

Stat 312: Lecture 20

Inference on Intercept

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Concepts

1. Least-squares estimator $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{x}$ is distributed as normal with mean β_0 and variance $\sigma^2 \bar{x}^2 / S_{xx}$.
2. Testing $H_0 : \beta_0 = 0$ is based on $T = \frac{\hat{\beta}_0 - \beta_0}{S_{\hat{\beta}_0}} \sim t_{n-2}$, where $S_{\hat{\beta}_0} = \sigma^2 \bar{x}^2 / S_{xx}$.
3. *Coefficient of determination* measures the proportion of variability explained by the fitted model and it is defined as

$$r^2 = 1 - \frac{SSE}{SST}$$

4. Since $SST = S_{yy}$ and $SSE = S_{yy} - S_{xy}^2 / S_{xx}$, we can see that $r^2 = S_{xy}^2 / (S_{xx} S_{yy})$. r is called the *sample correlation coefficient* and it will be studied in Lecture 21.

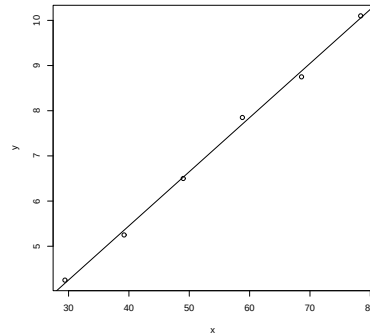


Figure 1: Hooke's law.

In-class problems

Example 1. Hooke's law states that the length change y in spring is proportional to applied force x , i.e. $y = \beta_1 x$

x (kg)	29.4	39.2	49.0	58.8	68.6	78.4
y (mm)	4.25	5.25	6.50	7.85	8.75	10.10

Fit the model $Y = \beta_0 + \beta_1 x + \epsilon$ and see if β_0 is in fact 0.

```
> ra<-cov(y,y)/cov(x,x)-
(cov(x,y)/cov(x,x))^2
> S0<-sqrt(mean(x^2)/4*ra)
> b0<-mean(y)-mean(x)*cov(x,y)/cov(x,x)
> b0/S0
[1] 4.013
> 2*pt(-4.013,4)
[1] 0.016
> x<-c(29.4,39.2,49.0,58.8,68.6,78.4)
> y<-c(4.25,5.25,6.50,7.85,8.75,10.10)
> summary(lm(y~x))
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.66 0.16 4.00 0.016
x 0.12 0.003 41.24 2.07e-06
Multiple R-Squared: 0.9977
```

Example 2. Suppose that data (x_j, y_j) satisfy a circular relationship $Y_j = \sqrt{10^2 - x_j^2} + \epsilon_j$. This is a nonlinear relationship so we expect not to have a good linear fit.

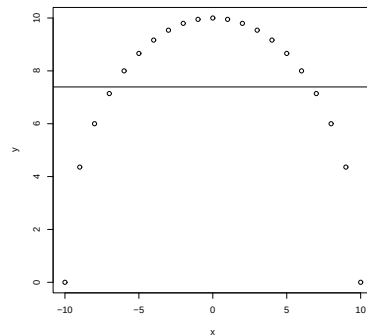


Figure 2: Complete lack of linear fit.

```
>x<- -10:10
>y<-sqrt(10^2 -x^2)
>summary(lm(y~x))
Coefficients: value Pr(>|t|)
(Intercept) .... 1.24e-09
x .... 4.47e-17
Multiple R-Squared: 1.378e-030
```

Self-study problems

Construct $100(1 - \alpha)\%$ confidence intervals for both β_0 and β_1 . Show $\text{Var}(\hat{\beta}_0) = \sigma^2 \bar{x}^2 / S_{xx}$. Example 12.9.