

Stat 312: Lecture 6

Confidence Intervals II.

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Concepts

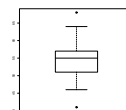
1. Let $X_i \sim N(\mu, \sigma^2)$ with known σ^2 and unknown μ . $100(1 - \alpha)\%$ confidence interval for μ is.

$$\hat{\mu}_L = \bar{x} - z_{\alpha/2} \cdot \sigma / \sqrt{n}, \quad \hat{\mu}_U = \bar{x} + z_{\alpha/2} \cdot \sigma / \sqrt{n}.$$

2. The sample size is inversely related to the width of confidence interval.
3. *Central Limit Theorem*. Let $X_1, \dots, X_n$ be a random sample with mean μ and variance σ^2 . For large n ,

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1).$$

4. If n is sufficiently large, approximate $100(1 - \alpha)\%$ confidence interval for μ is $\bar{x} \pm z_{\alpha/2} s / \sqrt{n}$, where s is the sample standard deviation.



```
[1] 5.230672
```

```
> mean(C1) - qnorm(0.975) * sd(C1) / sqrt(length(C1))
```

```
[1] 53.2286
```

```
> mean(C1) + qnorm(0.975) * sd(C1) / sqrt(length(C1))
```

```
[1] 56.18807
```

Ex. Toss $n = 100$ biased coins with $P(H) = p$. Suppose you observe 38 heads. Construct 95% CI of p .

```
> X <- rbinom(100, 1, 0.4)
```

```
> X
```

```
[1] 1 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0
```

```
[17] 0 0 1 1 0 1 0 1 1 0 1 0 1 0 1 0
```

```
[33] 1 1 0 0 0 0 1 0 1 0 0 0 0 0 1 0
```

```
[49] 0 1 0 0 0 0 1 1 0 1 0 1 1 0 0 1
```

```
[65] 0 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0
```

```
[81] 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 1
```

```
[97] 0 1 1 0
```

```
> sd(X)
```

```
[1] 0.4878317
```

```
> sqrt(0.38 * (1 - 0.38) / (100 - 1))
```

```
[1] 0.04878317
```

```
> 0.38 + 1.96 * 0.04878317 / sqrt(100)
```

```
[1] 0.389604
```

```
> 0.38 - 1.96 * 0.04878317 / sqrt(100)
```

```
[1] 0.370396
```

In-class problems

Continuing Exercise 6.25, construct 98% confidence interval.

Example 7.4. Response time $\sim N(\mu, \sigma^2)$, $\sigma = 25$. Find the sample size n that ensures 95% CI with a width of 10.

Example 7.6. Alternating current (AC) voltage data

```
> data(xmp07.06)
```

```
> attach(xmp07.06)
```

```
> str(xmp07.06)
```

```
'data.frame': 48 obs. of 1 variable:
```

```
 $ C1: int  62 50 53 57 ...
```

```
> boxplot(C1)
```

```
> mean(C1)
```

```
[1] 54.70833
```

```
> sd(C1)
```

Self-study problems

Example 7.8., Exercise 7.13., 7.19., 7.25.

In the above coin tossing example, check if $\bar{X}$ is MLE.