

2.

- a With $X = \#$ of T's in the sample, the estimator is $\hat{p} = \frac{x}{n}$; $x = 10$, so

$$\hat{p} = \frac{10}{20} = .50.$$

- b Here, $X = \#$ in sample which are not reverse polish, and $x = 17$ so $\hat{p} = \frac{17}{20} = .85$

4.

- a $E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2$; $\bar{x} - \bar{y} = 8.141 - 8.575 = .434$

- b $V(\bar{X} - \bar{Y}) = V(\bar{X}) + V(\bar{Y}) = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$$\sigma_{\bar{X} - \bar{Y}} = \sqrt{V(\bar{X} - \bar{Y})} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}; \text{ The estimate would be}$$

$$s_{\bar{X} - \bar{Y}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.66^2}{27} + \frac{2.104^2}{20}} = .5687.$$

- c $\frac{s_1}{s_2} = \frac{1.660}{2.104} = .7890$

- d $V(X - Y) = V(X) + V(Y) = \sigma_1^2 + \sigma_2^2$

$$\text{The estimate} = 1.66^2 + 2.104^2 = 7.1824$$

10.

- a $E(\bar{X}^2) = \text{Var}(\bar{X}) + [E(\bar{X})]^2 = \frac{\sigma^2}{n} + \mu^2$, so the bias of the estimator $\bar{X}^2$ is $\frac{\sigma^2}{n}$; thus $\bar{X}^2$ tends to overestimate μ^2 .

- b $E(\bar{X}^2 - kS^2) = E(\bar{X}^2) - kE(S^2) = \mu^2 + \frac{\sigma^2}{n} - k\sigma^2$, so with $k = \frac{1}{n}$,
 $E(\bar{X}^2 - kS^2) = \mu^2$.

16.

- a $E[\delta\bar{X} + (1-\delta)\bar{Y}] = \delta E(\bar{X}) + (1-\delta)E(\bar{Y}) = \delta\mu + (1-\delta)\mu = \mu$

- b $\text{Var}[\delta\bar{X} + (1-\delta)\bar{Y}] = \delta^2 \text{Var}(\bar{X}) + (1-\delta)^2 \text{Var}(\bar{Y}) = \frac{\delta^2 \sigma^2}{m} + \frac{4(1-\delta)^2 \sigma^2}{n}$

Setting the derivative with respect to δ equal to 0 yields

$$\frac{2\delta\sigma^2}{m} - \frac{8(1-\delta)\sigma^2}{n} = 0, \text{ from which } \delta = \frac{4m}{4m+n}.$$

20.

- a We wish to take the derivative of $\ln\left[\binom{n}{x} p^x (1-p)^{n-x}\right]$, set it equal to zero and

solve for p. $\frac{d}{dp}\left[\ln\left(\binom{n}{x}\right) + x\ln(p) + (n-x)\ln(1-p)\right] = \frac{x}{p} - \frac{n-x}{1-p}$; setting this

equal to zero and solving for p yields $\hat{p} = \frac{x}{n}$. For $n = 20$ and $x = 3$,

$$\hat{p} = \frac{3}{20} = .15$$

- b $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n}(np) = p$; thus $\hat{p}$ is an unbiased estimator of p.

- c $(1-.15)^5 = .4437$

22.

- a $E(X) = \int_0^1 x(\theta+1)x^\theta dx = \frac{\theta+1}{\theta+2} = 1 - \frac{1}{\theta+2}$, so the moment estimator $\hat{\theta}$ is

the solution to $\bar{X} = 1 - \frac{1}{\hat{\theta}+2}$, yielding $\hat{\theta} = \frac{1}{1-\bar{X}} - 2$. Since

$$\bar{x} = .80, \hat{\theta} = 5 - 2 = 3.$$

- b $f(x_1, \dots, x_n; \theta) = (\theta+1)^n (x_1 x_2 \dots x_n)^\theta$, so the log likelihood is

$n \ln(\theta+1) + \theta \sum \ln(x_i)$. Taking $\frac{d}{d\theta}$ and equating to 0 yields

$$\frac{n}{\theta+1} = -\sum \ln(x_i), \text{ so } \hat{\theta} = -\frac{n}{\sum \ln(X_i)} - 1. \text{ Taking } \ln(x_i) \text{ for each given } x_i$$

yields ultimately $\hat{\theta} = 3.12$.