

Stat 312: Lecture 17

Two sample t test

Moo K. Chung
mchung@stat.wisc.edu

November 9, 2004

- Let $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ be two independent samples from normal distributions with the same population variance, i.e.

$$X_i \sim N(\mu_X, \sigma^2) \text{ and } Y_j \sim N(\mu_Y, \sigma^2).$$

The pooled estimator of σ is

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}.$$

The test statistic for testing

$$H_0 : \mu_X = \mu_Y \text{ vs. } H_1 : \mu_X \neq \mu_Y$$

$$T = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{S_p \sqrt{1/n + 1/m}} \sim t_{n+m-2}.$$

Reject H_0 if $|T| > t_{\alpha/2, n+m-2}$.

Example. A study was conducted to compare the weights of cats and dogs. Weights of cats: 20, 21, 35, 13, 21, 10. Weights of dogs: 31, 10, 20, 40. Assume normality and equal variance for both cats and dogs. Is there any difference between the weights of cats and dogs?

```
> x<-c(20,21,35,13,21,10)
> y<-c(31,10,20,40)
> Sp<-sqrt((5*var(x)+3*var(y))/8)
[1] 10.52824
> t=(mean(x)-mean(y))/(Sp*sqrt(1/5+1/3))
> t
[1] -0.6827026
> qt(0.05,8)
[1] -1.859548
```

- Checking the equality of variance. This topic will be discussed later in detail.

```
> var(x)
[1] 75.2
```

```
> var(y)
[1] 170.25
> var.test(x,y)
```

F test to compare two variances

```
... p-value = 0.3975
alternative hypothesis: true ratio of variances
is not equal to 1 95 percent confidence interval
0.02967475 3.42920369
sample estimates: ratio of variances
0.4417034
```

The Welch-Satterthwaite t test is an alternative to the two sample t -test with equal variance, and is used when the assumption that the two populations have equal variances seems unreasonable. The problem with this method is that it is asymptotically a t distribution.

```
>t.test(x,y,alternative="two.sided",
conf.level=0.9)
Welch Two Sample t-test
data:x and y t = -0.7073,df = 4.778
p-value = 0.5124 alternative
hypothesis: true difference in means
is not equal to 0 90 percent
confidence interval:
-20.361647 9.861647
```

Review problems. Compute a CI for $\mu_X - \mu_Y$ in the previous Example. Example 9.7