

Stat 471: Homework 05

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November 12, 2003

Due Date: November 24, 1:20pm. No late assignments will be accepted or graded. There will be 10% bonus points for exceptional solutions.

1. Simulate bivariate normal

$$\mathbf{Z} = (X, Y)' \sim N(0, \begin{bmatrix} \sigma^2 & \rho \\ \rho & \sigma^2 \end{bmatrix})$$

when $(\rho, \sigma) = (0.9, 0.5)$ using the Gibbs sampler. Plot your Markov chains.

2. $(3, 2, 4, 2, 1)$ are observations for model

$$Y_i = \beta + \epsilon_i.$$

$\epsilon_i \sim N(0, 1)$ and prior $\beta|\gamma \sim N(\gamma, 1)$, $\gamma \sim N(2, 1)$. Estimate β based on the Gibbs sampling method.

3. From `strength.data`, fit the model

$$\text{grip}_i = \beta_0 + \beta_1 \text{arm}_i + \epsilon_i,$$

using the Gibbs sampler. $\epsilon_i \sim N(0, 3^2)$ and the priors are $\beta_0 \sim N(110, 3^2)$ and $\beta_1 \sim N(1, 1)$. Compare your result with the least squares estimation method we studied.

4. Download data <http://www.stat.wisc.edu/mchung/teaching/data/bainslice2> and run the following codes.

```
load brainslice2; intensity=reshape(brainslice2,256*124,1);
intensity=intensity(find((intensity >=50)&(intensity<=250)));
imagesc(brainslice2);colormap('bone');colormap;
hist(intensity,100);
```

You should get 19,660 values for `intensity` which are gray scale values between 50 and 250 of the magnetic resonance image (MRI) of the human brain. Let $y_1, \dots, y_{19660}$ be the observations. From the histogram below, we see that the image intensity can be modelled as the mixture of normals such that the probability density is given by

$$f(y) = pf_1(y) + (1 - p)f_2(y) : \text{two component Gaussian mixture model}$$

where $f_1 \sim N(\mu_1, \sigma_1^2)$ and $f_2 \sim N(\mu_2, \sigma_2^2)$. Assume $\mu_1 = 170, \sigma_1^2 = 20^2, \mu_2 = 210, \sigma_2^2 = 10^2$. Estimate parameter $0 < p < 1$ by maximizing the likelihood function using the EM algorithm. Use both deterministic and Monte-Carlo versions. What this parameter p measure?

