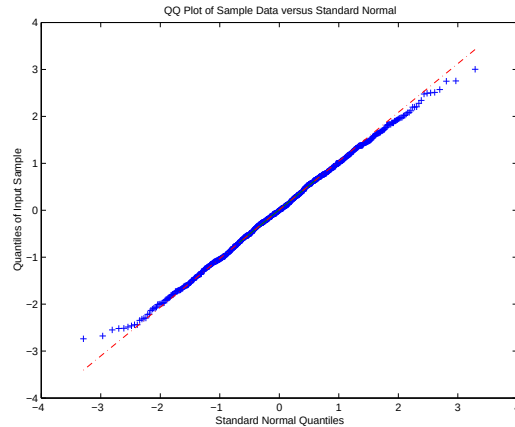
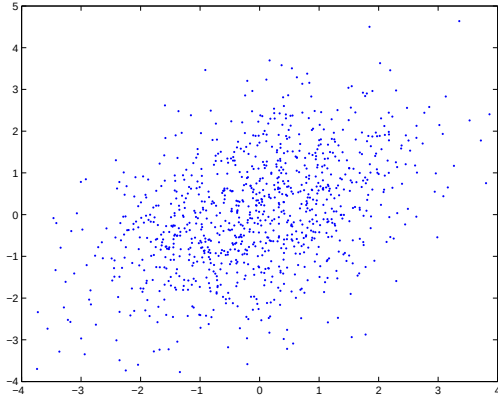


Stat 471: Lecture 7

Diagonalization.

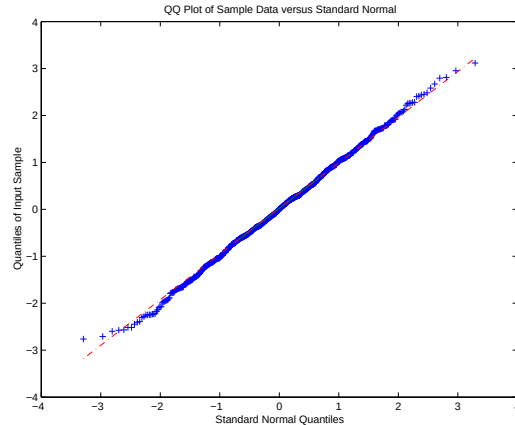
Moo K. Chung
mchung@stat.wisc.edu

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1. Go to www.infinityassociates.com to download MATLAB examples in the textbook.
2. Other factorization technique for covariance matrix V . Suppose V has positive eigenvalues λ_i and corresponding normalized eigenvectors e_i . Let $Q = (e_1, e_2, e_3)$. Then $QQ' = I$ and $V = Q \text{Diag}(\lambda_1, \lambda_2, \lambda_3) Q'$. So $V^{1/2} = Q \text{Diag}(\lambda_1^{1/2}, \lambda_2^{1/2}, \lambda_3^{1/2}) Q'$. For $z \sim N(0, I)$, $w = V^{1/2}z \sim N(0, V)$.

```
>>V=[2 1
      1 2]
>> [Q D]=eig(V)
Q = -0.7071    0.7071
      0.7071    0.7071
D = 1      0
      0      3
>> Q*D.^(1/2)*Q'
ans = 1.3660    0.3660
      0.3660    1.3660
>> V^(1/2)
ans = 1.3660    0.3660
      0.3660    1.3660
>>w=V^(1/2)*[z1 z2]';
```



```
>>hatV=cov(w(1,:),w(2,:));
>>z=hatV^(-1/2)*(w-mean(w,2));
>>qqplot(z(1,:));qqplot(z(2,:))
```

3. Review of lecture 6. For given bivariate data $w_i = (x_i, y_i)$, we computed the sample covariance matrix $\hat{V}$. Then we estimated the Cholesky factor of $\hat{V}$. Based on this, $w = V^{1/2}z + \mu$ becomes $z = \hat{V}^{-1/2}(w - \mu) \sim N(0, I)$. So we can test if w_i are from bivariate normal by checking if $z_i \sim N(0, 1)$.

4. Consider linear equation

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

There is no exact solution but it is possible to get a solution in the least-squares sense. Note that

$$2y_1 + y_2 - 1 = \epsilon_1, 4y_1 + y_2 - 1 = \epsilon_2.$$

Minimize the sum of squared errors, $\min(X\beta - c)'(X\beta - c)$, where $c = (1, 1)'$, $\beta = (y_1, y_2)'$ and $X = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$. $\frac{\partial}{\partial \beta} \beta' A = A$, $\frac{\partial}{\partial \beta} \beta' A \beta = 2A\beta$. Then it can be shown that $\hat{\beta} = (X'X)^{-1}X'c$.