

Stat 312: Lecture 12

Errors in hypothesis testing

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Type I error: you reject H_0 when H_0 is true.
 $P(\text{Type I error}) = P(\text{reject } H_0 | H_0 \text{ true}) = \alpha$. The resulting α is called the *significance level* of the test and the corresponding test is called a *level α test*. We will use test procedures that give α less than a specified level (0.05 or 0.01 usually).

I believe that dogs are as smart as people. Assume IQ of a dog follows $X_i \sim N(\mu, 10^2)$. IQ of 10 dogs are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. We want to test if dogs are as smart as people by testing

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100.$$

- (1) Let $\bar{X}$ be a test statistic and $R = (-\infty, 90)$ to be a rejection region. Let's compute the probability of making Type I error based on this testing procedure. Under the assumption H_0 is true,

$$X_i \sim N(100, 10^2).$$

Under this condition, $\bar{X} \sim N(100, 10)$ and

$$\alpha = P(\bar{X} < 90).$$

```
>help(pnorm)
```

```
...
```

```
Usage:
```

```
... pnorm(q, mean=0, sd=1, ...)
```

```
...
```

```
> pnorm(90,100,sqrt(10))
```

```
[1] 0.0007827011
```

By using this test procedure, it is highly unlikely to make Type I error.

- (2) Let's see what happens when we change the test procedure. When $R = (-\infty, 95)$ with the same test statistic $\bar{X}$,

$$\alpha = P(\bar{X} < 95).$$

```
> pnorm(95,100,sqrt(10))
```

```
[1] 0.05692315
```

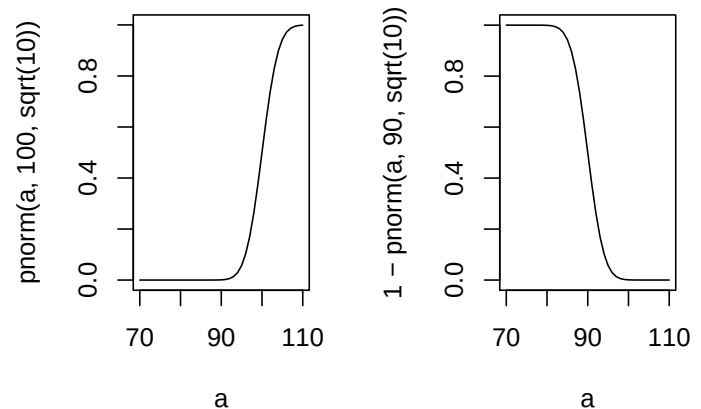


Figure 1: Errors in performing hypothesis testing based on test procedure if $\bar{X} < 90$, reject $H_0 : \mu = 100$. Left figure: type I error, right figure: type II error.

- (3) When $R = (-\infty, 99)$, $\alpha = P(\bar{X} < 99)$.

```
> pnorm(99,100,sqrt(10))
```

```
[1] 0.3759148
```

The test procedure based on rejecting H_0 if $\bar{X} < 99$ is producing large Type I error. Why?

Type II error: you do not reject H_0 when H_0 is false.

$$\begin{aligned} \beta &= P(\text{Type II error}) \\ &= P(\text{do not reject } H_0 | H_0 \text{ false}) \\ &= 1 - P(\text{reject } H_0 | H_1 \text{ true}) \end{aligned}$$

With the fixed test procedure of rejecting H_0 if $\bar{X} < 90$, let us compute β . When $\mu = 90$ is true,

```
> 1-pnorm(90,90,sqrt(10))
```

```
[1] 0.5
```

```
> 1-pnorm(90,95,sqrt(10))
```

```
[1] 0.9430769
> 1-pnorm(90,99,sqrt(10))
[1] 0.9977867
```

Now let us see how β changes when we change the rejection region. Assume $\mu = 90$ is true.

```
> a<- -20:20
> a<- a + 90
%Type II error --> right figure
> plot(a,1-pnorm(a,90,sqrt(10)),type='l')
%Type I error --> left figure
> plot(a,pnorm(a,100,sqrt(10)),type='l')
```

Review problems Exercise 8.1., 8.3., 8.5.