

STAT-324
Midterm II
April 5, 2005
9:30-10:40AM
Instructor: Moo K. Chung

Answer all questions showing all your work **neatly**.
Calculators are allowed. One page note is permitted

Name: _____
Student ID: _____

Pledge: On my honor, I have neither given nor received
unauthorized aid on this examination.

Signature: _____

Question	Points
1	
2	
3	
4	
Total	

1. A box contains 7 white balls and 3 black balls. A ball is chosen at random and replaced by the opposite color. Now again draw a ball.

(a) What is the probability that the second ball drawn is white? [5pts]

(b) What is the probability that the first ball drawn is black when the second ball drawn is white? [5pts]

2. The joint distribution of X and Y is given by

$$f(x, y) = \begin{cases} 2 & \text{for } y < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute the probability $P(\frac{1}{2} < X < 1, \frac{1}{2} < Y < 1)$. [5pts]

(b) Determine the marginal distributions of X and Y . [5pts]

(c) Are X and Y are independent? Prove your statement. [5pts]

3. A random sample $X_1, X_2, \dots, X_n$ comes from the distribution

$$f(x) = \lambda x^{\lambda-1}, 0 < x < 1.$$

(a) Write down the likelihood function for λ . [5pts].

(b) Derive the maximum likelihood estimator (MLE) of θ . [5pts].

4. The intelligence quotient (IQ) of 10 dogs are measured:

25, 21, 22, 17, 29, 25, 16, 20, 19, 22.

The measurements are assumed to follow a normal distribution with known variance 9.
The following R output is available.

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> qnorm(c(0.025, 0.05, 0.1))  
[1] -1.96 -1.64 -1.28
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(a) Construct a 95% confidence interval of the mean IQ. [5pts]

(b) Determine the minimum sample size needed for the width of a 95% confidence interval to be less than 2. [5pts]