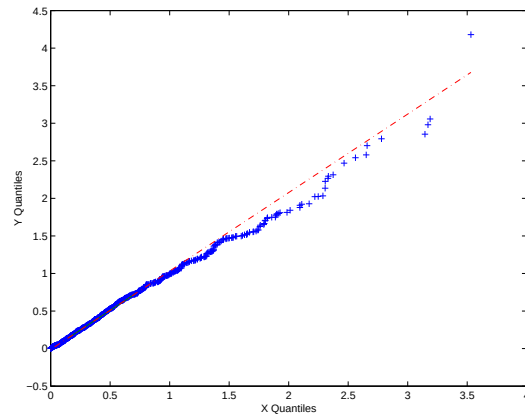
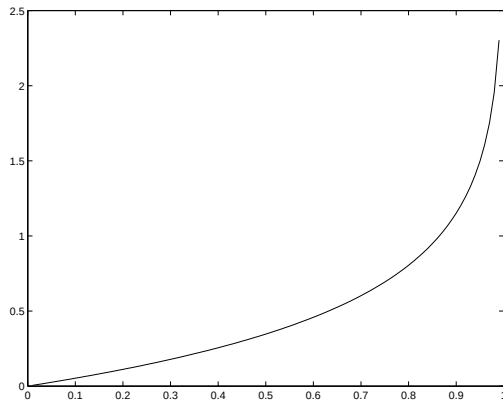


Stat 471: Lecture 4

Quantile-Quantile Plot

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We studied a couple of method for generating random numbers. For instance we studied two ways for generating exponential random numbers. *But how do you know two random number generators are equivalent.* One simple way is a graphical display called the quantile-quantile plot (Q-Q plot) (Wilk and Gnanadesikan, 1968).

1. Quantile point q_p for random variable X is the point such that

$$F_X(q_p) = P(X \leq q_p) = p.$$

So the quantile is a function of probability or proportion, i.e.

$$q_p = F_X^{-1}(p).$$

2. Plot the quantile function for the exponential random variable X with parameter $\lambda = 2$.

Solution. From lecture 02-5, $F^{-1}(p) = -\frac{1}{2} \ln(1-p)$.

```
p=[1:99]/100;
q=inline('log(1-p)/2');
plot(p,q(p));
```

3. If q_p^X and q_p^Y are quantile functions of random variables X and Y , Q-Q plot of X and Y is the plot of (q_p^X, q_p^Y) for all p .

4. Generate the Q-Q plot of $X \sim \exp(2)$ and $Y \sim \exp(2)$. Since they are identical distributions, you would expect line $y = x$ as the Q-Q plot.

```
U=rand(1000,1); X=-log(U)/2;
V=rand(1000,1); Y=-log(V)/2;
qqplot(U,V)
```

5. If q_p^X and q_p^Y are quantile functions of random variables X and Y , Q-Q plot of X and Y is the plot of (q_p^X, q_p^Y) for all p .
6. Check if the integral transform and the accept-reject method produce identical distribution for $\exp(2)$ random variable.

```
U=rand(1000,1); X=-log(U)/2;
f=inline('exp(-2*x)/2');
i=1; while i <= 1000
    U = rand; V = rand;
    if (U < 2*f(V)) Y(i)=V; i=i+1;
end;
qqplot(X,Y)
```

Warning: For the accept-reject method, the distributions f and g should be somewhat similar to have a sufficiently good algorithm. The range of f should be covered by g .

