

Stat 312: Lecture 14

Large sample test

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1. Testing mean μ with unknown variance σ^2 .

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

Test statistic: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$. Rejection region for level α test: $z \leq -t_{\alpha, n-1}$.

Ex. IQ of a dog, $X_i \sim N(\mu, \sigma^2)$, where σ is unknown. Test

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100$$

at level $\alpha = 0.05$.

```
> x<-c(30, 25, 70, 110, 40, 80, 50, 60,
100, 60)
> t=(mean(x)-100)/(sd(x)/sqrt(10))
> t
[1] -4.205156
> qt(0.05,9)
[1] -1.833113
```

A simpler method is to use command `t.test`.

```
> help(t.test)
...
t.test(x,alternative=c("two.sided", "less",
"greater"),conf.level = 0.95)
...
> t.test(x,mu=100,alternative="less",
conf.level=0.95)
One Sample t-test
data: x t = -4.2036, df = 9,
p-value = 0.001147
alternative hypothesis: true mean is
less than 100
95 percent confidence interval:
-Inf 78.8531
sample estimates: mean of x 62.5
```

2. Testing on mean μ when the sample size is large.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

Test statistic: $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$. Rejection region for level α test: $z \leq -z_\alpha$.

3. Let p be the proportion of a population with a specified property. Assume the sample size to be large ($n > 40$). Large-sample level α test for

$$H_0 : p = p_0 \text{ vs. } H_1 : p < p_0$$

test statistic: $z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$.

rejection region: $z < -z_\alpha$. Example 8.11. 47 out of 102 doctors did not know the generic name for the drug methadone. What is methadone? Methadone is a rigorously well-tested medication that is safe and efficacious for the treatment of narcotic withdrawal and dependence. Can you conclude that fewer than half of all doctors know the generic name for methadone? Test it at 1% level.

```
> p<-47/102
> (p-0.5)/sqrt(0.5*0.5/102)
[1] -0.792118
> qnorm(0.01)
[1] -2.326348
```

4. Small sample size. Example 8.13.

```
> pbinom(15,20,0.9)
[1] 0.04317450
> pbinom(16,20,0.9)
[1] 0.1329533
```

Homework V. Due Nov 11. 9:30pm. Exercise 8.18, 8.24, 8.26, 8.32, 8.38, 8.40. Second midterm (Nov 18) will cover Homework III. IV. and V. materials.