

Stat 471: Lecture 2

Integral Transform Method

Moo K. Chung
mchung@stat.wisc.edu

September 5, 2003

1. Uniform random numbers $U \sim Unif(0, 1)$. Using U , generate 1000 uniform random numbers $X \sim Unif(-1, 5)$.

```
>> u = rand(1000,1);
>> size(u)
ans =
      1000      1
>> x = 4*u-1;
>> rand
ans =
      0.4048
>> rand
ans =
      0.6271
>> s=rand('state')
s =
      0.8598
      0.1398
      .....
>> rand('state',s)
>> rand
ans =
      0.4048
```

2. Theorem (Integral transform method). For an increasing function g with inverse function g^{-1} , if $U \sim Unif(0, 1)$, $g^{-1}(U) \sim g$. *Proof.* See lecture note.
3. Generate 1000 random numbers whose probability density function is $f(x) = x$ for $(0, \sqrt{2})$ and 0 otherwise.

Solution. We need to generate a random variable whose cumulative distribution function (c.d.f.) is $F(x) = x^2/2$. The inverse is given by $F^{-1}(x) = \sqrt{2x}$.

```
>>sqrt(2*rand(1000,1))
```

4. Theoretically you can generate any random variables from the uniform distribution but the disadvantage of the integral transform method is that it is practical only if the c.d.f. are explicitly available. For instance the c.d.f. of a normal r.v. can not be expressed explicitly. If $X \sim N(0, 1)$, the c.d.f. of X is given by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{z^2}{2}\right) dz.$$

It can be shown that

$$\Phi^{-1}(x) \approx t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2}$$

for some constants a_i and b_i ($t^2 = -2 \log x$).

5. Generate 1000 random numbers from exponential distribution with parameter λ . *Solution.* $f(x) = e^{-\lambda x}/\lambda$. $F(x) = 1 - e^{-\lambda x}$. $F^{-1}(x) = -\frac{1}{\lambda} \ln(1 - x)$. Then $-\frac{1}{\lambda} \ln(1 - U) \sim exp(\lambda)$. Since $1 - U \sim Unif(0, 1)$ as well, $-\frac{1}{\lambda} \ln U \sim exp(\lambda)$. In the case $\lambda = 2$,

```
>> U=rand(1000,1);
>> X=-log(U)/2;
```

6. Read Chapter 4 and 5.