

30. The likelihood is $f(y; n, p) = \binom{n}{y} p^y (1-p)^{n-y}$ where
 $p = P(X \geq 24) = 1 - \int_0^{24} \lambda e^{-\lambda x} dx = e^{-24\lambda}$. We know $\hat{p} = \frac{y}{n}$, so by the invariance principle $e^{-24\lambda} = \frac{y}{n} \Rightarrow \hat{\lambda} = -\frac{\ln(\frac{y}{n})}{24} = .0120$ for $n = 20, y = 15$.

4.

- a $58.3 \pm \frac{1.96(3)}{\sqrt{25}} = 58.3 \pm 1.18 = (57.1, 59.5)$
 b $58.3 \pm \frac{1.96(3)}{\sqrt{100}} = 58.3 \pm .59 = (57.7, 58.9)$
 c $58.3 \pm \frac{2.58(3)}{\sqrt{100}} = 58.3 \pm .77 = (57.5, 59.1)$
 d 82% confidence $\Rightarrow 1 - \alpha = .82 \Rightarrow \alpha = .18 \Rightarrow \alpha/2 = .09$, so $z_{\alpha/2} = z_{.09} = 1.34$
 and the interval is $58.3 \pm \frac{1.34(3)}{\sqrt{100}} = (57.9, 58.7)$.
 e $n = \left[\frac{2(2.58)^2}{1} \right] = 239.62$ so $n = 240$.

10.

- a When $n = 15$, $2\lambda \sum X_i$ has a chi-squared distribution with 30 d.f. From the 30 d.f. row of Table A.6, the critical values that capture lower and upper tail areas of .025 (and thus a central area of .95) are 16.791 and 46.979. An argument parallel to that given in Example 7.5 gives $\left(\frac{2\sum x_i}{46.979}, \frac{2\sum x_i}{16.791} \right)$ as a 95% C.I. for $\mu = \frac{1}{\lambda}$. Since $\sum x_i = 63.2$ the interval is (2.69, 7.53).
 b A 99% confidence level requires using critical values that capture area .005 in each tail of the chi-squared curve with 30 d.f.; these are 13.787 and 53.672, which replace 16.791 and 46.979 in a.
 c $V(X) = \frac{1}{\lambda^2}$ when X has an exponential distribution, so the standard deviation is $\frac{1}{\lambda}$, the same as the mean. Thus the interval of a is also a 95% C.I. for the standard deviation of the lifetime distribution.

14.

- a $89.10 \pm 1.96 \frac{3.73}{\sqrt{169}} = 89.10 \pm .56 = (88.54, 89.66)$. Yes, this is a very narrow interval. It appears quite precise.
 b $n = \left[\frac{2(1.96)(4)}{2(0.5)} \right]^2 = 245.86 \Rightarrow n = 246$.

22.

$\hat{p} = .072$; the 99% upper confidence bound is:

$$\frac{.072 + \frac{(2.33)^2}{2(487)} + 2.33 \sqrt{\frac{(.072)(.928)}{487} + \frac{(2.33)^2}{4(487)^2}}}{1 + \frac{(2.33)^2}{487}} = \frac{.0776 + .0279}{1.0111} = .1043$$

26.

With $\theta = \lambda$, $\hat{\theta} = \bar{X}$ and $\sigma_{\hat{\theta}} = \sqrt{\frac{\lambda}{n}}$ so $\hat{\sigma}_{\hat{\theta}} = \sqrt{\frac{\bar{X}}{n}}$. The large sample C.I. is then

$\bar{x} \pm z_{\alpha/2} \sqrt{\frac{\bar{x}}{n}}$. We calculate $\sum x_i = 203$, so $\bar{x} = 4.06$, and a 95% interval for λ is

$$4.06 \pm 1.96 \sqrt{\frac{4.06}{50}} = 4.06 \pm .56 = (3.50, 4.62)$$