Nonparametric Statistics Homework I.

Due date: Two weeks from today. Worth 5% of your final grade. Please write clearly. Computer printouts should be included but do not dump everything.

1. We have 10 numbers $X_1 = 0.1, X_2 = 0.2, \cdots, X_9 = 0.9, X_{10} = 1.0$. Test if 10 numbers come from a standard normal distribution with mean 0.5 at 5% level.

2. You have a random binary sequence 0110110110110110 (16 numbers), which comes from a Bernoulli distribution. Test

$$H_0 : \mu = 0.4 \text{ vs. } H_1 : \mu > 0.4,$$

where μ is the population mean. Justify your answer by computing P-value.

3. Let $X \sim$ Unif$(0, 1)$. Let Y be a random variable whose probability density function is $f(x) = 2x$ for $0 < x < 1$. Generate 10,000 Y's from X. What formula did you use? Plot the histogram with 100 bins (do not print out 10,000 numbers).

4. Let $X_i \sim N(0, 1)$. Plot the probability density functions of $X_{(1)}$ and $X_{(10)}$ by computing the empirical distributions of $X_{(1)}$ and $X_{(10)}$ (Follow the example of Lecture 4).

5. From Problem 4, numerically estimate $\mathbb{E}X_{(1)}$ and $\mathbf{Var}X_{(1)}$.