

Stat 471: Homework 01

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Due Date: September 22, 1:20pm. No late assignments will be accepted or graded. There will be 10% bonus points for exceptional solutions.

1. (Computational) If $X \sim N(0, 1)$, the c.d.f. of X is given by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{z^2}{2}\right) dz.$$

It can be shown that

$$\Phi^{-1}(x) \approx t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2}$$

for constants $a_0 = 2.30753, a_1 = 0.27061, b_1 = 0.99229, b_2 = 0.04481$ ($t^2 = -2 \log x$) (Abramowitz and Stegun, Handbook of mathematical functions, 1964). Write a function that generate the standard normal random numbers using the integral transform method. Is your random number generator sufficiently good enough?

2. (Computational) Cauchy distribution is defined as $g(x) = \frac{1}{1+x^2}, x \in R$. Write a function that generate Cauchy random numbers. You may need to refer to a calculus textbook for the integral.
3. (Computational) Using the Cauchy random number generator, write a function that produce the standard normal random numbers using the accept-reject method. Compare the performance of the two standard normal random number generators using the Q-Q plot and other methods.
4. (Theoretical) When you generate the Q-Q plot of $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ for some arbitrary μ_i and σ_i , you will always get a straight line. Prove this fact by determining the equation of the line.