

Stat 312: Lecture 15

Two-sample t test

Moo K. Chung
mchung@stat.wisc.edu

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Concepts

1. Pooled sample variance:

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}.$$

2. Let $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ be two independent samples from normal distributions with the same population variance. The test statistic for testing

$$H_0: \mu_X = \mu_Y \text{ vs. } H_1: \mu_X \neq \mu_Y$$

$$T = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{S_p \sqrt{1/n + 1/m}} \sim t_{n+m-2}.$$

Reject H_0 if $|T| > t_{\alpha/2, n+m-2}$.

In-class problems

Example 1. A study was conducted to compare the weights of cats and dogs. Weights of cats: 20, 21, 35, 13, 21, 10. Weights of dogs: 31, 10, 20, 40. Assume that the population variance to be same for both cats and dogs. Is there any difference between the weights of cats and dogs?

```
> x<-c(20,21,35,13,21,10)
> y<-c(31,10,20,40)
> sqrt((5*var(x)+3*var(y))/8)
[1] 10.52824
> t=(mean(x)-mean(y))/(10.53*sqrt(1/5+1/3))
> t
[1] -0.6827026
> qt(0.05,8)
[1] -1.859548
```

If you use R, it is very easy to do two sample hypothesis testing.

```
>t.test(x,y,alternative="two.sided",
var.equal=TRUE,conf.level=0.9)
Two Sample t-test
```

```
data:x and y t = -0.7725, df = 8,
p-value = 0.462 alternative
hypothesis: true difference in means is not equal to
confidence interval:
-17.88739 7.38739
```

```
>t.test(x,y,alternative="two.sided",
conf.level=0.9)
```

```
Welch Two Sample t-test
data:x and y t = -0.7073,df = 4.778,
p-value = 0.5124 alternative
hypothesis: true difference in means is not equal to
confidence interval:
-20.361647 9.861647
```

Example 2. A cat named Tom catches average 3 mice per day with the sample standard deviation of 1 for 10 days. A cat named Jerry catches average 4 mice per day with the sample standard deviation of 1.2 for 8 days. Which cat do you think mice would hate the most at level 0.05?

```
> s<-sqrt((9*1^2+7*1.2^2)/(10+8-2))
> (3-4)/(s*sqrt(1/10+1/8))
[1] -1.930543
> qt(0.05,16)
[1] -1.745884
```

Self-study problems

Compute a CI for $\mu_X - \mu_Y$ in Example 1. Please read p. 370. section Pooled t -Procedures. Example 9.6., 9.7. using Concept 2.

Homework 5.

Due April 1. 11:00am. Exercise 8.50., 8.54., 9.2., 9.4., 9.20., 9.22. Use Concept 2 when you do two-sided t tests.