

STAT-324
Final Exam
May 11, 2005
12:25-2:25AM
Instructor: Moo K. Chung

Answer all questions showing all your work neatly.
Calculators are allowed. One page note is permitted

Name: _____
Student ID: _____

Pledge: On my honor, I have neither given nor received
unauthorized aid on this examination.

Signature: _____

Question	Points
1	
2	
3	
4	
5	
Total	

You may use the following R output for the answering exam problems.

```
> qnorm(c(0.005,0.01,0.025,0.05,0.1))
[1] -2.575829 -2.326348 -1.959964 -1.644854 -1.281552
> qt(0.025,5:11)
[1] -2.57 -2.44 -2.36 -2.30 -2.26 -2.22 -2.20
> qt(0.05,5:11)
[1] -2.01 -1.94 -1.89 -1.85 -1.83 -1.81 -1.79
> qt(0.01,5:11)
[1] -3.36 -3.14 -2.99 -2.89 -2.82 -2.76 -2.71
> qf(0.025,1,5:11)
[1] 0.001084778 0.001067109 0.001054611 0.001045308 0.001038115
0.001032389 [7] 0.001027723
> qf(0.05,1,5:11)
[1] 0.004344768 0.004273760 0.004223537 0.004186155 0.004157256
0.004134252 [7] 0.004115506
> qf(0.01,1,5:11)
[1] 0.0001735012 0.0001706780 0.0001686808 0.0001671942 0.0001660447
[6] 0.0001651297 0.0001643840
```

1. Two players A and B flip a biased coin alternately and the first player to obtain a head wins. The probability of obtaining a head is $p > 0$ at each toss. Suppose A flips the coin first.
 - (a) Find the probability that A wins the game. Derive your result [10pts].

(b) Find the expected number of tosses A must make till A wins. Derive your result [10pts].

2. (a) Suppose that X and Y are Bernoulli random variables with parameter $0 < p < 1$ such that $\text{cov}(X, Y) = 0$. Prove that X and Y are independent [10pts].

(b) Let $X_1, \dots, X_n$ be a random sample from the Bernoulli distribution with parameter p . Find an unbiased estimator for $(1 - p)^2$ [10pts].

3. 47 out of 102 doctors knew the generic name for the drug methadone. Compute a 98% confidence interval for the proportion of all doctors who **do not know** the generic name for methadone. [10 pts].

4. We wish to fit observations $(x_1, y_1), \dots, (x_n, y_n)$ with a linear model $Y = cx + \epsilon$. We assume that $\epsilon \sim N(0, \sigma^2)$.

(a) Estimate c by the least squares estimation. Determine if it is unbiased. Derive your results [10pts].

(b) Estimate c by the maximum likelihood method and compute the variance of the estimator. Derive your results [15pts].

- (c) If the sample correlation coefficient of the above paired data is denoted by r , what is the sample correlation coefficient of data $(x_1 - \bar{x}, y_1 - \bar{y}), \dots, (x_n - \bar{x}, y_n - \bar{y})$? $\bar{x}$ and $\bar{y}$ are the respective sample means of x_i 's and y_i 's. Prove your result [10pts].

- (d) Prove the sample correlation r is a number between -1 and 1 [10pts].

5. Three different types of boxes underwent compression strength test. The following data are measured compression strength (lb). Assume the normality of data.

Type I	65	79	72	68
Type II	78	77	68	77
Type III	60	62	74	

The following R output may be used to answer the following questions.

```
> type1<-c(65,79,72,68)
> type2<-c(78,77,68,77)
> type3<-c(60,62,74)
> z<-c(type1,type2,type3)
> w<-c(0,0,0,0,1,1,1,1,2,2,2)
> summary(lm(z~w))
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   73.158      3.092   23.663 2.05e-09 ***
w             -2.474      2.564   -0.965    0.36
Residual standard error: 6.738 on 9 degrees of freedom Multiple
R-Squared:  0.09376,    Adjusted R-squared:  -0.006933 F-statistic:
0.9312 on 1 and 9 DF,  p-value: 0.3598
```

- (a) Test if the true compression strength for box types I and III are equal at $\alpha = 0.1$ Assume the **equal** variance for different box types. Clearly write down the null and alternate hypotheses, the test statistic, and the statistical model you are using [10pts].

(b) Test if the true compression strength for all box types I, II and III are equal at $\alpha = 0.1$. Assume the **equal** variance for different box types. Clearly write down the null and alternate hypotheses, the test statistic, and the statistical model you are using [20pts].