

Stat 324: Lecture 02

Probability

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1. An *experiment* is any action whose outcome is subject to uncertainty (randomness). The *sample space* of an experiment is the set of all possible outcomes of the experiment. An *event* is a subset of the sample space. Two events are *mutually exclusive* if they have no outcomes in common.
2. Given a sample space S , probability, denoted by P , is a function that satisfies the following three conditions: (i) $P(A) \geq 0$, (ii) $P(S) = 1$, (iii) For mutually exclusive events $A_1, A_2 \in S$ $P(A_1 \cup A_2) = P(A_1) + P(A_2)$. Then we have the following properties

(a) If $A, B \in S$ and $A \subset B$, $P(A) \leq P(B)$.

(b) $P(A) = 1 - P(A^c)$.

(c) If A and B are mutually exclusive, then

$$P(A \cap B) = 0.$$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

3. Prove

$$P(A \cap B) \geq P(A) + P(B) - 1,$$

when A, B are two events in the same sample space.

proof. We have shown in the class that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

But $P(A \cup B) \leq 1$. So by rearranging the terms, $P(A \cap B) \geq P(A) + P(B) - 1$. This inequality is called *Bonferroni inequality*.

4. Show $P(A_1 \cup A_2 \cup A_3) \leq P(A_1) + P(A_2) + P(A_3)$.

proof. $P(A_1 \cup B) = P(A_1) + P(B) - P(A_1 \cap B) \leq P(A_1) + P(B)$. Now let $B = A_2 \cup A_3$. Then similarly $P(B) \leq P(A_2) + P(A_3)$. So $P(A_1 \cup A_2 \cup A_3) \leq P(A_1) + P(B) \leq P(A_1) + P(A_2) + P(A_3)$.

5. For events A and B , find formulas for the probabilities of the following events in terms of the quantities $P(A)$, $P(B)$ and $P(A \cap B)$ only.

(i) either A or B but not both

(ii) at most one of A or B

solution. (i) $P((A \cup B) \setminus (A \cap B)) = P(A \setminus B) + P(B \setminus A) = P(A) + P(B) - 2P(A \cap B)$.

(ii) Let D be the event of *at most one of A or B* . The complement of D is the event of *both A and B* , i.e. $D^c = A \cap B$. So $P(D) = 1 - P(D^c) = 1 - P(A \cap B)$.

Assigned Problems Read Chapter 2. Exercise 2.16., 2.20. Homework I. Submit assigned problems in Lecture 1-3 on January 27 9:30AM. No late assignment will be accepted.