

Stat 324: Lecture 05

Discrete Random Variables

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February 1, 2005

1. Given a sample space S , a *random variable* X is a rule that assigns a number to each element of S . X is a function that maps element $s \in S$ to a real number. In mathematical notation, $X : S \rightarrow \mathbb{R}$.
2. *Bernoulli random variable* X takes values only 0 or 1.
3. The *probability distribution* p of a random variable X is defined for every number x such that

$$p(x) = P(X = x) = P(s \in S : X(s) = x).$$

4. X is a Bernoulli random variable with parameter p if $P(X = 1) = p$ and $P(X = 0) = 1 - p$. It will be denoted by $X \sim \text{Bernoulli}(p)$. The distribution can be written as $p(x) = (1 - p)^{1-x}p^x$.
5. *Problem.* Toss a coin until the first head is obtained. Let X be the number of tosses until the first head is obtained. Find the probability distribution of X .
Solution. $P(X = x) = (1 - p)^{x-1}p$. This is called the geometric distribution.
6. The *cumulative distribution function* (cdf) of a discrete random variable X is defined as

$$F(x) = P(X \leq x) = \sum_{y \leq x} p(y).$$

7. Find cdf of the geometric distribution.
Solution. For positive integer x , $F(x) = 1 - (1 - p)^x$.
8. The *expected value* of a discrete random variable X is defined as $\mathbb{E}X = \sum xp(x)$. The expectation of the geometric distribution is $1/p$.
9. *Problem.* Two players A and B flip a biased coin alternately and the first player to obtain a head wins. The probability of obtaining a head is $p > 0$ at each toss. If A flips first, find the probability that A wins the game.
solution. Let X be the number of tosses A makes until he gets head.

$$\begin{aligned} P(A \text{ wins}) &= P(X = 1) + P(X = 2) + P(X = 3) + \dots \\ &= p + (1 - p)^2p + (1 - p)^4p + \dots \\ &= \frac{p}{1 - (1 - p)^2} \end{aligned}$$

Assigned Problems Exercise 3.18., Exercise 3.36.