

Stat 324: Lecture 08

Continuous distributions

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1. The *cumulative distribution function* F of a continuous random variable X is defined as

$$F(x) \leq P(X \leq x) = \int_{-\infty}^x f(x) dx.$$

From the fundamental theorem of calculus, the density function f can be obtained as $f(x) = F'(x)$.

2. For given p , the $100p$ -th *percentile* of the distribution of X is a number $\eta(p)$ such that

$$p = \int_{-\infty}^{\eta(p)} f(x) dx.$$

```
> x=0:40/10
> plot(x,dunif(x,1,3),type="s")
> qunif(0.3,1,3)
[1] 1.6
```

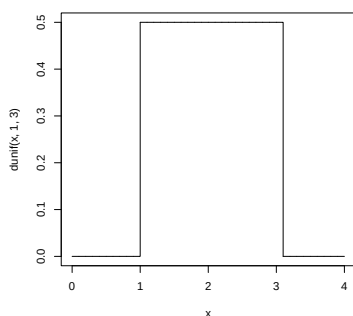


Figure 1: Density function of $Unif(1, 3)$.

3. The *expected value* of a continuous random variable X with pdf $f(x)$ is

$$\mathbb{E}X = \int_{-\infty}^{\infty} xf(x) dx.$$

$$\mathbb{E}h(X) = \int_{-\infty}^{\infty} h(x)f(x) dx.$$

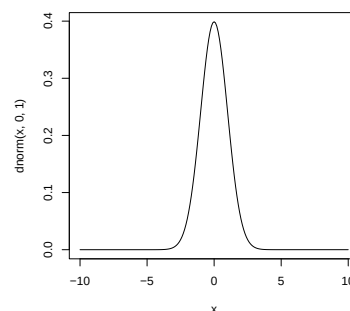


Figure 2: Density function of standard normal $N(0, 1)$

4. A continuous random variable X is a normal distribution with parameters μ and σ^2 if

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}.$$

We will denote this distribution

$$X \sim N(\mu, \sigma^2).$$

When $Z \sim N(0, 1)$, it is a standard normal distribution. The cdf of Z ,

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-x^2/2\sigma^2} dz.$$

```
>x=-100:100/10
>plot(x,dnorm(x,0,1),type="l")
```

5. The $100(1 - \alpha)$ -th percentile of $N(0, 1)$ will be denoted by z_α .

```
> alpha<-c(0,0.01,0.025,0.05,0.1)
> alpha
[1] 0.000 0.010 0.025 0.050 0.100
> qnorm(alpha,0,1)
[1]      -Inf -2.33 -1.96 -1.64 -1.28
> qnorm(1-alpha,0,1)
[1]      Inf  2.33  1.96  1.64  1.28
```

Assigned Problems Exercises 4.18., 4.32. Read section 4.3. The first midterm will cover upto Lecture 09.