

Stat 324: Lecture 17

Hypothesis testing

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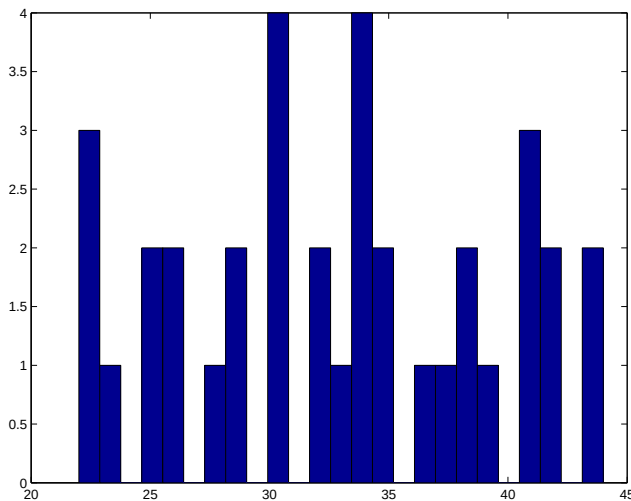


Figure 1: Histogram of the second midterm exam grade. $32.88 \pm 6.53(73 \pm 14\%)$, $n = 36$.

Concepts

1. *Motivating problem.* In 1905, a French psychologist Alfred Binet came up with the idea of testing intelligence. The average intelligence quotient (IQ) of the (human) population is by definition 100. We are interested in testing if dogs' IQ is 100. Assume IQ of a dog follows $X_i \sim N(\mu, 10^2)$. IQ of 10 dogs are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. We want to test if dogs are as smart as people by testing

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100.$$

One reasonable thing one may try is to see how high the sample mean is.

```
> x<-c(30,25,70,110,40,80,50,60,100,60)
> mean(x)
[1] 62.5
```

2. The *null hypothesis* H_0 is a claim about the value of a population parameter. The *alternate hypothesis* H_1 is a claim opposite to H_0 . A *test of hypothesis* is a method for using sample data to decide whether to reject H_0 . H_0 will be assumed to be true until the sample evidence suggest otherwise. A *test statistic* is a function of the sample data on which the decision is to be

based. A *rejection region* is the set of all values of a test statistic for which H_0 is rejected.

3. *Type I error:* you reject H_0 when H_0 is true. $P(\text{Type I error}) = P(\text{reject } H_0 | H_0 \text{ true}) = \alpha$. The resulting α is called the *significance level* of the test and the corresponding test is called a *level α test*. We will use test procedures that give α less than a specified level (0.05 or 0.01).

Since the average IQ of 10 dogs are lower than 100, one would be inclined to reject H_0 .

Let $\bar{X}$ be a test statistic and $R = (-\infty, 90]$ to be a rejection region. Let's compute the probability of making Type I error based on this testing procedure. Under the assumption H_0 is true,

$$X_i \sim N(100, 10^2).$$

Under this condition, $\bar{X} \sim N(100, 10)$ and

$$\alpha = P(\bar{X} \leq 90).$$

```
> pnorm(90,100,sqrt(10))
[1] 0.0007827011
```

By using this test procedure, it is highly unlikely to make Type I error. Let's see what happens when we change the rejection region.

When $R = (-\infty, 95]$, $\alpha = P(\bar{X} \leq 95)$.

```
> pnorm(95,100,sqrt(10))
[1] 0.05692315
```

When $R = (-\infty, 99]$, $\alpha = P(\bar{X} \leq 99)$.

```
> pnorm(99,100,sqrt(10))
[1] 0.3759148
```

The test procedure based on rejecting H_0 if $\bar{X} \leq 99$ will produce huge Type I error. Why?

Assigned problems. Exercise 8.10. (a) and (b) only, Exercise 8.26. HW 5 due April 14th. Solve the assigned problems in Lecture 15-18.