions are you making to perform the test? (10pts)