

Stat 324: Lecture 07

Binomial distributions

Moo K. Chung
mchung@stat.wisc.edu

February 8, 2005

1. Consider an experiment that give only two outcomes H or T . Let $P(H) = p$ and $P(T) = 1 - p$. This is a Bernoulli experiment. Now consider n independent experiments where each experiment gives only two outcomes. This is a *Binomial experiment*. Let X be the number of outcome H . This is called the *Binomial random variable* with parameter n and p . It will be denoted by $X \sim \text{Binomial}(n, p)$. Note that if X_i is the number of outcome H in the i -th experiment, $X = X_1 + X_2 + \cdots + X_n$.

2. *Binomial distribution*. The distribution is given by $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$.

$$\mathbb{E}X = np, \mathbb{V}X = np(1 - p).$$

```
> plot(dbinom(0:10, 10, 0.5), type="s")  
> plot(pbinom(0:10, 10, 0.5), type="s")
```

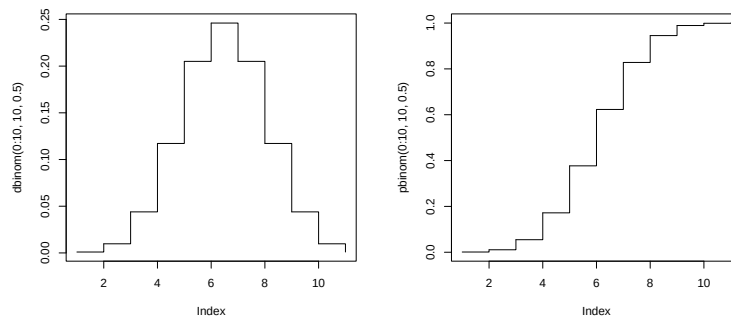


Figure 1: Left: distribution of $\text{Binomial}(10, 0.5)$, Right: cdf of $\text{Binomial}(10, 0.5)$

3. A random variable is *continuous* if its set of possible values is an entire interval of numbers. For a continuous random variable X , a probability distribution (density) of X is a function f that satisfies

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

for any a and b . The *cumulative distribution function* F of a continuous random variable X is defined as

$$F(x) \leq P(X \leq x) = \int_{-\infty}^x f(x) dx.$$

From the fundamental theorem of calculus, the density function f can be obtained as $f(x) = F'(x)$.

4. For given p , the $100p$ -th *percentile* of the distribution of X is a number $\eta(p)$ such that

$$p = \int_{-\infty}^{\eta(p)} f(x) dx.$$

Assigned Problems Exercise 3.56. Homework II due on Feb 10. 9:30AM. Submit solutions to assigned problems from Lecture 4-7 (7 problems). Do the following problem for Homework III. Exercise 4.6.